Application of the Hankel-Norm Optimal Reduction Algorithm in Order Reduction of High-Order Linear Power Systems

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ABSTRACT

This study focuses on addressing the computational challenges posed by high-order linear power system models, specifically targeting the New England system model, which is characterized by 66 state variables. The primary objective is to reduce the model's order while retaining its essential dynamic characteristics. To achieve this objective, the Hankel-norm Optimal Reduction (HOR) algorithm is employed, involving the resolution of Lyapunov equations to compute controllability and observability Gramians, extraction of singular value matrices, and execution of structural transformations to determine the optimal reduced order. Evaluation through impulse response and Bode analyses indicates that reduced-order models of orders 10, 11, and 12 closely approximate the original system's behavior. Notably, the 12th-order model demonstrates superior accuracy, exhibiting H_∞ norm errors of approximately 2.03×10^{-4}, 1.39×10^{-4}, and 1.73×10^{-4} for the 10th, 11th, and 12th orders, respectively. These findings suggest that the HOR algorithm effectively simplifies the model's complexity while ensuring robust control performance and precise dynamic analysis. The results of this study have significant implications for real-time simulation and the integration of reduced-order models with adaptive control and data-driven optimization strategies in modern power grids.

Keywords-model order reduction; high-order power system; Hankel-norm optimal reduction; reduction error; stable LTI system; power quality disturbance classification; signal processing in smart grids

I. INTRODUCTION

High-order linear power systems, characterized by a multitude of state variables, epitomize complex electrical networks in which individual components are interwoven through nonlinear and interdependent mechanisms that extend beyond elementary, pairwise interactions [1, 2]. These systems are often modeled using hypergraph representations and related simplicial complex frameworks, which facilitate the depiction of inherently unstable dynamic behaviors, continuous variations in load and renewable energy contributions, and non-ideal factors, such as measurement delays and operational disturbances [3, 4]. These high-dimensional systems have practical applications in real-time load forecasting, fault diagnosis, and grid operation optimization. This necessitates an integrative approach that combines advanced forecasting methodologies, modern signal processing algorithms, and fuzzy control paradigms to enhance grid stability and operational efficiency escalating renewable energy integration [5, 6]. Innovative strategies, such as the deployment of the CNN-BiLSTM model for load forecasting and the implementation of delay-based control techniques, have been introduced to mitigate operational risks and improve system predictability in grid management [7, 8].

Nevertheless, there are still significant challenges in effectively operating and harnessing high-order power systems due to their inherent nonlinearity, parameter heterogeneity, and the complex nature of modeling dynamic behaviors [9, 10]. Technical obstacles related to accurate state estimation, effective chaos management, and reducing measurement delays underscore the need for synchronized implementation of sophisticated control strategies alongside robust optimization algorithms [11, 12]. Furthermore, the growing need for scalability, system compatibility, and reliable predictive models in rapidly changing operational environments highlights the importance of developing multi-level control solutions that integrate seamlessly with comprehensive risk management protocols [13, 14].

In summary, the emerging landscape of high-order power systems requires ongoing research and innovative methodologies to overcome existing technical barriers and ensure the adaptive and robust integration of these systems into modern grid infrastructures [15, 16]. Future research should aim to further elucidate the complex nonlinear interdependencies and advance adaptive control techniques to mitigate system disturbances and enhance operational reliability [17]. Ultimately, a rigorous, holistic approach will pave the way for a power grid that is resilient, efficient, and scalable, and that can meet future energy demands in a sustainable manner [18, 19].

The Hankel-norm Optimal Reduction (HOR) algorithm is regarded as a key tool in reducing the order of high-order power systems, as it can optimize the Hankel norm error while preserving the core dynamic characteristics of the system. By reducing the system order, the computational load is significantly reduced while the numerical accuracy and stability of simulating complex system behaviors are maintained. This facilitates the analysis and design of advanced control solutions for large-scale power systems [20, 21]. The application of the HOR algorithm is notably demonstrated in fields such as digital filter design and automatic control systems, where precise modeling of complex dynamic phenomena is required. Due to its ability to reduce model order without compromising the system's fundamental characteristics, HOR enables engineers and researchers to implement highly reliable control strategies. This enhances the operational performance and rapid response to disturbances in modern power grids [22, 23].

