

Enhanced Time-of-Flight Estimation via Type-I Discrete Cosine Transform in Ultrasonic Flow Measurement

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ABSTRACT

This study investigated the possibility of using the Discrete Cosine Transform (DCT) method for Time-of-Flight (ToF) estimation in ultrasonic water flow measurements, as an improvement over traditional correlation-based techniques. A DN-20 ultrasonic water flow meter was employed to collect data across 13 distinct flow rates, calculating the travel time of signals through the medium. The average flow readings were compared against reference values obtained from an electromagnetic flow meter. The DCT method demonstrated superior performance, consistently yielding lower error percentages than the correlation method. At low flow rates, such as 16 L/h, the DCT method achieved an error of 4.62%, compared to the 4.75% obtained with the correlation method, while at higher flow rates, such as 4000 L/h and 5000 L/h, the DCT method maintained minimal errors of 0.02% and 0.06%, respectively, versus the 0.05% and 0.07% attained with the correlation method. The DCT method's computational efficiency, derived from its frequency domain analysis, significantly reduced the required computational resources, as well as its robustness to noise, ensuring more accurate ToF estimates in noisy environments. Overall, the DCT method offered a more accurate, efficient, and robust alternative for ToF estimation in ultrasonic flow measurements. These findings suggest potential improvements in various applications requiring precise flow measurement, with future research focusing on integrating the DCT method into embedded systems and extending its applicability to other types of flow measurements.

Keywords- correlation; time of flight; DCT; flow meter

I. INTRODUCTION

Ultrasound technology has been utilized in various scientific and engineering fields, including environment sensing of autonomous mobile robots, material evaluation, and fluid flow measurement [1-5]. In order to analyze and comprehend the behavior of this system, the ultrasound wave motion has been examined, focusing on the similarities or differences between transmitting and receiving ultrasonic signals as well as their various characteristics.

The duration of an ultrasonic signal's journey from a source via a medium to a target is known as the ToF [6, 7]. The signal that the transducer (an ultrasonic sensor) can receive is described by:

$$S_R(t) = A(t)S_T(t - \tau) + \eta(t) \quad (1)$$

where $S_T(t)$ is the transmitted signal, $\eta(t)$ is a Gaussian White Noise (GWN), τ is the expected ToF, and $A(t)$ is the ultrasonic echo envelop expressed by:

$$A(t) = A_0 \left(\frac{t-\tau}{T} \right)^\alpha \exp \left(\frac{t-\tau}{T} \right) \quad (2)$$

where A_0 , a , and T are shape parameters.

The ultrasonic flow meter is a flow sensor that does not require direct contact, utilizing sound waves to estimate flow rate, ensuring constant substance measurement. It was primarily designed for liquid flow measurement, employing the Doppler effect and transit time to determine the velocity of the liquid as well as the flow volume. Authors in [8] examined the effectiveness of different discrete random walk models in turbulent flows, considering irregular Root-Mean-Square (RMS) velocity fluctuations and turbulence time scales. Authors in [4] employed a discrete extended Kalman filter to estimate ToF variations, identifying perfect orthogonality between the source and target, along with an acoustic interference-free measurement environment, as optimal conditions for performance evaluation. Additionally, in [9] a three-phase flow meter was introduced, capable of accurately

measuring the flow rates of oil-gas-water mixtures deploying multiple techniques, such as electromagnetic resonance, imaging technology, and ultrasonic sound methods.

In a discrete-time system, the ToF was assessed [10]. The results revealed that choosing an excessive sampling rate will not necessary obtain a small ToF dispersion range. In [11], authors proposed a thermal cross correlation-based method for flow monitoring and calibration, with their findings indicating that the system's configuration affected the calibration coefficient.

II. MATERIALS AND METHODS

Both the widely recognized cross-correlation method and the newly proposed DCT technique are discussed in this section.

A. Cross Correlation Method

The cross-correlation between two signals can be readily calculated in the typical discrete time scenario as [12]:

$$R_{XY}(\tau) = \sum_{-\infty}^{\infty} X(m)Y(m - \tau) \tag{3}$$

The main goal of the cross-correlation method was to identify the greatest $R_{XY}(\tau)$ at a particular time shift of τ . Considering a continuous system, the ToF can be identical to τ . Nonetheless, while investigating a system modeled as discrete time, τ is a number of shifting steps in place of the time, so ToF might be predicted as:

$$ToF = \tau T_s \tag{4}$$

where T_s is the sampling period. Further details on the applications of the cross-correlation technique in engineering can be found in the literature [13-16].

