Application of Stochastically Balanced Model Reduction to Simplify LTI Models of Air Core Transformers

Nguyen Dang Khang
Hanoi University of Industry, Hanoi, Vietnam
khangnd@haui.edu.vn

Hoang Quoc Xuyen
Hanoi University of Industry, Hanoi, Vietnam
xuyen_hq@haui.edu.vn

Dang Dinh Chung
Hanoi University of Industry, Hanoi, Vietnam
chungdd@haui.edu.vn

Duong Quoc Tuan
Thai Nguyen University of Technology, Thai Nguyen, Vietnam
duongquoctuan-tdh@tnut.edu.vn (corresponding author)

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ABSTRACT

This study investigates the effectiveness of the Stochastically Balanced Model Reduction (SBMR) method for reducing the order of complex Linear Time-Invariant (LTI) systems, aiming to strike an optimal balance between model fidelity and computational complexity. To address this challenge, an SBMR procedure was developed by solving the Lyapunov and Riccati equations to determine the controllability and observability Gramian matrices. Subsequently, Singular Value Decomposition (SVD) was employed to extract Hankel singular values and construct projection matrices, thereby establishing a reduced-order state-space model. The experimental results obtained from a 10th-order air core transformer model reveal that the 5th-order reduced model exhibits an almost perfect match with the original system in terms of impulse response, magnitude, and phase across the entire frequency range, achieving an H∞ error of 5.795817×10^{-3}. In contrast, although the 4th-order model preserves the system characteristics in certain time and frequency intervals, it demonstrates significant deviations ($H\infty = 3.348487\times10^{-2}$) in other regions. Overall, the findings confirm the feasibility and effectiveness of SBMR in simplifying LTI systems while retaining essential dynamic properties, paving the way for its potential application in modern control systems with lower computational costs.

Keywords-stochastically balanced model reduction; LTI system; model reduction; air core transformer; minimum phase

I. INTRODUCTION

The modeling and identification of LTI systems are crucial for understanding system dynamics, optimizing performance, and enhancing control strategies. These systems serve as the foundation in various engineering applications, facilitating their effective analysis and design [1-3]. LTI system significance can be also underscored through system identification techniques [1, 2]. The latter are applied in control systems to accurately capture dynamic behavior, optimize characteristic parameters, and improve input–output reliability based on models [3]. Although LTI systems provide a robust framework for modeling, it is essential to consider the limitations imposed by real-world complexities, such as nonlinearity and time-varying properties, which may necessitate alternative modeling approaches [4].

The relationship between model accuracy, number of states or variables, and system complexity is multifaceted. As models become more accurate, they often require an increased number of variables to encapsulate system complexity, leading to higher overall complexity. The latter can manifest in various forms, including increased computational demands and challenges related to interpretability [5-7]. Conversely, while higher accuracy and complexity may enhance model fidelity, they can also impede its practical applications and understanding. Thus, a balance between model detail and usability is required.

Model Order Reduction (MOR) is a substantial technique for simplifying complex LTI systems while preserving their main characteristics. By reducing the model order, engineers can achieve faster simulations and more manageable designs, which is particularly advantageous in applications, such as control system design and grid integration [8–14]. MOR simplifies high-order models, facilitating easier analysis and simulation, enhancing computational efficiency, and preserving the intrinsic properties of the original system. MOR applications in LTI systems are varying, ranging from control system design and power system optimization to data-driven MOR approaches [8-14]. Despite the significant benefits of MOR, it is imperative to consider that excessive simplification may overlook critical issues, potentially leading to inaccuracies in system behavior under certain conditions. Balancing reduction and fidelity remains an ongoing challenge in this field.

SBMR is a model reduction technique designed to simplify large-scale LTI systems while maintaining essential dynamic characteristics. This method leverages controllability and observability Gramian matrices to identify and retain the most critical states of the system, thereby reducing its order. SBMR application in model reduction offers benefits across various domains, including circuit design, data assimilation, and enhanced computational efficiency [15-18].

