Assessing the Newly Carbon-Backed Cryptocurrency Downside Risk: Insights from Value at Risk and Expected Shortfall Estimations

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ABSTRACT

This study presents a method for predicting volatility and assessing the risk and expected shortfall for carbon-backed cryptocurrencies, investigating whether accounting for long memory in cryptocurrency volatilities, asymmetry, and fat-tailed returns improves forecasting and risk quantification. Various long-memory GARCH-class models were investigated under normal and skewed Student-t distributions and the one-ahead value at risk and expected shortfall. The results show that FIEGARCH under skewed Student t-distribution is a strong fit for carbon credit cryptocurrency volatilities, outperforming other FIGARCH models under normal and skewed Student t-distributions. The model consistently produced accurate values of risk and expected shortfalls for both short- and long-term trading positions in and out of the sample. This research provides unique insights into carbon credit cryptocurrencies and offers practical implications for operational risk management for portfolio managers, cryptocurrency traders, and eco-friendly cryptocurrency market regulators, enhancing their ability to manage risk in the evolving cryptocurrency market.

Keywords-carbon-backed cryptocurrency; volatility; value at risk; expected shortfal; forecasting

I. INTRODUCTION

The swift rise of cryptocurrencies has substantially changed the landscape of digital finance, and the emergence of Carbon-Backed Cryptocurrencies (CBCs) represents a novel approach to connecting financial markets with environmental sustainability. Compared to traditional cryptocurrencies, CBCs integrate carbon credits as the underlying value integrated into a blockchain-based system, incentivize carbon offsets, and tackle climate change. For example, Klimado is a CBC launched in October 2021 by the Klimadao digital platform to accelerate the price appreciation of carbon emissions by purchasing and retiring carbon offsets. However, like any other financial asset, CBCs are subjected to market risks and volatility, making it crucial to effectively assess its risk exposure.

In traditional financial markets, risk management practices such as Value-at-Risk (VaR) and Expected Shortfall (ES) are vital in quantifying the potential losses of portfolios. Similarly, implementing these risk metrics into CBCs helps provide a structured method for understanding, quantifying, and managing price volatility, especially during extreme market conditions. VaR, a statistical tool commonly used to measure market risk, estimates the potential loss in the value of an asset or portfolio over a defined time horizon given a certain level of confidence. For CBCs, VaR can be employed to quantify the likelihood of losses due to market shifts, investor climate attention, and climate policy changes that may influence CBC prices. Although VaR provides valuable insights into risk under normal market conditions, it has limitations, particularly in capturing the impact of extreme events or tail risks [1, 2]. Here, ES becomes a crucial measure, addressing the shortcomings of

VaR by estimating the average loss that could occur beyond the VaR threshold, specifically during extreme market downturns or crises. This was explained by the Basel Committee on Banking Supervision (BCBS), which revised its standards for minimum capital requirements for market risk and recommended the shift from VaR to ES. In the context of CBCs, VaR and ES are particularly relevant because of the high level of volatility often associated with both the cryptocurrency markets and climate policy changes or their relative regulation policies governing carbon offset initiatives. These risk factors may induce high volatility in CBC prices, making them essential in providing a more comprehensive risk quantification. By applying these metrics, traders, developers, and policymakers can better understand the potential downside risks associated with CBCs, ensuring that they are better equipped to make appropriate investment decisions that align with financial and environmental goals.

In the literature, various studies have estimated VaR and ES for cryptocurrencies using forecasting models under various parametric specifications for return distributions [3-6]. Recent studies have also underscored the existence of long memory and persistence behavior in cryptocurrency volatilities [7, 8]. This study is within this research line. Although it assumes parametric models for the conditional mean and variance, it adopts a different standpoint. Given the relevance of VaR and ES in quantifying cryptocurrency risk, this study investigates and examines their performance for innovative CBCs. Various long-memory GARCH-class models are evaluated using normal and skewed Student-t distributions. This study attempts to answer the following research question: Does accounting for asymmetry, long memory, and fat-tailed returns enhance the CBC forecasting ability and risk quantifications using the VaR and ES? This study makes several contributions to the literature on eco-friendly cryptocurrency risk assessment. To the best of our knowledge, it represents the first empirical analysis that quantifies the risk of CBCs. Furthermore, it implements a range of GARCH-class models with two alternative return volatility distributions and performs a back-testing procedure for both short- and long-trading positions. For this reason, the largest and most comprehensive dataset available was used, covering the four highest capitalized CBCs, with the analysis period beginning from each cryptocurrency's launch date.