Furthermore, the system inherently exhibited random characteristics due to factors, such as ultrasonic sensor signal generation, turbulence flow, vibration, and temperature variations. Consequently, in this study, all signals, both transmitted and received, had a random part in order to get a better realization of the physical system. Hence, the transmitted signal from the sensor-1, S_{T1} , the received signals by the sensor-2, S_{R2} , the transmitted signal from the sensor-2, S_{T2} , and the received signals by the sensor-1, S_{R1} are defined as:

$$\begin{aligned} S_{T1}[n] &= S_T[n] + \eta_1[n] \\ S_{R2}[n] &= S_T[n - n_1] + \eta_1[n - n_1] + \eta_2[n] \\ S_{T2}[n] &= S_T[n] + \eta_3[n] \\ S_{R1}[n] &= S_T[n - n_2] + \eta_3[n - n_2] + \eta_4[n] \end{aligned} \tag{5}$$

where $\eta_i[n]$ is the related GWN resulted from a Wiener process. All samples of this procedure are independent and identically distributed. Thereafter, the cross-correlation functions of $S_{R1}[n]$ and $S_{R2}[n]$ were calculated by means of the expectation operator, $E[S_{R1}[n] S_{R2}[n]]$:

$$\begin{aligned} E[S_{R1}[n]S_{R2}[n]] &= \\ E[(S_T[n - n_2] + \eta_3[n - n_2] + \eta_4[n])(S_T[n - n_1] & \\ + \eta_1[n - n_1] + \eta_2[n])] &= E[S_T[n - n_2]S_T[n + \tau - n_1]] \\ + E[S_T[n - n_2]\eta_1[n + \tau - n_1]] &+ E[S_T[n - n_2]\eta_2[n + \tau]] \\ + E[\eta_3[n - n_2]S_T[n + \tau - n_1]] & \\ + E[\eta_3[n - n_2]\eta_1[n + \tau - n_1]] &+ E[\eta_3[n - n_2]\eta_2[n + \tau]] \\ + E[\eta_4[n - n_2]S_T[n + \tau - n_1]] &+ E[\eta_4[n]\eta_1[n + \tau - n_1]] \\ + E[\eta_4[n]\eta_2[n + \tau]] & \end{aligned} \tag{6}$$

Generally, the cross-correlation function is equal to the product of the time average of the periodic signal and the expected value of the random signal. There is no correlation between these two types of signal. Due to this, the cross-correlation of the two variables is equal to the multiplication of each of their respective mean values [12].

By means of that, (6) can be expressed as:

$$\begin{aligned} E[S_{R1}[n]S_{R2}[n]] &= \\ E[S_T[n - n_2]S_T[n + \tau - n_1]] &+ 4\mu^2 + 4\mu < S_T[n] > \end{aligned} \tag{7}$$

where μ is the expected value of GWN, and $< S_T[n] >$ is the time average of $S_T[n]$. In the case of $\mu = 0$, (7) yields the following result:

$$E[S_{R1}[n]S_{R2}[n]] = E[S_T[n - n_2]S_T[n + \tau - n_1]] \tag{8}$$

This represents the autocorrelation function of S_T , and thus:

$$E[S_T[n - n_2]S_T[n + \tau - n_1]] = R_{S_T,S_T}[\tau + n_2 - n_1] \tag{9}$$

The autocorrelation function's characteristics state that $|R_{S_T,S_T}[\tau]| \leq R_{S_T,S_T}[0]$. This implies that when $\tau = n_1 - n_2$, which corresponds to the ToF , the correlation between the received signals reaches its maximum.

B. Discrete Cosine Transform, Type I

The DCT is widely used in digital image processing and time series analysis [17, 18]. One form of the type-I DCT kernel is defined as [19]:

$$K_c(m, n) = \cos(\pi \frac{mn}{N}) \tag{10}$$

where $m, n = 0, 1, \dots, N$.