Modeling air core transformers as high-order LTI systems involves the integration of various electrical and thermal characteristics via state-space models. These models account for component properties, such as resistance, capacitance, and inductance, which are important for analyzing transformer performance under diverse conditions [19-21]. Although such models provide valuable insights into transformer behavior, they are complicated by the intricacies of real-world applications, where environmental factors and material properties can significantly influence performance.

The present work deployed the SBMR technique [15-18] to reduce the order of the air core transformer model described in [19]. Subsequently, simulations and comparative analyses were conducted using MATLAB to assess the performance of the original high-order system versus the reduced-order model. By comparing the time-domain and frequency-domain responses between the original and reduced-order models (particularly the 5th-order model), the current paper demonstrates that the core dynamic characteristics, such as the minimum-phase property and stability, are effectively preserved while computational complexity is significantly reduced. These contributions enable selecting a reduced-order model that meets specific application requirements, facilitating control system efficiency and reliability improvement.

II. STOCHASTICALLY BALANCED MODEL REDUCTION ALGORITHM FOR LINEAR TIME-INVARIANT SYSTEMS

The SBMR algorithm for LTI systems is based on balancing the system's controllability and observability through the Gramian matrices, which are obtained by solving the Lyapunov and Riccati equations. Subsequently, SVD is deployed to extract the Hankel singular values, which are then used to determine the projection matrices that form the basis of the reduced-order system. The SBMR technique is implemented as follows [15-18]:

Algorithm input: A minimal LTI system, which is both minimum-phase and stable, represented by:

$$G(s):\begin{cases}\dot{x}(t)=Ax(t)+Bu(t)\\ y(t)=Cx(t)+Du(t)\end{cases}\Leftrightarrow$$

$$\Leftrightarrow G(s):=C(sI-A)^{-1}B+D \tag{1}$$

where $x(t)$ is the state vector, $u(t)$ is the input vector, $y(t)$ is the output vector, A is the state matrix, B is the input matrix, C is the output matrix, and D is the feed-through matrix.

Algorithm output: A reduced-order system that is also minimal, preserves the minimum-phase and stability properties, and is expressed as:

$$G_r(s):\begin{cases}\dot{x}_r(t)=A_rx_r(t)+B_ru(t)\\ y_r(t)=C_rx_r(t)+D_ru(t)\end{cases}\Leftrightarrow$$

$$\Leftrightarrow G_r(s):=C_r(sI-A_r)^{-1}B_r+D_r \tag{2}$$

where $x_r(t)$ is the reduced-order state vector, $y_r(t)$ is the output vector, and A_r, B_r, C_r, D_r are the state-space matrices for the reduced model.

The steps of the algorithm are:

1. Compute the controllability Gramian P by solving the Lyapunov equation:

$$AP+BB^T+PA^T=0 \tag{3}$$

2. Perform the Cholesky factorization of P as given in:

$$P=S^TS \tag{4}$$

3. Compute the observability Gramian Q by solving the Riccati equation:

$$A^TQ+\left(C-W^TQ\right)^T inv\left(DD^T\right)\left(C-W^TQ\right)+Q^TA=0 \tag{5}$$

where $W=PC^T+BD^T$

4. Perform the Cholesky factorization of Q as given in:

$$Q=R^TR \tag{6}$$

5. Apply SVD as shown in:

$$SR^T = UZV^T \tag{7}$$

where $Z = diag(\sigma_1, \sigma_2, \ldots, \sigma_n)$ with $\sigma_1 \ge \sigma_2 \ge \ldots \ge \sigma_n \ge 0$.

6. Select the desired reduced order r.
7. Construct the projection matrices as in:

$$T_{\text{left}} = Z_1^{-1/2} V_1^T R \tag{8}$$

$$T_{\text{right}} = S^T U_1 Z_1^{-1/2} \tag{9}$$

where $Z_1 \in \mathbb{R}^{r \times r}$ and U_1, V_1 are the corresponding sub-matrices of U and V.