TABLE I. LITERATURE REVIEW

Ref.	Data	Methods	Main conclusions
[8]	105 major cryptocurrencies	Generalized Random Forests (GRF) forecasting method and VaR and ES estimations	The GRF adapted to quantile prediction had superior performance over other established methods such as quantile regression, GARCH-type models, and CAViaR models.
[9]	The five highly capitalized cryptocurrencies	VaR and ES estimations: Various long-range GARCH models with treatment of outliers	Long-range GARCH models perform better than the standard GARCH model.
[10]	Bitcoin and foreign exchange rates	(eGARCH)-Extreme Value Theory (EVT)-Gumbel copula model	The model is fitting well the data for Bitcoin and FX rates enhancing the VaR and ES estimations.
[5]	The five highly capitalized cryptocurrencies	ARMA-GARCH under an alpha stable return innovation	VaR and ES dynamic modeling outperform their static counterparts.
[7]	Bitcoin	Various long-memory GARCH-class models	Fractionally integrated models are effective in predicting cryptocurrency prices during times of crisis.
[11]	191 cryptos	DCC-GARCH model	DCC-GARCH performs well in forecasting cryptocurrency downside risks.
[8]	Major cryptocurrencies	Dual long memory GARCH model ARFIMA-FIGARCH	Dual long memory in cryptos' returns and volatility and structural breaks under skewed Student t distributions fit the cryptocurrency market well
[12]	Largest cryptos	The new Range VaR (RVaR)	The Range VaR under normal distribution outperforms other standard VaR and ES estimation methods.
[13]	Major traditional cryptocurrencies	GAS-GARCH model	The GAS model is a suitable alternative for forecasting VaR and ES
[14]	Major traditional cryptocurrencies	Markov-Switching GARCH model and VaR and ES estimations	MS-GARCH models provide more accurate VaR and ES forecasts than their single-regime counterparts.
[6]	Major Cryptocurrencies	Various GARCH class model	The GJR-GARCH model was superior during the in-sample period, while the CGARCH and TGARCH models were optimal in the out-of-sample
[3]	Bitcoin	Student t and NGARCH model	The model with heavy-tailed distribution performs better.
[15]	The 16 largest cryptocurrencies	Various asymmetric GARCH models	Cryptocurrencies are well described by a TGARCH model, exhibiting strong asymmetrical responses to shocks.
[16]	Bitcoin and other commodities	Various GARCH-class models	The standard GARCH model fits well with the Bitcoin volatility and generates accurate VaRs and ES.
[17]	Ethereum crypto	VaR and ES with Generalized Hyperbolic Distributions (GHD)	The model fits well the data and estimates accurately the VaR and ES.
[18]	Major cryptocurrencies	Artificial Intelligence and Deep Learning	LSTM can predict both Ethereum and Bitcoin prices.
[19]	Cryptocurrency blockchain	Finite-dimensional system of Ordinary Differential Equations (ODEs)	Infinite- and finite-dimensional systems incorporate a dynamic pricing approach in the modeling process.
[20]	Bitcoin	PGARCH model	Persistence behavior in the Bitcoin volatility and a significant role of the trading volume.

II. LITERATURE REVIEW

Table I presents the main studies focused on cryptocurrency forecasting and risk assessments. The literature review emphasizes several critical points. First, previous studies focused mainly on leading cryptocurrencies such as Bitcoin and Ethereum, but no previous study has focused on green or eco-friendly cryptocurrencies. Despite their role in enhancing environmental sustainability and mitigating carbon emissions, the risk of CBCs has not yet been explored. Second, although various methods are used to predict the volatility of cryptocurrency and assess risk, the results do not reach a common conclusion on the best-fitting model. The results obtained are substantially varying between cryptocurrency markets and sample periods. Third, only a few studies [5, 8, and 9] examined the dependence on long memory, the fat tail phenomenon in cryptocurrency volatility, and its impact on VaR and ES accuracy. This study aimed to fill these research gaps.

III. DATA AND MODELS

A. The Long Memory GARCH-type Models

The FIGARCH model [21] can be written as follows:

$$[\varphi(L)(1-L)^d]\varepsilon_t^2 = w + [1-\beta(L)](\varepsilon_t^2 - \sigma_t^2)$$

and:

$$\sigma_t^2 = w + \beta(L)\sigma_t^2 + [1-\beta(L)]\varepsilon_t^2 - [\varphi(L)(1-L)^d]\varepsilon_t^2$$
$$= w[1-L]^{-1} + \lambda(L)\varepsilon_t^2$$

where L is the Lag operator and d is the fractional integration parameter: $\lambda(L) = \sum_{i=1}^{\infty} \lambda_i L^i$ and $0 \leq d \leq 1$. The fractional integration parameter is expressed as follows:

$$(1-L)^d = 1 - \sum_{k=1}^{\infty} C_k\,(d)L^k,$$

where $C_1(d) = d,\ C_2(d) = \frac{1}{2d(1-d)}$, etc.

B. VaR and ES Estimations Under Normsal and Skewed Student t-Distributions

One-day-ahead VaR and ES are estimated for the CBCs' short and long trading positions using the FIGARCH and FIEGARCH models under normal and skewed Student-t distributions. The main goal is to examine whether accounting for skewed returns and long memory in the CBC volatilities can enhance the VaR and ES estimations. VaR under normal and skewed distributions is expressed as follows [22-24]:

Consider that the CBCs' returns $r_{i,t}$ are following the process:

$$r_{i,t} = \mu_t + \varepsilon_t \tag{1a}$$

$$\mu_t = \mu + \sum_{i=1}^{m} \xi r_{t-i} + \sum_{j=1}^{n} \Theta_j\, \varepsilon_{t-j} \tag{1b}$$