The DCT transformation matrix C is of size $(m+1) \times (n+1)$ [20] and is given by:

$$C(m, n) = \sqrt{\frac{2}{N}} k_m k_n \cos\left(\pi \frac{mn}{N}\right) \tag{11}$$

where:

$$k_i = \begin{cases} 1, & \text{if } i \neq 0 \text{ or } i \neq N \\ \frac{1}{\sqrt{2}}, & \text{if } i = 0 \text{ or } i = N \end{cases} \tag{12}$$

If the discrete $N+1$ samples are represented by a vector x , the DCT of this vector can be computed as:

$$X_c = Cx \tag{13}$$

or, in the open form:

$$X_c(m) = \sqrt{\frac{2}{N}} \sum_{n=0}^N k_m k_n \cos\left(\pi \frac{mn}{N}\right) x(n) \tag{14}$$

Since C is a unitary matrix, the inverse DCT can be calculated as:

$$x(n) = \sqrt{\frac{2}{N}} \sum_{m=0}^{N-1} k_m k_n \cos(\pi \frac{mn}{N}) X_c(m) \tag{15}$$

The vectors X_c and x are called DCT pairs.

Similarly, the pairs of Discrete Sine Transform (DST) can be expressed as:

$$X_s(m) = \sqrt{\frac{2}{N}} \sum_{n=1}^{N-1} \sin(\pi \frac{mn}{N}) x(n) \tag{16}$$

$$x(n) = \sqrt{\frac{2}{N}} \sum_{m=1}^{N-1} \sin(\pi \frac{mn}{N}) X_s(m) \tag{17}$$

The time-shifted DCT denoted as X_c^+ can be expressed in terms of X_c and X_s :

$$\begin{aligned} X_c^+(m) = & \cos(\frac{m\pi}{N}) X_c(m) + k_m \sin(\frac{m\pi}{N}) X_s(m) \\ & + \sqrt{\frac{1}{N}} [(-\frac{1}{\sqrt{2}}) \cos(\frac{m\pi}{N}) x(0) + (\frac{1}{\sqrt{2}} - 1) x(1) \\ & + (-1)^m (\frac{1}{\sqrt{2}} - 1) \cos(\frac{m\pi}{N}) x(N) \\ & + (-1)^m (\frac{1}{\sqrt{2}}) x(N+1)] \end{aligned} \tag{18}$$

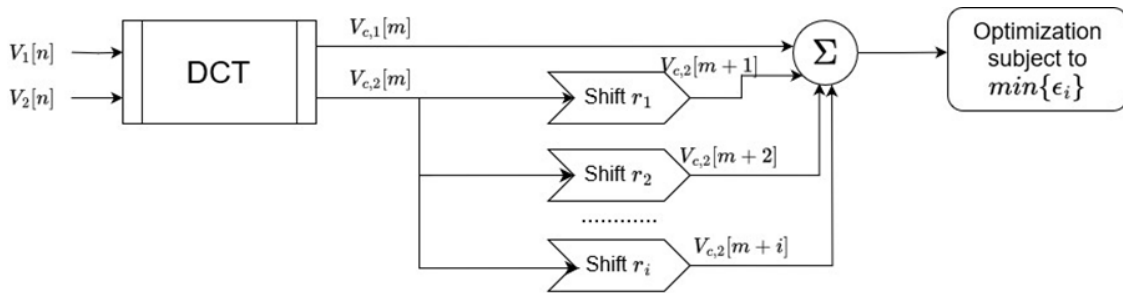


Fig. 1. Block diagram representation of the proposed method.

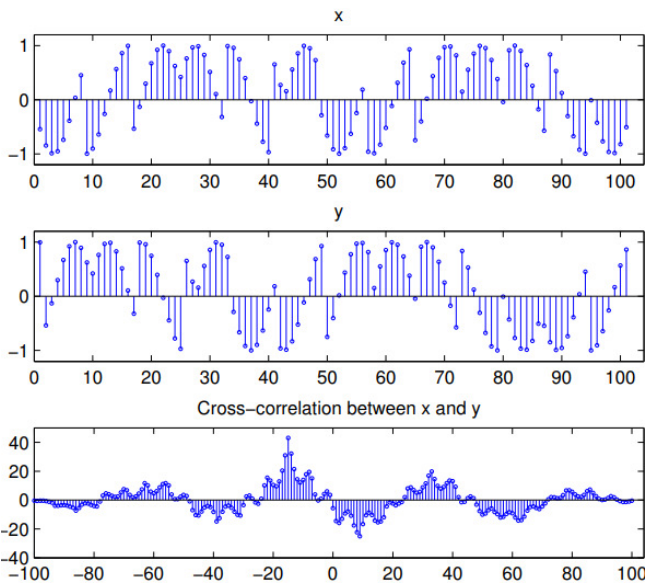


Fig. 2. Cross-correlation results of ultrasonic signals obtained from a flow-meter.