8. Construct the reduced-order model according to:

$$A_r = T_{\text{left}} A T_{\text{right}}; B_r = T_{\text{left}} B; C_r = C T_{\text{right}}; D_r = D \tag{10}$$

III. MODEL ORDER REDUCTION OF AN AIR CORE TRANSFORMER USING STOCHASTICALLY BALANCED MODEL REDUCTION

Figure 1 represents the electrical circuit of an air core transformer.

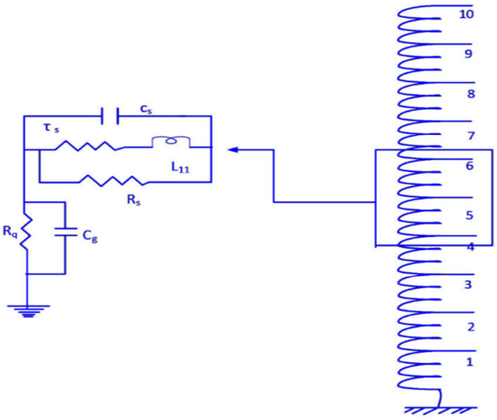

Fig. 1. Equivalent electrical circuit model for the air core transformer.

This device is modeled as a 10th-order LTI system based on component characteristics, such as resistance, capacitance, inductance, and other relevant electrical parameters [19]. The transfer function of the system is given by:

$$G(s) = \frac{Num(s)}{Den(s)} \tag{11}$$

where $Num(s)$ and $Den(s)$ are calculated by:

$$\begin{aligned} Num(s) &= s^9 + 510.1s^8 + 1.106\times10^5 s^7 + \\ &+1.33\times10^7 s^6 + 9.690\times10^8 s^5 + 4.393\times10^{10} s^4 + \\ &+1.223\times10^{12} s^3 + 1.980\times10^{13} s^2 + 1.652\times10^{14} s + \\ &+5.211\times10^{14} \end{aligned} \tag{12}$$

$$\begin{aligned} Den(s) &= s^{10} + 529.81s^9 + 1.202\times10^5 s^8 + \\ &+1.527\times10^7 s^7 + 1.191\times10^9 s^6 + 5.892\times10^{10} s^5 + \\ &+1.842\times10^{12} s^4 + 2.513\times10^{13} s^3 + 3.784\times10^{14} s^2 + \\ &+1.965\times10^{15} s + 3.330\times10^{15} \end{aligned} \tag{13}$$

The SBMR algorithm [15-18] was implemented in MATLAB, and the system order was subsequently reduced to 4th and 5th order. This process yielded reduced-order models characterized by corresponding state-space matrices and transfer functions.

4th-order reduced system:

$$\boldsymbol{A} = \begin{bmatrix} -4.724 & 4.611 & 0.4483 & -17.16 \\ -6.86 & 3.292 & -20.46 & 23.95 \\ 3.821 & 13.07 & -6.68 & 17.59 \\ -4.85 & -3.861 & 5.448 & -29.6 \end{bmatrix}$$

$$\boldsymbol{B} = \begin{bmatrix} 1.212 \\ 0.1527 \\ -0.6907 \\ 0.6859 \end{bmatrix}$$

$$\boldsymbol{C} = [0.7127 \quad -0.01172 \quad 0.3311 \quad 0.5348]$$

$$\boldsymbol{D} = 0$$

$$\boldsymbol{G}_4(s) = \frac{s^3 + 16.65s^2 + 571.5s + 2094}{s^4 + 37.72s^3 + 444.8s^2 + 9325s + 1.102\times10^4}$$

$$\boldsymbol{G}_4(s) = \frac{s^3 + 16.65s^2 + 571.5s + 2094}{s^4 + 37.72s^3 + 444.8s^2 + 9325s + 1.102\times10^4}$$