It is assumed that $\varepsilon_t = z_t\sigma_t$ is following a FIGARCH(1,d,1) process and the CBCs returns follow a skewed Student-t distribution [25] if:

$$f\big((z_t|k,v)\big) =$$
$$\begin{cases} (2k + \left(\frac{1}{k}\right)(sg(k, sz_t + m)|v) & \text{if } z_t < -m/s \\ (2k + \left(\frac{1}{k}\right)(sg(k, sz_t + m)/k|v) & z_t \geq -m/s \end{cases} \tag{2}$$

where $(sg(.)|v)$ refers to the symmetrical Student distribution, and k is the asymmetry parameter. Thus, the VaRs for long and short trading positions are given as:

$$\alpha = P\big(r_t < VaR_{t,L}\big) = P\left(\frac{r_t - \mu_t}{\sigma_t} < \frac{VaR_{t,L} - \mu_t}{\sigma_t}\right) \tag{3a}$$

$$\alpha = P\big(r_t > VaR_{t,S}\big) = P\left(\frac{r_t - \mu_t}{\sigma_t} > \frac{VaR_{t,S} - \mu_t}{\sigma_t}\right) \tag{3b}$$

where $VaR_{t,L}$ and $VaR_{t,S}$ refer to the VaRs for long and short positions, respectively. Using a skewed distribution, the VARs can be expressed as follows:

$$VaR_{t,L} = \mu_t + st_\alpha(v,k)\,\sigma_t \tag{4a}$$

$$VaR_{t,S} = \mu_t + st_{1-\alpha}(v,k)\,\sigma_t \tag{4b}$$

where the $st_\alpha(v,k)$ and $st_{1-\alpha}(v,k)$ are the left and the right α% quintiles for the skewed Student t-distribution. Subsequently, the one-day ahead VaR estimations are:

$$\widehat{VaR_{t,L}} = \widehat{\mu_t} + st_\alpha(v,k)\,\widehat{\sigma_t} \tag{5a}$$

$$\widehat{VaR_{t,S}} = \widehat{\mu_t} + st_{1-\alpha}(v,k)\,\sigma_t \tag{5b}$$

C. Assessments of the VaR Estimations

The Kupiec [26] and the Dynamic Quintile (DQ) regression [26] tests were used to evaluate the accuracy of the VaR and ES estimations. The Kupiec test estimates the probability of observing a loss greater than the VaR amount. It is a likelihood ratio test (LR_{UC}) to examine whether the failure rate of the model is statistically equal to the expected one

D. Data and Preliminary Analysis

This study used daily data for the four highly capitalized CBCs, KLIMADAO (Klima), MOSS-CO2 (Moss), Save Planet Earth (SPE), and TOUCAN-carbon (Toucan), collected from coindesk.com. The time series covers the period from October 19th, 2019, to February 24th, 2025, yielding 1,225 observations. The daily logarithmic of price changes was used as follows: $r_t = 100\ln(P_t/P_{t-1})$. The first 725 observations were used for in-sample estimations, while the remaining 500 observations were reserved for out-of-sample estimations. Descriptive statistics are displayed in Table II. It can be observed that all CBC average returns are negatively signed. Klima and Moss exhibit the highest level of risk as measured by the standard deviation. All CBCs exhibit substantial deviations from the normal distribution. Klima and SPE are skewed to the left while Moss and Toucan are skewed to the right. The Jarque–Bera test strongly rejects the normal distribution.

TABLE II. DESCRIPTIVE STATISTICS

	Klima	**Toucan**	**Moss**	**SPE**
Descriptive statistics				
Mean	-0.685183	-0.184238	-0.322187	-0.187819
Median	-0.450345	-0.185276	-0.358768	-0.010642
Maximum	98.60107	48.13296	122.1477	36.69798
Minimum	-100.1107	-55.29208	-28.20125	-38.17215
Std. Dev.	9.468325	5.080644	6.429085	3.665382
Skewness	-0.080864	0.055178	6.974165	-0.805032
Kurtosis	44.38014	36.63672	123.0418	26.51108
Jarque-Bera	84404.18	55770.61	719884.3	27374.78
Probability	0.000000	0.000000	0.000000	0.000000
Unit root and stationarity tests				
ADF	-12.45***	-18.86***	-23.44***	-36.43***
ERS	0.307***	0.117***	0.001***	0.589***
KPSS	0.756***	0.155***	0.079***	0.487***

J-B refers to Jarque Bera normality test. KPSS is the Kwiatkowski–Phillips–Schmidt–Shin (KPSS) stationarity test. ADF is the augmented Dickey-Fuller test ERS is the Elliott, Rothenberg and Stock unit root test

These outcomes imply that it is more useful to consider another alternative distribution to account for these substantial deviations from the Gaussian distribution. The Ljung-Box statistic for the squared returns revealed that the assumption of white noise can be rejected, making the assumption that the CBCs' returns are autocorrelated. These outcomes are supported by Figure 1, reporting the time path of the CBC returns showing volatility clustering, and Figure 2, which reports the Quintile-Quintile (Q-Q) distributions against the Gaussian distribution. The bottom of the table shows that all CBCs are stationary at their first difference levels.

