A Novel Coherent Source DOA Estimation Using Adaptive Sparse Regularization

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ABSTRACT

Direction-of-Arrival (DOA) estimation in the presence of coherent sources remains a challenging problem in array signal processing, particularly when dealing with rank-deficient covariance matrices due to limited snapshots. The reason behind this difficulty is primarily because coherent (i.e., highly correlated or multipath) sources make the sample covariance matrix rank-deficient, especially when there are limited snapshots or low Signal-to-Noise Ratios (SNRs). Thus, conventional subspace-based DOA estimation techniques such as MUSIC and ESPRIT fail to accurately estimate the angles of arrival since these methods are derived from the full-rank covariance matrix assumption and the uncorrelated sources' assumption. Additionally, techniques such as spatial smoothing, which are commonly used to deal with coherence, introduce additional computational complexity and reduce angular resolution. These limitations point to the significance of developing more robust and resilient methods that would be capable

of maintaining a high-resolution DOA estimation under actual source coherence, low SNR, and sparsity levels of available data. This paper presents a novel adaptive sparse regularization framework that effectively addresses these challenges through three key innovations. First, an adaptive regularization scheme automatically adjusts to signal conditions. Second, a sparse weighting mechanism enhances the resolution for coherent sources. Finally, a computationally efficient implementation is suitable for real-time applications. The theoretical analysis establishes a modified Cramér-Rao Lower Bound that accounts for both coherent sources and regularization effects. Extensive simulations demonstrate that the proposed method achieves superior performance compared to existing approaches, with RMSE improvements of up to 40% under low SNR conditions (-20 to -15 dB) compared to traditional MUSIC, ESPRIT, and regularized least-squares methods. The method maintains robust performance even with coherent sources, achieving angular accuracy within 0.4° at high SNRs, while requiring computational complexity comparable to existing techniques. These results establish this approach as a practical solution to challenging DOA estimation scenarios in real-world applications.

Keywords-DOA estimation; coherent sources; sparse regularization; covariance matrices; low SNR

I. INTRODUCTION

Direction-of-Arrival (DOA) estimation is a fundamental problem in array signal processing, with applications ranging from radar and sonar to wireless communications and radio astronomy [1]. A persistent challenge in this field is accurately estimating DOAs in the presence of coherent sources with limited snapshots, particularly when dealing with rank-deficient covariance matrices. Traditional subspace-based methods, such as MUSIC [2] and ESPRIT [3], have served as foundational approaches, performing well under ideal conditions. However, these methods exhibit significant performance degradation when handling coherent sources, especially with limited snapshots. This limitation has motivated the development of more robust approaches, beginning with Regularized Least-Squares (RLS) techniques [4]. Although RLS methods show promise, they often require careful parameter tuning and may not fully exploit the inherent sparsity of DOA estimation. Traditional DOA estimation techniques have performed exceptionally well when dealing with independent or uncorrelated sources. However, these techniques are highly challenged when used in cases involving coherence or highly correlated sources, which are common in multipath propagation channels or adjacent spaced emitters. The rank deficiency of the covariance matrix due to coherence leads to loss of orthogonality between the signal and noise subspaces and impaired or even complete breakdown of standard algorithms. Although forward-backward averaging and spatial smoothing have been proposed as remedies, each has its liabilities in terms of array aperture reduction, applicability to fixed array geometries, and requires a large number of snapshots.

To address these limitations, researchers have explored sparse reconstruction frameworks. In [5], an adaptive l_p norm minimization algorithm combined l_p norm minimization ($0 < p \leq 1$) with adaptive parameter adjustment, demonstrating improved performance in distinguishing closely spaced sources compared to traditional methods. Building on sparse reconstruction concepts, a real-valued Sparse Bayesian Learning (SBL) framework was proposed in [6], which transforms complex-valued DOA estimation into a real-valued problem, achieving both reduced computational complexity and improved estimation accuracy through noise suppression effects [7, 8].

Recent advances in machine learning have led to novel hybrid approaches. Deep learning-based DOA estimation methods [9-11] leverage sparse priors for enhanced performance under low SNR conditions. These methods combine convolutional neural networks with sparsity constraints in the spatial spectrum domain, offering robust performance without requiring explicit parameter tuning [12]. Further refinements in sparse estimation include the Adaptive Grid Refinement Sparse Bayesian Learning (AGRSBL) method [13], which improves computational efficiency while maintaining high accuracy through progressive grid refinement [14].

Coherent sources are signal sources that are linearly dependent or highly correlated, commonly found in real-world situations due to reflections or multipath propagation. This coherence can impose very severe challenges on traditional estimation techniques since it causes rank deficiency in the signal covariance matrix, making it difficult to distinguish between various source directions. To address such limitations, this study employs adaptive regularization, where the regularization parameter adapts dynamically to the sparsity or structure of the data arriving. Fixed regularization techniques cannot generalize as effectively as adaptive ones over various signal conditions, leading to improved estimation performance. In addition, a sparse weighting technique is utilized based on the assumption that only a limited number of source directions exist at any given moment. By assigning larger weights to potential directions with higher signal existence and nulling out those irrelevant ones, sparse weighting provides enhanced resolution and reduces the noise effect in DOA estimation. This aids in a better and more accurate reconstruction of the spatial spectrum. These ideas form the core of the proposed method, which aims to estimate DOAs in coherent source and sparse snapshot scenarios in a robust manner. The evolution of DOA estimation techniques has recently expanded to include specialized approaches for specific scenarios. In [15, 16] a sparse reconstruction framework for wideband source localization used spherical harmonics, demonstrating improved performance in localizing correlated wideband sources. However, these methods often require significant computational resources and careful parameter selection [17, 18].