5th-order reduced system:

$$\boldsymbol{A} = \begin{bmatrix} -3.136 & 4.993 & -1.867 & 0.5433 & 0.693 \\ -8.96 & 2.787 & -17.4 & 0.5515 & 5.415 \\ 1.619 & 12.54 & -3.471 & -6.949 & 18.76 \\ -2.948 & -3.403 & 2.676 & -8.414 & -26.65 \\ 4.865 & 1.17 & -7.09 & 54.21 & -43.38 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 1.056 \\ 0.3583 \\ -0.4751 \\ 0.4997 \\ -0.4764 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 0.7127 & -0.01172 & 0.3311 & 0.5348 & -0.2969 \end{bmatrix}$$

$$\mathbf{D} = 0$$

$$\mathbf{G}_5(s) =$$

$$= \frac{s^4 + 38.36s^3 + 1669s^2 + 2.048\times10^4 s + 1.988\times10^5}{s^5 + 55.62s^4 + 2412s^3 + 2.417\times10^4 s^2 + 4.887\times10^5 s + 1.269\times10^6}$$

From the time-domain response plots displayed in Figure 2, the following observations can be made:

- In time intervals from 0 to 0.2 sec and from 2.8 sec onward, the 4th-order reduced system (red curve) closely tracks the original system (blue curve). However, between 0.2 and 2.8 sec, the 4th-order model exhibits discrepancies relative to the 10th-order system . Therefore, the 4th-order model may be considered a viable substitute for the original system in time-domain applications, where its behavior approximates that of the original, allowing for a more compact model.
- Across the entire time domain, the impulse response of the 5th-order reduced system (green curve) aligns with that of the 10th-order original system. Consequently, the 5th-order model is proposed as a replacement for the high-order model in time-domain applications, offering a simplified implementation and reduced computational cost.

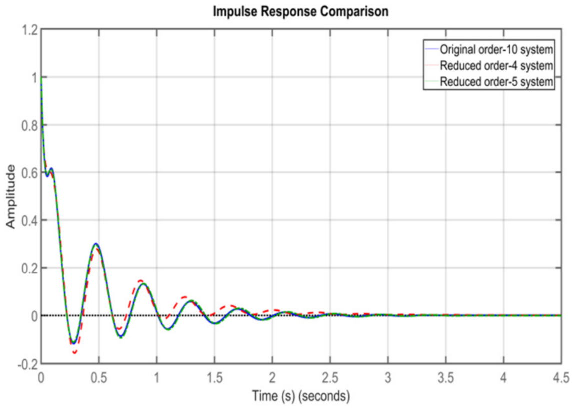

Fig. 2. Impulse responses of the original system and the 4th-order and 5th-order reduced systems.

The examination of the magnitude response plots (in dB) versus frequency (rad/s), as depicted in Figure 3, reveals that:

- For frequencies below 10^2 rad/s, the magnitude response of the 4th-order model deviates from that of the original system despite both exhibiting the same peak response. However, for frequencies above 10^2 rad/s, the responses of the 4th-order and 10th-order systems coincide. The 4th-order model may be utilized as a substitute for the original system in applications on magnitude responses at frequencies above 10^2 rad/s.
- Across the entire frequency range, the magnitude response of the 5th-order model closely matches that of the original system, indicating that this model is suitable for replacing the 10th-order system in magnitude-based frequency applications.

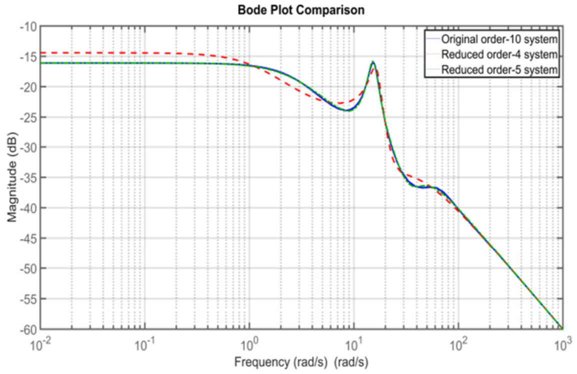

Fig. 3. Magnitude responses (dB) of the original system and the reduced-order systems (4th and 5th order).