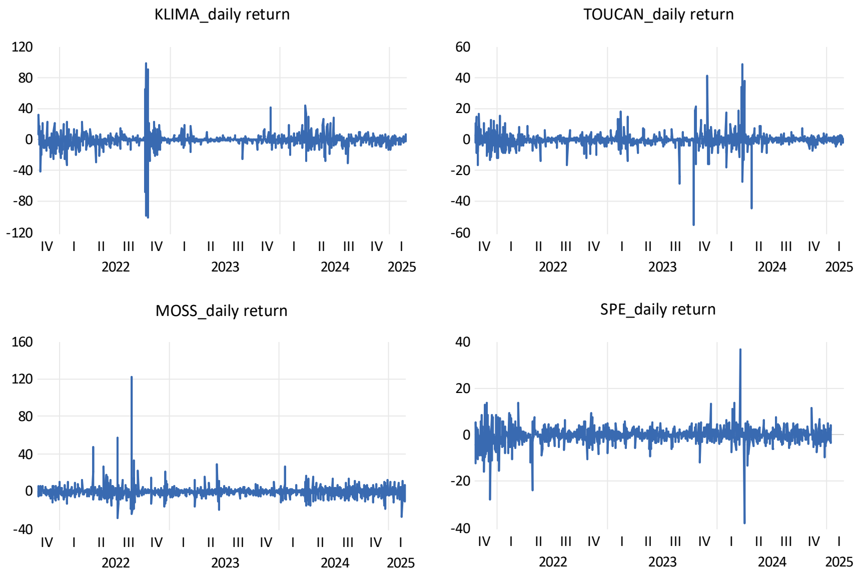

Fig. 1. CBC returns.

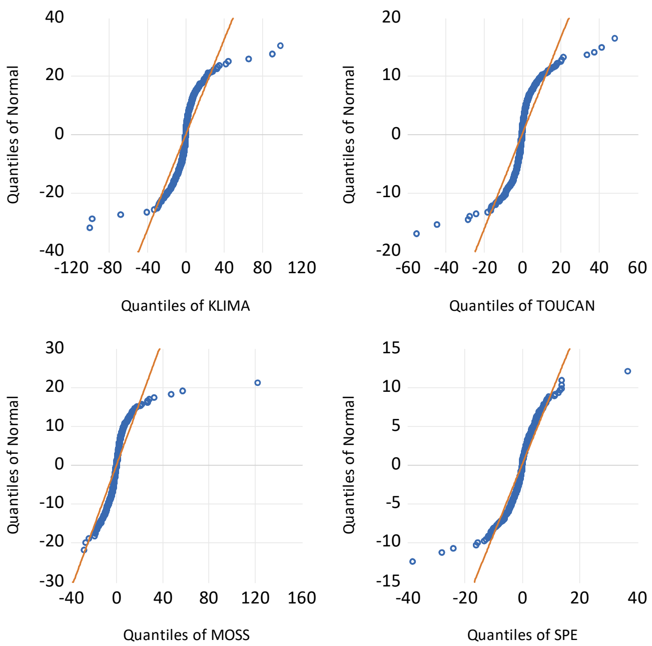

Fig. 2. Q-Q plots vs. normal distribution.

IV. EMPIRICAL RESULTS

A. The Estimating Results of the GARCH-class Models

Tables III and IV report the estimated results of the FIGARCH and FIEGARCH models under the normal and the skewed Student t-distributions of returns. The normal distribution is used as a reference. As a preliminary step, several long-memory tests were used to examine the presence of long memory in the CBC mean and volatility. The test results show that long memory is absent in the mean equation, while it is evidenced for volatility. The results show that the volatility of all CBCs is fractionally integrated. The FIGARCH model captures the long-memory phenomenon in all CBC returns' volatility. The fractional parameter is significant at the 1% level, confirming the existence of assertive persistence behavior. FIGARCH under skewed Student t-distribution performs better than the normal distribution, as shown in the log-likelihood values. All CBCs exhibit fait-tailed return distributions, as shown by the significance of the tail parameter. Moreover, all the CBCs except Toucan are asymmetric. Klima and SPE are skewed to the left, whereas Moss is skewed to the right. The results also show that the null hypothesis of a correct model specification cannot be rejected for all CBCs since the Ljung-box statistics and the Residual-Based Diagnostics (RBD test) confirm the absence of serial correlation and remaining ARCH effects in the 20^{th} lag of the squared residuals.

Overall, the FIGARCH model under skewed Student t-distribution performs well and can capture the main facts of long memory, asymmetry, and fat tails in the CBC return distributions. This result is in line with prior studies, showing the relevance of the long memory process in cryptocurrency volatility modeling and the estimation of risk using the VaR and ES methods [7-9, 12].