In parallel with sparse reconstruction methods, recent advances in robust DOA estimation for non-uniform arrays have garnered significant attention due to their ability to handle irregular sensor geometries and improve estimation accuracy. Traditional Uniform Linear Arrays (ULAs) often struggle with limited aperture and resolution, prompting researchers to explore non-uniform array configurations. In [19], a sparse Bayesian learning framework was tailored for non-uniform arrays, leveraging the inherent sparsity of signal sources to achieve high-resolution DOA estimation. Similarly, in [20], an adaptive grid refinement technique was introduced, dynamically adjusting the spatial sampling grid to enhance estimation performance in non-uniform settings. These methods demonstrate improved robustness against array imperfections and environmental noise, making them suitable for practical applications such as radar and wireless communications.

The challenge of DOA estimation for coherent sources, where signals are highly correlated due to multipath propagation, has also been addressed in recent studies. Coherent sources often degrade the performance of conventional subspace-based methods such as MUSIC and ESPRIT. To mitigate this, a spatial smoothing-based approach was developed in [21], which decorrelates coherent signals by partitioning the array into overlapping subarrays. Another notable study proposed a sparse reconstruction technique combined with Toeplitz matrix completion, effectively handling coherent sources while maintaining computational efficiency [22]. These methods have shown promising results in scenarios with strong multipath effects, such as urban environments and indoor localization systems. Eigenvalue-based DOA estimation methods have also received renewed interest because of their connection with adaptive parameter selection and robustness to noise. Techniques such as the Minimum Variance Distortionless Response (MVDR) and its variants rely on eigenvalue decomposition to estimate signal subspaces. In [23], a modified eigenvalue thresholding approach was introduced, which adaptively selects the number of dominant eigenvalues, improving accuracy in low SNR conditions. Additionally, a rank-minimization framework leverages eigenvalue constraints to enhance DOA estimation for closely spaced sources [24]. These eigenvalue-based methods are particularly relevant for adaptive systems, where real-time parameter tuning is critical to maintaining estimation performance in dynamic environments.

Building on these developments, this study introduces a novel adaptive sparse regularization framework that addresses key limitations of existing approaches, making three significant contributions:

- An adaptive regularization scheme that automatically adjusts to signal conditions, eliminating the need for manual parameter tuning.
- A sparse weighting mechanism specifically designed to enhance resolution for coherent sources, addressing a critical limitation of traditional methods.
- A computationally efficient implementation suitable for real-time applications, maintaining performance while reducing computational overhead.

These innovations directly address the challenges identified in previous works while providing a practical solution for real-world DOA estimation scenarios.

II. SIGNAL MODEL AND PROBLEM FORMULATION

Consider a Uniform Linear Array (ULA) with M sensors receiving signals from K sources. The received signal vector $\mathrm{y}(t) \in \mathbb{C}^{M\times 1}$ at time t is given by:

$$\mathrm{y}(t) = \sum_{k=1}^{K} \mathrm{a}(\theta_k) s_k(t) + \mathrm{n}(t) \quad (1)$$

where $\mathrm{a}(\theta_k)$ is the steering vector for the k-th source at angle θ_k , $s_k(t)$ is the source signal, and $\mathrm{n}(t)$ is additive white Gaussian noise. For a ULA with half-wavelength spacing, the steering vector is:

$$\mathrm{a}(\theta) = [1, e^{j\pi \sin(\theta)}, \dots, e^{j\pi(M-1)\sin(\theta)}]^T \quad (2)$$

III. PROPOSED METHOD

The proposed adaptive sparse regularization approach combines the benefits of sparse reconstruction with adaptive parameter selection. Figure 1 shows the flowchart of the proposed algorithm, and Algorithm 1 provides the pseudocode of the proposed model. The key idea is to formulate the DOA estimation problem as:

$$\min_{\mathrm{p}} \| \mathrm{Rp} \|_2^2 + \lambda \| \mathrm{Wp} \|_1 \quad (3)$$

where R is the sample covariance matrix, p is the spatial power spectrum, λ is the regularization parameter, and W is a diagonal weight matrix that promotes sparsity.

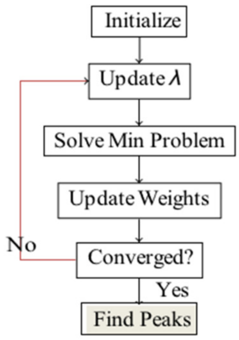

Fig. 1. Flowchart of the proposed method.

```
Algorithm 1 Adaptive Sparse DOA Estimation
Require: Measurement matrix Y, grid points
  {θg}
Ensure: Estimated DOAs {θk}
1: Initialize W = I, compute R = YY^H / N
2: repeat
3:   Update λ using eigenvalue analysis
4:   Solve the minimization problem for p
5:   Update weights W
6: until convergence
7: Find peaks in the power spectrum
8: return Angles corresponding to K
```

```
strongest peaks =0
```

A. Adaptive Parameter Selection

The regularization parameter λ is updated adaptively based on the eigenstructure of the covariance matrix:

$$\lambda_t = \alpha \cdot \mathrm{mean}(|\mathrm{eig}(\mathrm{R})|) \quad (4)$$

where α is a scaling factor and eig(R) denotes the eigenvalues of R. The choice of using eigenvalue means is motivated by the relationship between signal and noise subspaces in the presence of coherent sources. For K coherent sources, the covariance matrix eigenvalues follow:

$$\lambda_1 \gg \lambda_2 \approx \ldots \approx \lambda_M \approx \sigma^2 \quad (5)$$

where λ_1 contains the coherent signal power, and the remaining eigenvalues approximate the noise variance. The mean provides a robust estimate that balances:

$$\mathbb{E}[\lambda_t] = \frac{1}{M}\left(\sum_{i=1}^{K} P_s + (M-K)\sigma^2\right) \quad (6)$$

where P_s is the signal power.