The analysis of the phase response plots (in degrees) versus frequency (rad/s), as shown in Figure 4, demonstrates that:

- For frequencies below 10^{-1} rad/s and above 10^2 rad/s, the phase responses of the 4th-order and 10th-order systems are approximately similar. However, in the frequency range of 10^{-1} rad/s-10^2 rad/s, the phase responses differ between the 4th-order and 10th-order systems. Therefore, the 4th-order model may be considered as a substitute for the original system in phase response applications within the 10^{-1} rad/s-10^2 rad/s frequency band.
- Over the entire frequency range, the phase response of the 5th-order reduced system exactly matches that of the original system, suggesting that this model is preferable for replacing the high-order system in phase-based frequency applications.

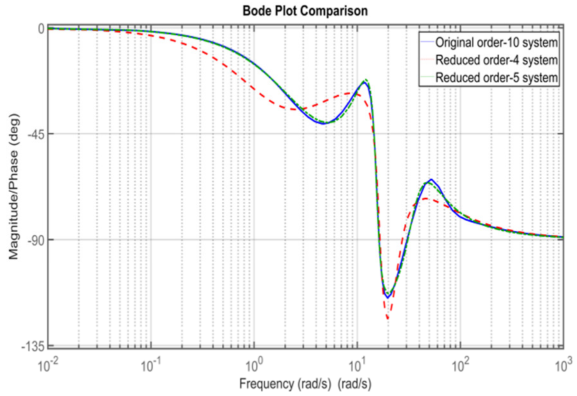

Fig. 4. Phase responses of the original system and the 4th-order and 5th-order reduced systems.

In terms of the H∞ reduction error, the 4th-order and 5th-order models exhibit errors of 3.348487×10^{-2} and 5.795817×10^{-3}, respectively, when compared to the 10th-order original system. This clearly demonstrates that the 5th-order model offers accuracy over the 4th-order model.

IV. GENERAL EVALUATION

The SBMR method offers several advantages in reducing complex LTI systems. It can preserve core dynamic characteristics, such as minimum-phase behavior and stability, thereby ensuring that the reduced-order model reflects the behavior of the original system. The results indicate that the reduced model, the 5th-order model, nearly replicates the time and frequency responses of the original system, demonstrating its potential for application in control systems.

SBMR also presents certain limitations. The process of solving the matrix equations to compute the Gramians requires complex computations, which can be challenging for large-scale or structurally intricate systems. The selection of the reduction order (r) is a sensitive factor, as an inappropriate choice may lead to significant discrepancies relative to the original system. In practical applications, for systems with nonlinear characteristics or those subject to external disturbances, SBMR performance may be constrained, necessitating further improvements and complementary approaches to enhance both the accuracy and applicability of the reduced-order model.

Overall, the research results demonstrate that the SBMR method effectively preserves the core dynamic characteristics, specifically, the minimum-phase property and stability of high-order LTI systems when applied to the air core transformer model. The 5th-order reduced model exhibits the closest dynamic behavior to the original system in both time and frequency domains, as well as in terms of $H\infty$ reduction error. Consequently, users may consider employing the 5th-order reduced model as a substitute for the original 10th-order system to obtain a more compact, lower-order model, which in turn leads to faster response, reduced computational complexity, and simplified system implementation.

V. CONCLUSION

In this study, the Stochastically Balanced Model Reduction (SBMR) technique for Linear Time-Invariant (LTI) systems modeled from air core transformers was implemented and its effectiveness was evaluated. By comparing the time-domain and frequency-domain responses between the original 10th-order model and the reduced-order models (4th and 5th order), the experimental results demonstrated that SBMR can preserve the essential dynamic characteristics of the system while significantly reducing computational complexity.

The 5th-order reduced model accurately replicates the impulse, magnitude, and phase responses of the original system across the entire frequency range, achieving a lower $H\infty$ reduction error compared to the 4th-order model. In contrast, although the 4th-order model performs adequately in certain time and frequency intervals, notable discrepancies in its response relative to the 10th-order system in specific time periods and frequency bands limit its applicability in high-accuracy scenarios.

These findings not only confirm the feasibility of using SBMR to simplify complex LTI systems, but also provide practical guidance for selecting an appropriate reduced-order model based on specific application requirements. Moreover, this research allows for integrating model reduction techniques with error quantification methods, contributing to enhanced computational efficiency and reliability in modern control systems.

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