TABLE III. IN-SAMPLE FIGARCH MODEL ESTIMATIONS UNDER NORMAL AND SKEWED STUDENT T-DISTRIBUTIONS

	KLIMA		**TOUCAN**		**MOSS**		**SPE**	
	Normal	**Sk. St**	**Normal**	**Sk. St**	**Normal**	**Sk. St**	**Normal**	**Sk. St**
	Model estimations							
$AR(1)$	0.115** (2.03)	0.025** (2.01)	0.002** (2.00)	-0.221*** (3.02)	-0.286*** (-5.61)	-0.214*** (-6.88)	-0.055 (-1.11)	-0.097*** (-2.93)
$Cst\ (v)$	18.93** (2.20)	1.835** (2.19)	3.746*** (4.03)	6.326*** (5.94)	9.468 (5.51)	12.88*** (4.01)	2.809** (2.15)	6.70*** (3.41)
$d-figarch$	0.658*** (7.79)	0.564*** (6.71)	0.492*** (-5.143)	0.714*** (17.86)	0.999*** (50.29)	0.898*** (13.36)	0.337 (4.25)	0.537*** (6.86)
$Arch$	-0.325** (-1.91)	-0.991***	-0.958*** (-5.66)	0.280*** (6.91)	-0.754*** (-6.93)	-0.488*** (-3.87)	-0.673*** (-6.00)	-0.847*** (-34.93)
$Garch$	-0.386*** (-2.38)	-0.992*** (-3.23)	-0.961*** (-5.84)	0.496*** (6.07)	-0.022*** (-3.46)	-0.063*** (-6.11)	-0.631*** (-6.56)	-0.793 (-21.11)
$tail(v)$	-	2.96*** (3.01)	-	2.49*** (25.4)	-	2.727*** (16.88)	-	2.77*** (18.47)
$Asymmetry$	-	-0.026*** (-2.69)	-	-0.018 (-0.61)	-	0.024*** (3.62)	-	-0.019*** (-2.63)
$\ln(l)$	-4016.5	-3580.05	-3402.61	-2868.41	-3615.62	-3.268.87	-3054.76	-2.842.79
	Diagnostic tests							
AIC	6.61	6.00	4.88	5.76	5.88	5.53	5.17	4.81
Shibata	6.61	6.00	4.88	5.76	5.88	5.53	5.17	4.81
Hannan − Quinn	6.62	6.02	4.89	5.77	5.89	5.55	5.18	4.83
$Q^2(20)$	10.36 [0.91]	0.21 [0.99]	3.85 [0.92]	0.48 [0.99]	8.55 [0.96]	8.13 [0.97]	17.33 [0.95]	0.74 [0.99]
RBD(20)	13.04 [0.94]	11.21 [0.92]	4.39 [0.94]	4.41 [0.95]	8.58 [0.96]	8.43 [0.97]	6.65 [0.99]	6.54 [0.98]

TABLE IV. IN-SAMPLE FIEGARCH ESTIMATIONS UNDER NORMAL AND SKEWED STUDENT T-DISTRIBUTIONS

	Klima		**Toucan**		**Moss**		**SPE**	
	Normal	**Sk. St**	**Normal**	**Sk. St**	**Normal**	**Sk. St**	**Normal**	**Sk. St**
	Model estimations							
$AR(1)$	0.134*** (2.096)	0.014 (0.53)	0.073** (2.34)	0.052** (2.31)	-0.274*** (-5.38)	-0.221*** (-7.39)	-0.038 (-1.87)	0.020 (0.36)
$Cst\ (v)$	5.286* (6.32)	9.79*** (7.50)	3.40*** (10.10)	3.88*** (9.01)	3.97*** (10.51)	3.13*** (4.42)	4.51*** (12.37)	5.82*** (7.67)
$-figarch$	0.2218 (1.86)	0.711*** (6.45)	0.157*** (3.03)	0.172*** (4.01)	0.278*** (3.74)	-0.43*** (-3.75)	0.90*** (32.77)	0.73*** (5.17)
$Arch$	-0.717*** (-6.93)	-0.941*** (-7.41)	-0.431*** (-2.96)	-0.421*** (-3.11)	1.58*** (3.53)	0.51* (1.77)	-0.147* (-1.47)	-0.58** (-2.04)
$Garch$	0.888*** (11.64)	0.971*** (-7.14)	0.665*** (3.30)	0.569*** (4.41)	-0.170*** (-2.60)	-0.100*** (-2.88)	-0.68*** (-4.22)	0.203** (1.97)
$Egarch\ 1$	0.292* (1.90)	0.438*** (3.82)	0.193 (1.31)	0.121*** (2.36)	-0.056 (-1.30)	0.561*** (6.19)	0.161*** (3.31)	0.132** (1.70)
$Egarch\ 2$	0.86** (2.27)	1.49*** (4.94)	0.473*** (2.39)	0.441** (2.26)	0.536*** (5.79)	-0.01 (-0.46)	0.050* (1.76)	0.779*** (3.16)
$tail(v)$	-	2.04*** (6.84)	-	1.66*** (6.89)	-	2.96*** (11.58)	-	2.32*** (11.48)
$Asymmetry$	-	0.04*** (2.48)	-	0.01*** (3.48)	-	-0.01 (-0.46)	-	0.03 (1.47)
$\ln(l)$	-3980.481	-3613.242	-3526.035	-3486.11	-3578.308	-3391.788	-3031.114	-2827.64
	Diagnostic tests							
AIC	6.52	5.92	5.77	5.42	5.86	5.56	5.13	4.79
Shibata	6.52	5.92	5.77	5.42	5.86	5.56	5.13	4.79
Hannan − Quinn	6.53	5.94	5.79	5.44	5.87	5.57	5.15	4.81
$Q^2(20)$	36.15 [0.89]	0.274 [0.99]	2.26 [0.99]	2.21 [0.98]	5.40 [0.99]	6.06 [0.99]	12.61 [0.85]	1.901 [0.98]
RBD(20)	12.55 [0.93]	11.25 [0.92]	4. 99 [0.94]	4.49 [0.95]	8.01 [0.99]	8.43 [0.97]	6.69 [0.99]	6.44 [0.98]

$Q^2(20)$ refers to the Ljung-Box statistics for the residuals and squared residuals at the 20th lag, respectively.
AIC is the Akaike Information Criteria.
The figures between parentheses correspond to the t-students.
RBD(20) designates the RBD for conditional heteroscedasticity at the 20th lag.
l is the log likelihood value.