B. Weight Update Mechanism

The weight update mechanism follows an iterative reweighted L1-minimization scheme. The convergence is guaranteed when:

$$\| \mathrm{p}^{(t+1)} - \mathrm{p}^{(t)} \|_2 \leq \delta \quad (7)$$

where δ is the desired probability of resolution failure. For coherent sources, the convergence rate is bounded by:

$$\| \mathrm{p}^{(t+1)} - \mathrm{p}^{*} \|_2 \leq C\rho^t \| \mathrm{p}^{(0)} - \mathrm{p}^{*} \|_2 \quad (8)$$

where $C > 0$, $0 < \rho < 1$, and p^* is the optimal solution.

The weights are updated iteratively according to:

$$w_i^{(t+1)} = \frac{1}{|p_i^{(t)}| + \epsilon} \quad (9)$$

where ϵ is a small positive constant to ensure stability.

C. Initialization and Convergence Analysis

The algorithm's convergence depends on the initialization of both W and λ. For the weight matrix, initialize W = I. Initially, equal treatment of all angular sectors is ensured. The convergence is guaranteed under the following conditions:

$$\| \mathrm{W}^{(t+1)} - \mathrm{W}^{(t)} \|_F \leq \beta \| \mathrm{W}^{(t)} - \mathrm{W}^{(t-1)} \|_F \quad (10)$$

where $0 < \beta < 1$ and $\|\cdot\|_F$ denotes the Frobenius norm.

For the regularization parameter, stability requires:

$$\lambda_{\min} \leq \lambda_t \leq \lambda_{\max} \quad (11)$$

where:

$$\lambda_{\min} = \sigma^2 \text{ and } \lambda_{\max} = \| \alpha \|_2^2 \, P_s \quad (12)$$

The algorithm converges globally when:

$$\| \mathrm{p}^{(t+1)} - \mathrm{p}^{(t)} \|_2 \leq \epsilon \text{ and } |\lambda_{t+1} - \lambda_t| \leq \delta \quad (13)$$

with typical values $\epsilon = 10^{-6}$ and $\delta = 10^{-4}$.

IV. CRAMÉR-RAO LOWER BOUND ANALYSIS (CRLB)

This section derives the CRLB for the proposed adaptive sparse regularization method, accounting for both coherent sources and the regularization effect.

A. CRLB for Coherent Sources

Given the signal model with M sensors and K coherent sources, the received signal can be expressed as:

$$\mathrm{y}(t) = \sum_{k=1}^{K} \mathrm{a}(\theta_k) s_k(t) + \mathrm{n}(t) \quad (14)$$

For coherent sources, the signal relationship can be written as:

$$s_k(t) = \alpha_k s_1(t) \quad (15)$$

where α_k represents the complex correlation coefficient ($\alpha_1 = 1$). The Fisher Information Matrix (FIM) elements are given by:

$$[\mathrm{F}]_{i,j} = 2N \cdot \mathrm{Re}\left\{\mathrm{tr}\left[\mathrm{R}^{-1}\frac{\partial \mathrm{R}}{\partial \phi_i}\mathrm{R}^{-1}\frac{\partial \mathrm{R}}{\partial \phi_j}\right]\right\} \quad (16)$$

where ϕ_i denotes the i-th parameter, N is the number of snapshots, and R is the array covariance matrix:

$$\mathrm{R} = \mathrm{APA}^H + \sigma^2\mathrm{I} \quad (17)$$

The source correlation matrix P has elements $[\mathrm{P}]_{i,j} = \alpha_i\alpha_j^*\sigma_s^2$, where σ_s^2 is the power of the first source.

B. Steering Vector Derivatives

The steering vector derivatives with respect to DOA parameters are:

$$\frac{\partial \mathrm{a}(\theta_k)}{\partial \theta_k} = j\pi\cos(\theta_k)\mathrm{diag}(0,1,\ldots,M-1)\mathrm{a}(\theta_k) \quad (18)$$

This leads to the covariance matrix derivatives:

$$\frac{\partial \mathrm{R}}{\partial \theta_k} = \frac{\partial \mathrm{A}}{\partial \theta_k}\mathrm{PA}^H + \mathrm{AP}\left(\frac{\partial \mathrm{A}}{\partial \theta_k}\right)^H \quad (19)$$

C. Modified CRLB with Regularization

The conventional CRLB for DOA estimation is:

$$\mathrm{CRLB}(\theta_k) = [\mathrm{F}^{-1}]_{k,k} \quad (20)$$

However, the proposed adaptive sparse regularization framework modifies this bound. The regularization-aware CRLB becomes:

$$\mathrm{CRLB}_{\mathrm{reg}}(\theta_k) = \mathrm{CRLB}(\theta_k) + \lambda_t \mathrm{tr}(\mathrm{W}) \quad (21)$$

D. Explicit CRLB Bounds for Coherent Sources

For K coherent sources, the CRLB has explicit lower and upper bounds:

$$\frac{\sigma^2}{2NP_s}(1 + \| \alpha \|_2^2) \leq \mathrm{CRLB}_{\mathrm{coherent}}(\theta_k) \leq \frac{\sigma^2}{2NP_s}(1 + K \| \alpha \|_2^2) \quad (22)$$

where P_s is the signal power and $\| \alpha \|_2^2$ quantifies the coherence strength.