The proposed method is illustrated in Figure 1. It begins with the transmission of an ultrasonic signal through a medium, where the signal is received and captured and is then processed using the DCT. Within the DCT block, the signal is transformed from the time domain into the frequency domain. The time shift τ is estimated by analyzing the frequency components and determining the phase shift. Using this estimated time shift and known parameters of the ultrasonic system, the flow rate can be accurately calculated.

In this study, the ToF between two signals was estimated using both the well-known correlation approach and the proposed DCT method. Data from an ultrasonic flow meter were recorded in discrete time. Figures 2 and 3 present the correlation and DCT technique results, respectively. In Figure 2, x denotes the reference ultrasonic signal, while y represents the measured ultrasonic signal. In Figure 3, the DCT coefficients of these signals are represented as X_c and Y_c , respectively. Due to the discrete nature of the data, the approaches produced an error-free ToF estimation.

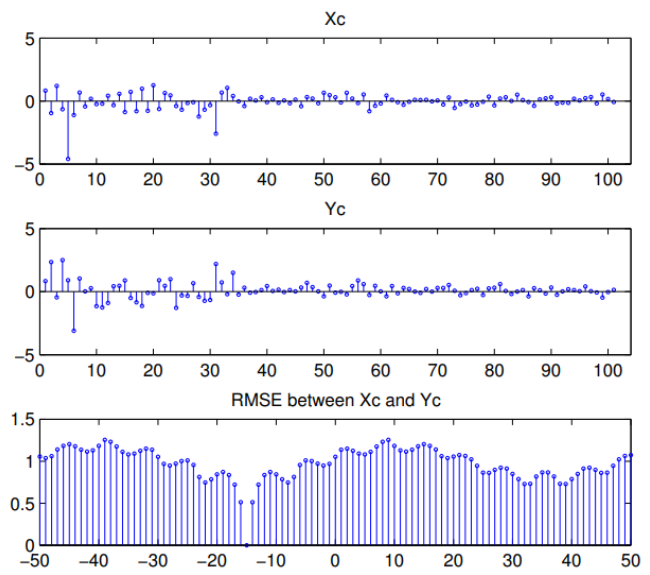


Fig. 3. DCT results of ultrasonic signals obtained from a flow-meter

The DCT primary advantage over the correlation method is that it requires fewer calculation steps to estimate the flight time. Conversely, in an actual implementation, a process of conversion from analog to digital signals is required. Thus, to guarantee a high degree of accuracy for the proposed method, the sampling frequency and the matching resolution play a crucial role.

III. RESULTS AND DISCUSSION

A DN-20 type (pipe with a diameter of 20 mm) ultrasonic water flow meter was used in the experimental setup to collect the data needed for analysis. Measurements were taken at 13 distinct water flow levels. For each level, multiple readings were recorded, and the average of these measurements was taken into consideration as representative value for comparison. The reference flow rates were measured using an electromagnetic flow meter.

To enhance the accuracy of the ToF estimation, the minimum error value at τ_0 was first determined, using the three-point Lagrange interpolation technique. The algorithm is displayed in Figure 4.

Algorithm 1: Lagrange Interpolation for Local Minimum	
Input: Three points $(\tau_{-1}, f_{-1}), (\tau_0, f_0), (\tau_1, f_1)$	
Step 1: Compute Lagrange basis polynomials:	
$L_{-1}(\tau) = \frac{((\tau - \tau_0)(\tau - \tau_1))}{((\tau_{-1} - \tau_0)(\tau_{-1} - \tau_1))}$	
$L_0(\tau) = \frac{((\tau - \tau_{-1})(\tau - \tau_1))}{((\tau_0 - \tau_{-1})(\tau_0 - \tau_1))}$	
$L_1(\tau) = \frac{((\tau - \tau_{-1})(\tau - \tau_0))}{((\tau_1 - \tau_{-1})(\tau_1 - \tau_0))}$	
Step 2: Construct the interpolating polynomial: $P(\tau) = f_{-1}L_{-1}(\tau) + f_0L_0(\tau) + f_1L_1(\tau)$	
Step 3: Differentiate $P(\tau)$ to find critical points: Compute $P'(\tau) = \frac{d}{d\tau}P(\tau)$	
Step 4: Solve $P'(\tau) = 0$ to find τ_{min} : $\tau_{min} = -\frac{(f_{-1}(\tau_0 - \tau_1)^2 + f_0(\tau_{-1} - \tau_1)^2 + f_1(\tau_{-1} - \tau_0)^2)}{(f_{-1}(\tau_0 - \tau_1) + f_0(\tau_{-1} - \tau_1) + f_1(\tau_{-1} - \tau_0))}$	
Step 5: Verify if τ_{min} is a local minimum: If $P''(\tau_{min}) > 0$, then τ_{min} is a local minimum.	
Output: $\tau_{min}, P(\tau_{min})$	

Fig.4. Lagrange interpolation algorithm for the determination of minimum τ .