Table IV reports the FIEGARCH estimations under normal and skewed student t-distributions. These estimations show that the fractional integration parameter is significant for all the CBCs under normal and skewed Student t-distributions. All CBCs have positive long memory parameters, except Moss under the skewed Student-t, where the long memory parameter is negatively signed. This result implies that all CBCs exhibit a strong persistence in their volatility behavior. This result corroborates previous studies showing the significance of skewed-tailed distributions and their relevance when modeling volatility [7, 8]. The autoregressive parameter in the mean equation is significant. The parameters of the EGARCH components are significant for all CBCs, except for Toucan and Moss under the Gaussian distribution. The tail parameter is significant for all the estimated FIEGARCH models under skewed Student t-distributions. Moreover, two out of four (Klima and Toucan) cryptocurrency volatilities exhibit substantial asymmetry, as indicated by the significance of the asymmetry parameters. When looking at the log-likelihood values, it is observed that the FIEGARCH model under the skewed Student t-distribution outperforms its counterpart under normal distribution for all the CBCs. This result implies that accounting for long memory, non-Gaussianity, fat tails, and asymmetries in the CBCs return distributions substantially improves the accuracy of GARCH-class modeling of CBCs. Diagnostic tests reveal that the null hypothesis of a correct model specification for all CBCs cannot be accepted. The Ljung-box statistics and the RBD tests confirm the absence of serial correlation and remaining ARCH effects in the 20^{th} lag of the squared residuals. Overall, the FIGARCH model under a skewed Student t-distribution performs well and can capture the main stylized facts of long memory, asymmetry, and fat tails in the CBC return distributions.

B. Forecasting Performance Evaluation

This subsection evaluates the in-sample one-day-ahead forecasting performance of the FIGARCH and FIEGARCH models, considering both normal and skewed Student-t return distributions. The primary objective is to identify the best model for the underlying framework of VaR and ES estimations, both in-sample and out-of-sample. To achieve this, several criteria are employed, namely the Mean Squared Error (MSE), the Mean Absolute Prediction Error (MAPE), the logarithmic loss function, and the Mincer-Zarnowitz regression (M-Z) [27]. The M-Z is a commonly used method to assess forecast performance. It involves a linear regression that compares the actual realized volatility of the CBCs to their forecasts to analyze both the accuracy and reliability of the forecasting model in a one-day-ahead context. Table V presents the results of the in-sample one-day-ahead forecasting assessments. The results indicate that the FIEGARCH model utilizing the skewed Student's t-distribution outperforms its counterpart based on the normal distribution, as well as the FIGARCH model under both normal and skewed Student's t-distributions, for one-day-ahead forecasts. The logarithmic loss function is minimized for the FIEGARCH model on all CBCs. The M-Z regression results show that the highest coefficient of determination is achieved for all CBCs when the FIEGARCH model with skewed distributions is employed as the forecasting method. This conclusion is supported by the MSE and MAPE criteria, which also reveal minimized values corresponding to the FIEGARCH model with skewed distribution specifications. Based on these findings, the FIEGARCH model is well-suited for accounting for the key facts of CBC volatility and for generating day-ahead forecasts. The FIEGARCH model was applied under a skewed distribution to estimate one-day-ahead VaR and ES for both in-sample and out-of-sample periods.

TABLE V. ONE-DAY-AHEAD FORECASTING ASSESSMENTS

	α	β	R^2	MSE	MAPE	LL
FIGARCH under skewed Student-t distribution						
Klima	0.01	0.99	0.12	1.25	11.63	4.34
Toucan	0.04	0.86	0.07	14.32	14.62	5.76
Moss	0.67	1.03	0.13	95.3	32.21	7.12
SPE	0.44	0.97	0.08	6.62	29.02	6.35
FIEGARCH under skewed Student-t distribution						
Klima	0.02	1.01	0.14	1.22	11.22	4.30
Toucan	0.03	0.87	0.08	14.31	14.06	5.75
Moss	0.66	1.02	0.16	94.39	30.97	7.11
SPE	0.43	0.98	0.09	5.96	28.39	6.30

α and β refer to the intercept and the slope of the M-Z regression. R^2 is the coefficient of determination and *LL* designates the logarithmic loss function.

C. In-sample VaR and ES Estimations

Table VI shows the in-sample one-day ahead VaR and ES estimations for the FIEAGARCH model under normal and skewed Student t-distributions. VaR and ES are estimated for short- and long-term trading positions for five quintiles ranging from 0.95 to 0.9975. Shaded areas indicate the rejection of the null hypothesis at the given significance levels for the LRUC and DQ tests. It can be observed that the FIEGARCH model under the skewed Student t-distribution consistently outperforms its counterpart under the normal distribution. The null hypothesis is not rejected for almost all CBCs, quintiles, and trading positions, as indicated by the gray-shaded areas. This underscores the potential of the FIEAGARCH model under the skewed Student t-distribution to outperform the risk metrics model. The one-day ahead VaR and ES for short and long positions were estimated using the RM. To preserve space, the results are not reported here, but are available upon request to the corresponding author. These in-sample results highlight the practical relevance of the FIEGARCH model with skewed return distributions in generating accurate VaR and ES estimations for a one-day time horizon for short and long trading positions of carbon credit cryptocurrencies, providing valuable insights for finance and risk management practitioners.

D. The Out-of-Sample VaR and ES Estimations

Table VII presents the out-of-sample one-day-ahead VaR and ES estimations. It can be observed that the FIEGARCH model under the normal distribution performs poorly, particularly for the 0.995 and 0.9975 quantiles in both short- and long-term trading positions. This indicates that the model fails to account for extreme market events when assuming a normal distribution. However, it performs better for long trading positions than for short ones. The shaded areas in the table show a success rate of 50% for the model overall, while the success rate for long trading positions is 57.5%.