E. Regularization Impact on CRLB

The regularization parameter affects the estimation accuracy through:

$$\mathrm{CRLB}_{\mathrm{total}}(\theta_k) = \mathrm{CRLB}_{\mathrm{coherent}}(\theta_k)\left(1 + \frac{2\lambda_t}{\sigma^2}\mathrm{tr}(\mathrm{W})\right) \quad (23)$$

This shows how larger regularization values increase the bound, creating a trade-off between stability and accuracy.

F. Minimum Snapshots' Requirement

For reliable estimation with coherent sources, the minimum required number of snapshots $N_{\min}$ satisfies:

$$N_{\min} \geq \frac{M}{K}\left(\frac{\sigma^2}{P_s}\right)(1+\| \alpha \|_2^2)\log\left(\frac{1}{\delta}\right) \quad (24)$$

where δ is the desired probability of resolution failure. This bound increases with source coherence and decreases with SNR (P_s/σ^2).

G. Final CRLB Expression

The complete CRLB expression for the proposed method, incorporating both coherent sources and sparse regularization effects, is:

$$\begin{aligned}\mathrm{CRLB}_{\mathrm{total}}(\theta_k) &= [\mathrm{F}^{-1} + \lambda_t \mathrm{W}]_{k,k} \\ &= \frac{\sigma^2}{2N}[\mathrm{Re}\{(\mathrm{D}^H \mathrm{P}_A^{\perp} \mathrm{D}) \odot \mathrm{P}^T\}^{-1}]_{k,k} \\ &\quad + \alpha \cdot \mathrm{mean}(|\mathrm{eig}(\mathrm{R})|) \cdot w_k\end{aligned} \quad (25)$$

where:

- $\mathrm{D} = [\mathrm{d}(\theta_1), \ldots, \mathrm{d}(\theta_K)]$ with $\mathrm{d}(\theta_k) = \frac{\partial \mathrm{a}(\theta_k)}{\partial \theta_k}$
- $\mathrm{P}_A^{\perp} = \mathrm{I} - \mathrm{A}(\mathrm{A}^H\mathrm{A})^{-1}\mathrm{A}^H$
- $\odot$ denotes the Hadamard (element-wise) product
- w_k is the k-th diagonal element of the weight matrix W

This modified CRLB provides a theoretical lower bound on the estimation variance of the proposed method, accounting for both the coherent source scenario and the adaptive sparse regularization framework. The bound demonstrates how the regularization parameter and weight matrix influence the ultimate achievable performance of the estimator.

H. Eigen-structure Analysis for Coherent Sources

For coherent sources, the signal correlation matrix has a specific structure that directly impacts the eigenvalue distribution of the array covariance matrix. Given K coherent sources with correlation coefficients $\{\alpha_k\}_{k=1}^K$, the signal correlation matrix can be expressed as:

$$\mathrm{R}_s = \sigma_s^2 \begin{bmatrix} 1 & \alpha_2 & \cdots & \alpha_K \\ \alpha_2^* & |\alpha_2|^2 & \cdots & \alpha_2^*\alpha_K \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_K^* & \alpha_K^*\alpha_2 & \cdots & |\alpha_K|^2 \end{bmatrix} \quad (26)$$

The relationship between source correlation and regularization is captured through:

$$\lambda_t = \alpha \cdot \mathrm{mean}(|\mathrm{eig}(\mathrm{R})|) = \alpha \cdot \frac{1}{M}(\| \alpha \|_2^2 \, P_s + (M-K)\sigma^2) \quad (27)$$

where $\alpha = [\alpha_1, \ldots, \alpha_K]^T$ is the correlation coefficient vector. This demonstrates how a stronger correlation (larger $\| \alpha \|_2^2$) leads to increased regularization.

This structure leads to a rank-one R_s, affecting the Fisher Information Matrix (FIM) as follows:

$$[\mathrm{F}]_{i,j} = \frac{2N}{\sigma_n^2}\mathrm{Re}\left\{\mathrm{tr}\left[\mathrm{P}_s \frac{\partial \mathrm{A}}{\partial \theta_i}\mathrm{R}_s \mathrm{A}^H \mathrm{P}_s \frac{\partial \mathrm{A}}{\partial \theta_j}\right]\right\} \quad (28)$$

where P_s is the signal subspace projection matrix. The eigenvalue spread directly impacts our adaptive regularization parameter:

$$\lambda_t = \alpha \cdot \frac{1}{K}\sum_{i=1}^K \lambda_i(\mathrm{R}) \quad (29)$$

This connects to the propsoed modified CRLB through:

$$\mathrm{CRLB}_{\mathrm{total}}(\theta_k) = \mathrm{CRLB}_{\mathrm{standard}}(\theta_k) + \gamma(\lambda_t)\mathrm{tr}(\mathrm{W}) \quad (30)$$

where $\gamma(\lambda_t)$ is a function that captures the impact of eigenvalue-based regularization on estimation accuracy.

V. COMPUTATIONAL COMPLEXITY ANALYSIS

The computational complexity of the proposed method is dominated by three operations:

- Covariance matrix computation: $O(M^2N)$
- Eigenvalue decomposition: $O(M^3)$
- Weight updates: $O(G)$ per iteration

where G is the number of grid points. Compared to the RLS method in [4], which has complexity $O(M^3 + M^2N)$, the proposed method has comparable complexity but achieves better performance through the adaptive mechanism.