The data presented in Table I highlighted the performance comparison between the traditional correlation method and the novel DCT method for estimating flow rates in ultrasonic water flow measurements. The results revealed several key insights favoring the DCT method. Although in certain cases -such as at flow rates of 25 L/h and 200 L/h- the correlation method exhibited lower error rates than the DCT method, the DCT

consistently demonstrated lower error percentages. This improved precision is important in various applications, such as water resource management, industrial process control, and medical diagnostics, where accurate flow measurements are critical. For example, at a typical flow rate of 16 L/h, the correlation method achieved an error of 4.75%, whereas the DCT yielded a slightly lower error of 4.62%. Similarly, at higher flow rates, such as 4000 L/h and 5000 L/h, the DCT method maintained a minimal error of 0.02% and 0.06% respectively, compared to the correlation method's errors of 0.05% and 0.07%. These small yet significant differences underscored the DCT method's superior precision across a wide range of flow rates, suggesting that it can be more reliably used in critical applications where even minor inaccuracies can have substantial impacts.

TABLE I. RESULTS OBTAINED FROM DCT AND CORRELATION METHOD

Collected data		Correlation		DCT	
Standard (L/h)	Reading (L/h)	Estimation (L/h)	Absolute error (%)	Estimation (L/h)	Absolute error (%)
16	16.21	16.98	4.75	16.96	4.62
25	25.44	25.86	1.65	25.98	2.12
40	40.36	41.47	2.75	41.34	2.44
60	61.12	61.08	0.07	61.15	0.05
80	81.97	82.67	0.85	82.31	0.42
120	122.36	122.1	0.21	122.16	0.16
200	203.62	203.57	0.02	203.5	0.06
400	405.83	401.51	1.06	401.97	0.95
1000	1015.78	1002.31	1.33	1004.2	1.14
1400	1426.11	1401.46	1.73	1402.01	1.69
2800	2819.55	2818.43	0.04	2818.7	0.03
4000	4050.17	4048.22	0.05	4049.36	0.02
5000	5076.12	5072.75	0.07	5073.07	0.06

Additionally, the DCT method's computational efficiency presented a significant advantage over the correlation method due to the fewer required computational resources. This is attributed to its inherent ability to represent signals in the frequency domain with fewer coefficients, which is beneficial in real-time applications where rapid processing is crucial. For example, in automated industrial systems, where flow rates need to be monitored and controlled in real-time, the reduced computational load of DCT can lead to faster response times and lower power consumption, enhancing the system's overall performance and reliability.

Another aspect favoring the DCT method is its robustness in noisy environments. Ultrasonic signals are often contaminated with noise, which can adversely affect the accuracy of ToF measurements. In practical scenarios, such as in medical ultrasound imaging or flow measurement in industrial settings where background noise is prevalent, the DCT method's ability to maintain accuracy despite noise can lead to more reliable measurements.

IV. CONCLUSION

This study validated the effectiveness of the Discrete Cosine Transform (DCT) method as an alternative to the traditional correlation-based approach for Time-of-Flight (ToF) estimation in ultrasonic water flow measurements. Using a DN-

20 ultrasonic flow meter and comparing the results across 13 different flow rates against a reference electromagnetic low meter. The DCT method consistently exhibited lower error percentages at both low and high flow rates. For instance, at a low flow rate of 16 L/h, the DCT method achieved an error of 4.62%, compared to 4.75% with the correlation method. At higher flow rates, such as 4000 L/h and 5000 L/h, the DCT method maintained very low errors of 0.02% and 0.06%, respectively, outperforming the correlation method's 0.05% and 0.07%. These improvements are significant in applications where measurement precision is critical.

Additionally, the DCT method proved to be computationally more efficient, requiring fewer resources. Its robustness in noisy environments ensured accurate ToF estimates even in challenging conditions, further solidifying its practical value. These benefits make the DCT method a valuable tool for improving operational efficiency and sustainability. Future research should focus on integrating the DCT method into embedded systems and exploring its applicability to other types of flow measurements, broadening its utility across different industries.

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