TABLE VI. ONE-DAY AHEAD IN-SAMPLE VAR AND ES ESTIMATIONS

	Klima			**Toucan**			**Moss**			**SPE**		
Model: AR(1)-FIGARCH(1,d.1)- Normal distribution												
Short trading position												
	LR_{UC}	**DQ**	**ES**	**LR_{UC}**	**DQ**	**ES**	**LR_{UC}**	**DQ**	**ES**	**LR_{UC}**	**DQ**	**ES**
0.95	4.92	6.52	18.56	8.17***	8.45	13.2	4.3**	8.11	16.9	0.94	13.8**	6.84
0.975	0.37	3.31	20.61	0.46	4.86	16.6	1.29	5.29	18.9	0.08	7.1	8.13
0.99	2.40	14.74	26.44	2.80*	15.1***	20.1	10.3**	18.5***	23.7	1.33	6.47	10.9
0.995	4.43	6.91	29.01	11.7***	35.1***	21.2	24.7**	52.9***	25.5	6.3**	21.7***	11.1
0.9975	9.85	17.20**	31.87	15.59***	56.1***	23.5	34.1**	91.5***	28.2	5.8***	9.37	13.3
Long trading position												
0.95	5.58***	6.52	-16.70	21.2***	18.4***	-15.8	4.92**	10.2	-11.2	4.46**	12.5***	-9.8
0.975	0.22	2.32	-18.25	5.38***	6.54	-18.9	0.62	7.97	-11.7	0.19	17.1***	-10.2
0.99	0.59	6.13	-19.62	0.06	8.02	-23.2	5.23**	21.7***	-13.6	7.0***	57.9***	-11.5
0.995	3.16	19.87**	-20.34	2.34	20.7***	-23.9	4.47**	21.8***	-14.6	13.8**	99.7***	-12.6
0.9975	7.57***	12.5***	-20.0	10.3**	52.4***	-23.8	9.8***	17.2***	-15.2	28.8**	27.33***	-12.4
Model: AR(1)-FIEGARCH(1,d.1) - Skewed Student distribution												
Short trading position												
0.95	0.25	9.08	14.94	0.08	9.60	1.83	0.02	4.30	14.0	1.64	12.4**	5.82
0.975	1.11	6.67	20.42	0.39	3.92	1.52	0.93	5.09	19.2	1.15	13.6**	8.22
0.99	0.004	0.53	25.98	1.41	1.44	1.35	1.67	6.70	29.3	2.33	2.12	14.7
0.995	0.002	0.11	25.03	0.32	2.61	1.26	5.8***	20.6**	35.9	1.76	1.46	20.4
0.9975	0.41	0.36	35.70	0.29	2.96	2.15	2.21	3.13	44.9	0.00	0.03	20.5
Long trading position												
0.95	1.59	9.84	-14.03	0.00	2.22	-9.67	0.02	0.64	-9.95	1.98	5.15	-7.1
0.975	0.06	16.6***	-14.70	0.08	10.8	-12.4	0.19	6.36	-11.4	0.06	19.3	-9.9
0.99	0.94	1.11	-21.40	0.37	6.30	-19.3	2.64	2.33	-15.1	1.33	69.7***	-12.3
0.995	0.21	0.29	-23.54	0.0001	0.16	-27.6	0.21	0.26	-15.2	0.15	0.23	-22.2
0.9975	0.002	0.03	-23.03	0.33	0.46	-35.2	0.41	0.37	-19.4	0.35	0.32	-31.1

TABLE VII. ONE-DAY AHEAD OUT-OF-SAMPLE VAR AND ES ESTIMATIONS

	Kilma			**Toucan**			**Moss**			**SPE**		
Model: AR(1)-FIEGARCH(1,d.1)- Normal distribution												
Short trading position												
	LR_{UC}	**DQ**	**ES**	**LR_{UC}**	**DQ**	**ES**	**LR_{UC}**	**DQ**	**ES**	**LR_{UC}**	**DQ**	**ES**
0.95	6.29***	10.1	22.2	7.86***	11.2*	13.9	9.6***	11.3*	18.3	0.94	11.7**	6.1
0.975	0.00	9.26	26.7	0.22	3.77	16.3	0.001	5.22	20.8	0.23	8.59	8.6
0.99	1.67	6.66	29.4	4.18**	14.51**	18.5	10.3***	18.7**	23.8	0.11	6.57	11.1
0.995	3.16*	4.80	34.9	13.0***	46.3***	20.1	15.2***	32.4**	27.5	2.34	3.58	12.9
0.9975	12.3***	22.4	34.9	17.8***	97.7***	22.5	27.2***	75* **	29.3	3.9**	6.04	15.3
Long trading position												
0.95	3.73*	5.10	-20.1	25.1***	21.2***	-14.9	4.92**	5.41	-10.3	1.94	10.3	-9.1
0.975	0.44	5.38	-25.3	4.26**	4.95	-16.1	0.01	2.51	-10.9	0.64	18.2**	-10.5
0.99	2.40	7.42	-20.4	0.004	0.62	-20.2	0.04	6.41	-13.9	12.7**	48.7***	-11.3
0.995	4.43**	6.91	-22.2	2.07	3.18	-21.5	2.07	20.2**	-14.8	16***	38.3***	-12.9
0.9975	12.3***	22.5	-22.9	5.52**	8.72	2.25	5.52**	8.7***	-16.1	28.1***	90.1***	-13.1
Model AR(1)-FIEGARCH(1,d.1) - Skewed Student distribution												
Short trading position												
0.95	0.13	10.23	2.15	0.57	11.7*	9.73	0.46	8.01	15.2	1.98	11.09***	6.0
0.975	0.37	8.95	1.96	0.62	3.11	13.2	0.37	6.09	19.9	1.62	3.48	8.9
0.99	0.04	0.76	2.01	0.04	0.79	16.1	1.07	6.27	29.3	5.08***	4.03	16.5
0.995	0.21	0.26	2.32	0.21	0.27	21.0	1.19	1.87	37.1	0.70	0.67	18.7
0.9975	0.0001	0.02	2.14	1.88	1.38	41.4	1.03	1.40	51.1	0.35	0.32	24.0
Long trading position												
0.95	5.04*	13.95**	-15.9	0.46	2.73	-9.91	0.08	1.63	-10.2	0.80	8.49	-7.8
0.975	0.62	5.97	-14.4	0.08	3.13	-12.1	0.06	5.58	-11.7	0.39	9.15	-10.4
0.99	0.007	1.11	-18.22	0.43	0.76	-21.2	2.67*	2.37	16.8	0.06	7.84	-14.8
0.995	0.004	0.29	-22.6	0.21	0.29	-30.8	0.83	0.77	-19.4	0.70	0.67	-21.6
0.9975	0.001	0.37	-19.62	0.96	0.37	-35.5	0.41	0.37	-22.7	1.74	1.3	-31.8