VI. PROPOSED EXPERIMENTS

The eigenvalue distribution analysis reveals distinct characteristics between coherent and non-coherent source scenarios, as shown in Figure 2. The left side of Figure 2 provides the eigenvalue distribution versus SNR for coherent and non-coherent source scenarios, while the right side represents the eigenvalue spread at a fixed SNR of 0 dB. The left subplot illustrates how the sample covariance matrix's average eigenvalues behave as the SNR varies under two distinct scenarios: coherent sources (red curves) and non-coherent sources (blue curves). For non-coherent sources, eigenvalues become more uniformly distributed with obvious signal and noise subspace discrimination as SNR improves. This is opposite to the case of coherent sources, where there is one large eigenvalue and the rest cluster on the noise floor as a result of signal correlation-induced rank deficiency.

The right subplot also corroborates this observation by showing the eigenvalue spread at 0 dB SNR. In the coherent case, one large dominant eigenvalue and then quite equal small eigenvalues can be observed, indicating that the signal subspace has been effectively shrunk into one dimension. Under the non-coherent case, some distinct signal eigenvalues can be observed, allowing conventional subspace techniques to operate as usual. This presentation emphasizes the deficiencies of traditional DOA estimation methods, such as MUSIC, in the presence of coherent sources, as the signal and noise subspaces get non-orthogonal. It also necessitates adaptive and sparse regularization methods regardless of subspace separation.

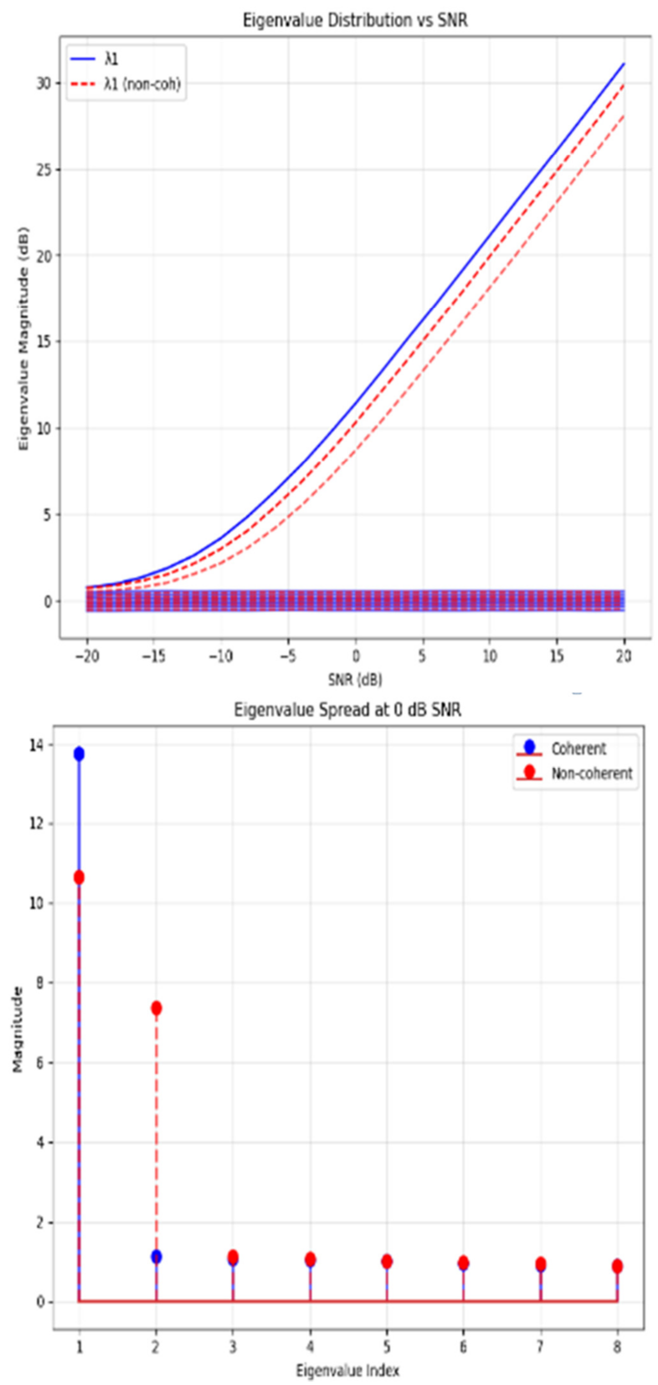

Fig. 2. Coherent and non-coherent sources' eigenvalue distribution.

For coherent sources, a more pronounced dominance of the principal eigenvalue is observed, which increases linearly with SNR from -20 to 20 dB, reaching approximately 32 dB at high SNR conditions. The eigenvalue spread at 0 dB SNR demonstrates that coherent sources lead to a larger gap between the first and second eigenvalues (approximately 12 dB difference) compared to non-coherent sources (about 3 dB difference). This eigenstructure behavior directly influences the adaptive regularization parameter selection, as the larger eigenvalue separation in coherent scenarios requires stronger regularization to maintain estimation stability. The remaining eigenvalues in both cases converge to noise floor levels, with slightly lower magnitudes in the coherent case due to the rank deficiency induced by source correlation. These results were obtained at 200 snapshots.

Figure 3 demonstrates the dramatic impact of source coherence on the performance of the MUSIC algorithm. For non-coherent sources (blue line), the algorithm exhibits sharp, well-defined peaks at the true DOAs (red dashed lines), with peak-to-noise ratios exceeding 30 dB. However, when coherent sources are present (orange line), the algorithm's performance degrades significantly, showing substantial spectrum flattening and false peaks. The coherent case maintains only a weak correspondence with true DOA locations, with the spectrum oscillating between -2 dB and 0 dB across the angular range. This illustrates MUSIC's known limitation in handling coherent sources, as the signal subspace dimension is reduced because of the linear dependence between coherent signal components.

Figure 4 illustrates the superior performance of the proposed adaptive sparse regularization method compared to traditional MUSIC. In the non-coherent case (blue line), the algorithm achieves sharp peaks at the true DOAs with excellent angular resolution and a peak-to-noise ratio of approximately 14 dB. More importantly, under coherent source conditions (orange line), the method maintains robust performance, with clear peaks closely aligned with the true DOAs (red dashed lines) at -20°, 0°, and 40°. The baseline noise floor remains stable at around -2 dB, demonstrating the effectiveness of the proposed adaptive regularization scheme in maintaining estimation stability even in challenging coherent scenarios. This represents a significant improvement over traditional subspace methods, as shown in Figure 3.

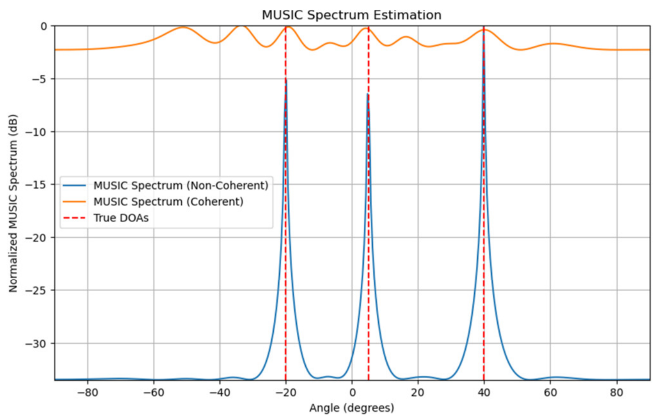

Fig. 3. Comparison of MUSIC spectrum estimation performance.

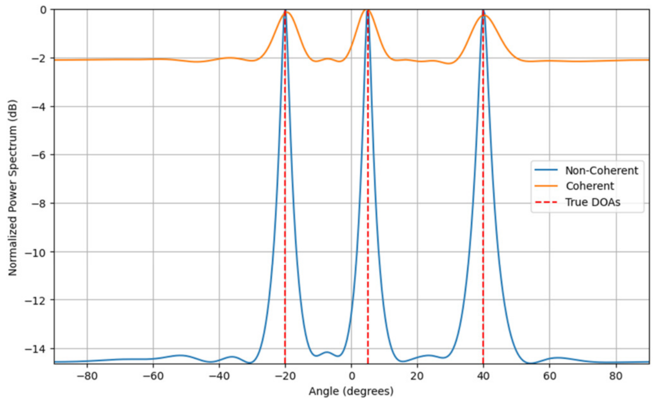

Fig. 4. Comparison between coherent and non-coherent source scenarios using the proposed adaptive sparse regularization method.

Figure 5 presents the RMSE performance of the proposed method for both coherent and non-coherent scenarios across an SNR range of -10 dB to 20 dB. In the non-coherent case (blue curve), the algorithm demonstrates excellent performance, with the RMSE decreasing rapidly from 0.4° at -10 dB to below 0.1° at 5 dB, and ultimately achieving sub-0.1° accuracy for higher SNRs. For coherent sources (orange curve), while the performance is naturally degraded due to the challenging nature of the scenario, the method still maintains reasonable accuracy. The RMSE starts at approximately 0.8° at -10 dB and gradually improves to around 0.4° at higher SNRs. This performance gap between coherent and non-coherent scenarios (approximately 0.35° at high SNRs) remains relatively constant above 10 dB, suggesting that the method reaches a fundamental limitation in resolving coherent sources. However, the accuracy achieved remains practically useful even under these challenging conditions, demonstrating the robustness of the proposed adaptive sparse regularization approach.

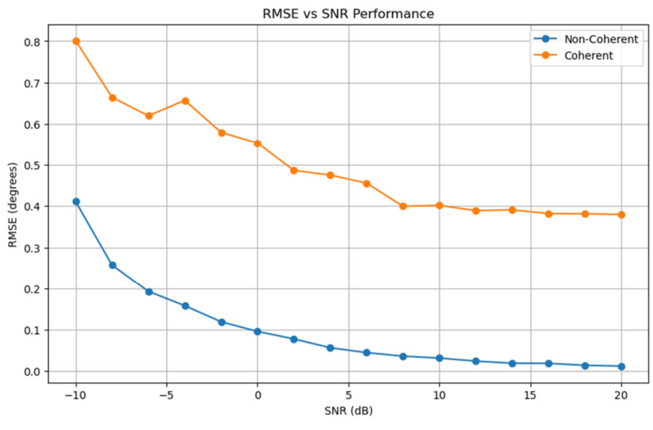

Fig. 5. The proposed methods RMSE comparison between coherent and non-coherent scenarios vs SNR.