*, **, and *** refer to the significance at the 10, 5, and 1% levels respectively.
DQ designates the Dynamic Quantile τest. ES is the expected shortfall. LR_{UC} is the Kupiec test.

In contrast, the performance results change when the FIEGARCH model is evaluated under the skewed Student t-distribution. In this case, the model performs exceptionally well for all carbon credit cryptocurrencies for both short- and long-term trading positions. Using the LRUC and DQ tests, the model fails in three instances, yielding a success rate of 92.5%

for short positions and 97.5% for long trading positions. Overall, it can be concluded that the FIEGARCH model under the skewed Student t-distribution is highly effective for one-day ahead VaR and ES forecasting for all CBCs. By accounting for long memory in the volatility of CBCs and the fat-tailed nature of returns, the model significantly enhances its ability to assess downside risk.

V. CONCLUSIONS AND POLICY IMPLICATIONS

This study assesses the downside risk for CBCs using the value at risk and the expected shortfall. This study examined whether accounting for long memory and fat-tailed return distributions can improve the accuracy of risk assessment. Various long-memory GARCH-class models were estimated under normal and skewed Student t-distributions for CBC time series covering the period from October 2021 to February 2025. The forecasting and risk assessment analysis highlights some relevant outcomes. First, the FIEGARCH model under skewed Student t-distribution performed better than the normal distribution, confirming that accounting for fat-tails in the CBC returns and long memory in the volatility enhances the forecasting ability of the model. Second, the in-sample-VaR and ES estimations show that FIEGARCH under skewed Student distributions returns generates the most accurate one-day-ahead VaR and ES for all the CBCs, selected quintiles, and short and long trading positions, with a performance rate of 90% and 95% for short and long positions, respectively. Regarding out-of-sample forecasting, the model performed exceptionally well for all carbon credit cryptocurrencies in both short and long trading positions. The model failed in three instances, yielding a success rate of 92.5% for short positions and 97.5% for long trading positions.

The results offer several policy and managerial implications. First, the presence of a long-memory process in the CBC's behavior may shed light on the efficiency of the carbon credit cryptocurrency market. Second, CBC traders and portfolio managers must account for long memory in CBC behavior, which can help them forecast volatility and assess their portfolio risk. Accounting for fat-tailed returns and long memory of CBCs is essential in any similar forecasting exercise. Since they are tied to environmental sustainability, CBC risk managers are invited to integrate environmental risk and climate policy changes to improve the VaR and ES estimates. Furthermore, the use of alternative distributions that account for fat-tailed returns and asymmetry is highly recommended when exploring CBC volatility modeling and risk quantification. Finally, credit carbon regulators could implement transparency requirements to attenuate the risk of sharp price fluctuations and improve the VaR and ES quantification models. By requiring greater transparency in the market, regulators can help reduce the risk of sudden price changes and improve the accuracy of risk assessments, leading to a more stable and predictable market.

This research may be extended in several ways. Comparing CBCs with other traditional cryptocurrencies may offer new insight into the performance of long-term volatility modeling and cryptocurrency risk assessment. Intraday data may be helpful, since using long time series may increase the number of observations and strengthen the results by avoiding non-convergence in the estimation process. Estimating intraday VaR and ES may be helpful to portfolio managers when allocating and designing the risk of cryptocurrency portfolios. This could provide a more accurate and timely assessment of risk, leading to better risk management. Estimating the VaR and ES of the CBC for other forecast time horizons helps to examine how their performance is evolving. This can provide a more comprehensive view of the market and help make more accurate risk assessments. Finally, using other alternative distributions such as the Generalized Error Distribution (GED), the Gaussian Mixture Distribution (GMM), and the Extreme Value Theory (EVT) that consider deviations from the normal distribution may be another research avenue to explore. This could lead to more accurate risk assessments and better risk management strategies.

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