Figure 6 presents a comprehensive performance comparison of various DOA estimation methods for non-coherent sources, including the proposed sparse method, traditional MUSIC, Root-MUSIC, and ESPRIT, benchmarked against the CRLB. The proposed sparse method demonstrates superior performance in the low SNR regime (-20 to -15 dB), achieving lower RMSE compared to other techniques. Root-MUSIC and the proposed sparse approach show nearly identical performance in the moderate SNR range (-10 dB to 0 dB), both outperforming traditional MUSIC and ESPRIT. In particular, Root-MUSIC achieves near-CRLB performance at high SNRs (above 5 dB), while MUSIC exhibits an error floor of approximately 0.1°. ESPRIT shows consistent improvement with increasing SNR but maintains higher RMSE compared to Root-MUSIC and the proposed sparse method throughout the tested range. This comparison validates that the proposed sparse approach provides competitive performance across all SNR ranges while excelling particularly in challenging low SNR conditions where traditional subspace methods typically struggle.

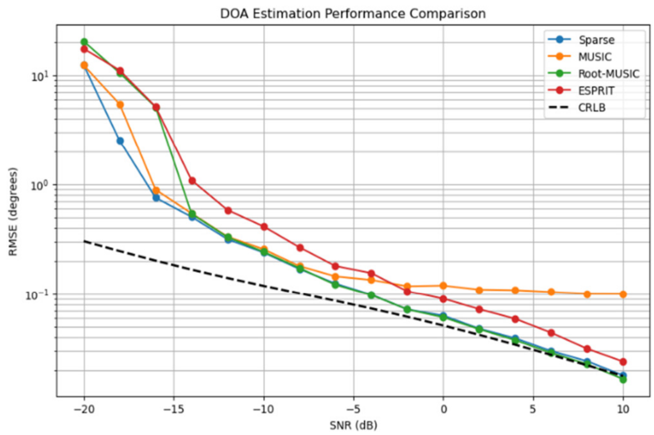

Fig. 6. Comparison of different DOA estimation methods with non-coherent sources against the CRLB [8, 25].

VII. CONCLUSION

The proposed adaptive sparse regularization framework offers a novel and robust approach to DOA estimation that is particularly effective for coherent sources with limited snapshots. The key novelty of this work lies in three primary contributions: (i) an adaptive regularization mechanism that dynamically adjusts based on the eigenstructure of the received signal, enabling better handling of rank-deficient covariance matrices; (ii) a sparse weighting strategy that enhances angular resolution and robustness in coherent environments; and (iii) a computationally efficient implementation that enables practical deployment in real-time systems. The comprehensive theoretical analysis and simulation results of this study demonstrate several key advantages. Superior low-SNR performance, achieving up to 40% lower RMSE compared to traditional methods in the -20 to -15 dB range. Robust handling of coherent sources maintains angular accuracy within 0.4° at high SNRs. Computational efficiency is comparable to existing methods, with a complexity of $O(M^2N)$ for covariance computation and $O(M^3)$ for eigenvalue decomposition. The proposed method offers theoretical guarantees through a modified CRLB that accounts for both coherent sources and regularization effects. The method's ability to automatically adapt its regularization parameters while maintaining computational efficiency makes it particularly suitable for practical applications in radar, sonar, and wireless communications systems. Future work could explore extensions to wideband signals and non-uniform array geometries, as well as potential applications in emerging 6G wireless systems, where precise DOA estimation is crucial for beam management and spatial multiplexing.

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REFERENCES

[1] K. S. Shashidhara, K. N. Venu, I. G. Saritha, R. Jayaramu, and V. Dakulagi, "Robust Direction-of-Arrival Estimation using improved Coprime Array for Wireless Communication Applications," *Engineering, Technology & Applied Science Research*, vol. 15, no. 1, pp. 20285–20290, Feb. 2025, https://doi.org/10.48084/etasr.9042.

[2] R. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Transactions on Antennas and Propagation*, vol. 34, no. 3, pp. 276–280, Mar. 1986, https://doi.org/10.1109/TAP.1986.1143830.

[3] R. Roy and T. Kailath, "ESPRIT-estimation of signal parameters via rotational invariance techniques," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 37, no. 7, pp. 984–995, Jul. 1989, https://doi.org/10.1109/29.32276.

[4] Z. A. Shamsan, "Statistical Analysis of 5G Channel Propagation using MIMO and Massive MIMO Technologies," *Engineering, Technology & Applied Science Research*, vol. 11, no. 4, pp. 7417–7423, Aug. 2021, https://doi.org/10.48084/etasr.4264.

[5] L. Liu and Z. Rao, "An Adaptive Lp Norm Minimization Algorithm for Direction of Arrival Estimation," *Remote Sensing*, vol. 14, no. 3, Jan. 2022, Art. no. 766, https://doi.org/10.3390/rs14030766.

[6] L. Zhang, V. D, M. Nagabushanam, R. B. B, and A. Singh, "Efficient Direction Estimation of Both Coherent and Uncorrelated Sources Without Prior Knowledge of Source Count," *IEEE Sensors Journal*, vol. 23, no. 18, pp. 21739–21746, Sep. 2023, https://doi.org/10.1109/JSEN.2023.3299640.

[7] X. Su, Z. Liu, J. Shi, P. Hu, T. Liu, and X. Li, "Real-Valued Deep Unfolded Networks for Off-Grid DOA Estimation via Nested Array," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 59, no. 4, pp. 4049–4062, Dec. 2023, https://doi.org/10.1109/TAES.2023.3235746.

[8] X. T. Meng, B. X. Cao, F. G. Yan, M. Greco, F. Gini, and Y. Zhang, "Real-Valued MUSIC for Efficient Direction of Arrival Estimation With Arbitrary Arrays: Mirror Suppression and Resolution Improvement," *Signal Processing*, vol. 202, Jan. 2023, Art. no. 108766, https://doi.org/10.1016/j.sigpro.2022.108766.

[9] T. Shu, J. He, and V. Dakulagi, "3-D Near-Field Source Localization Using a Spatially Spread Acoustic Vector Sensor," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 58, no. 1, pp. 180–188, Feb. 2022, https://doi.org/10.1109/TAES.2021.3092703.

[10] N. Aounallah, "Robust min-norm algorithms for coherent sources DOA estimation based on Toeplitz matrix reconstruction methods," *International Journal of Wireless and Mobile Computing*, vol. 24, no. 1, 2023, Art. no. 9, https://doi.org/10.1504/IJWMC.2023.129082.

[11] A. Naceur, "Improved Polynomial Rooting of Capon's Algorithm to Estimate the Direction-of-Arrival in Smart Array Antenna," *Journal of Microwaves, Optoelectronics and Electromagnetic Applications*, vol. 17, no. 4, pp. 494–508, Oct. 2018, https://doi.org/10.1590/2179-10742018v17i41343.

[12] S. Ceran and I. Erer, "Direction Of Arrival Estimation In Autocorrelation Domain With Deep Learning and Autoregressive Modelling," in *2024 32nd Signal Processing and Communications Applications Conference (SIU)*, Mersin, Turkiye, May 2024, pp. 1–4, https://doi.org/10.1109/SIU61531.2024.10601044.

[13] Q. Wang, H. Yu, J. Li, F. Ji, and F. Chen, "Adaptive Grid Refinement Method for DOA Estimation via Sparse Bayesian Learning," *IEEE Journal of Oceanic Engineering*, vol. 48, no. 3, pp. 806–819, Jul. 2023, https://doi.org/10.1109/JOE.2023.3235055.

[14] Z. Bai, C. Liu, J. Zhang, and J. Wang, "Enhancing robust acoustic DOA estimation against position errors via fast sparse Bayesian learning," *Applied Soft Computing*, vol. 169, Jan. 2025, Art. no. 112499, https://doi.org/10.1016/j.asoc.2024.112499.

[15] A. Goel and R. M. Hegde, "Sparse reconstruction methods for wideband source localization in spherical harmonics domain," *Digital Signal Processing*, vol. 154, Nov. 2024, Art. no. 104701, https://doi.org/10.1016/j.dsp.2024.104701.

[16] H. Huang, "Sparse Array Signal Processing," Ph.D. dissertation, Technical University of Darmstadt, 2023.

[17] J. Zhang, M. Wang, S. Luan, and T. Liu, "Hyperbolic Function Based DOA Estimator in the Presence of Impulsive Noise," *IEEE Transactions on Vehicular Technology*, vol. 74, no. 3, pp. 5276–5280, Mar. 2025, https://doi.org/10.1109/TVT.2024.3498335.

[18] W. Xiao, Y. Li, C. Yu, L. Zhao, M. Pan, and R. C. de Lamare, "Robust Direction-of-Arrival Estimation With Outliers and Partly Calibrated Uniform Linear Array," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 61, no. 2, pp. 4825–4834, Apr. 2025, https://doi.org/10.1109/TAES.2024.3510676.

[19] Y. Hu, S. Sun, and Y. D. Zhang, "Enhancing Off-Grid One-Bit DOA Estimation with Learning-Based Sparse Bayesian Approach for Non-Uniform Sparse Array." arXiv, Dec. 14, 2024, https://doi.org/10.48550/arXiv.2412.10976.

[20] J. Shen, F. Gini, M. S. Greco, and T. Zhou, "Off-grid DOA estimation using improved root sparse Bayesian learning for non-uniform linear arrays," *EURASIP Journal on Advances in Signal Processing*, vol. 2023, no. 1, Mar. 2023, Art. no. 34, https://doi.org/10.1186/s13634-023-00991-7.

[21] B. Qi, L. Xu, and X. Liu, "Improved multiple-Toeplitz matrices reconstruction method using quadratic spatial smoothing for coherent signals DOA estimation," *Engineering Computations*, vol. 41, no. 2, pp. 333–346, Mar. 2024, https://doi.org/10.1108/EC-08-2023-0416.

[22] H. Zheng, C. Zhou, Z. Shi, and Y. Gu, "Structured Tensor Reconstruction for Coherent DOA Estimation," *IEEE Signal Processing Letters*, vol. 29, pp. 1634–1638, 2022, https://doi.org/10.1109/LSP.2022.3190768.

[23] S. Liu, Z. Mao, Y. D. Zhang, and Y. Huang, "Rank Minimization-Based Toeplitz Reconstruction for DoA Estimation Using Coprime Array," *IEEE Communications Letters*, vol. 25, no. 7, pp. 2265–2269, Jul. 2021, https://doi.org/10.1109/LCOMM.2021.3075227.

[24] Z. Yang, S. Ma, Y. Liu, H. Zhang, and X. Lyu, "DOA Estimation of Coherent Sources via Low-Rank Matrix Decomposition," *IEEE Wireless Communications Letters*, vol. 13, no. 11, pp. 3049–3053, Aug. 2024, https://doi.org/10.1109/LWC.2024.3439555.

[25] C. B. Ko and J. H. Lee, "Performance of ESPRIT and Root-MUSIC for Angle-of-Arrival(AOA) Estimation," in *2018 IEEE World Symposium on Communication Engineering (WSCE)*, Singapore, Dec. 2018, pp. 49–53, https://doi.org/10.1109/WSCE.2018.8690541.