Recent studies have also indicated that the HOR algorithm outperforms traditional methods, such as Singular Value Decomposition (SVD)-based approaches or time-matching techniques, in balancing numerical accuracy and computational efficiency. Additionally, frequency-selected projection methods have shown high stability when applied to large-scale power systems, underscoring the role of HOR in optimizing the order reduction process for applications in integrated circuit design and system analysis [24].

Recognizing the complexity of high-order power systems with numerous state variables and the effective order reduction capability of the HOR algorithm, the authors have applied this method to the New England power system model [21, 24] in order to validate the efficiency of the order reduction process.

The main contribution of this paper is the empirical demonstration of the effective application of the pre-developed HOR algorithm on a real-world, high-order power system model from CEPEL (Brazil), a linear power system with a large number of state variables. Through analyses of impulse response, Bode plots, and error metrics, the study shows that HOR can reduce the system order from 66 to lower orders while preserving the dynamic characteristics of the original system and significantly reducing the computational load. The obtained results provide a solid empirical basis for applying order reduction techniques to optimize the analysis and design of control strategies, thereby contributing to enhanced operational efficiency in modern power grids.

II. HANKEL-NORM OPTIMAL REDUCTION ALGORITHM

HOR is a model order reduction technique designed to derive a lower-order representative model while preserving the essential dynamic characteristics of the original system. This method is based on solving Lyapunov equations to determine the controllability and observability Gramians, which are used to evaluate the contribution of each state via the Hankel Singular Values (HSV). Through state-space transformation and partitioning, HOR optimizes the approximation error in the Hankel norm between the original system and its reduced-order counterpart, ensuring stability and performance in both control and signal processing. The HOR method involves the following input and output specifications:

- Input: HOR requires a system modeled in state-space form (A, B, C, D) with order n , where the system is asymptotically stable and minimal.
- Output: The outcome of the reduction process is a new system of order r (with $r < n$) that guarantees stability and minimizes the Hankel norm approximation error relative to the original system.

The steps for implementing the HOR algorithm are as follows [20, 21]:

1. Solve the Lyapunov equations given in (1) and (2) to determine the corresponding controllability Gramian P and observability Gramian Q.

$$AP + BB^T + PA^T = 0 \quad (1)$$

$$A^\top Q + QA = -C^\top C \quad (2)$$

2. Determine the number of states to retain, i.e., select the reduced order r such that $r < n$. This selection is based on analyzing the HSV.
3. Transform the matrices P and Q into (3) to extract the matrix S containing the HSV, where each diagonal element of S is greater than the [threshold value].

$$P = Q = \begin{bmatrix} S & \\ & \partial_{r+1} I \end{bmatrix} \quad (3)$$

4. Transform the system A, B, C, D according to the structure given in (4).

$$[A, B, C, D] = \left[\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, [C_1 \quad C_2], D\right] \quad (4)$$

5. Determine the orthogonal matrix J and the transformation matrix K according to (5) and (6).

$$J = -inv(C_2^T)B_2 \quad (5)$$

$$A^{\top}Q + QA = -C^{\top}C \quad (6)$$

6. Construct the reduced-order model according to (7), (8), and (9).

$$A_r = K^{-1}\left(\partial_{r+1}{}^2 A_{11}^T + SA_{11}S_1 - \partial_{r+1}C_1^T J B_1^T\right); \quad (7)$$

$$B_r = K^{-1}(SB_1 + \partial_{r+1}C_1^T J); \quad (8)$$

$$C_r = C_1 S - \partial_{r+1} J B_1^T; D_r = D + \partial_{r+1} J \quad (9)$$

Through these steps, the HOR algorithm provides an effective tool for reducing system order, thereby simplifying complex models while maintaining the necessary accuracy and performance for practical applications.

III. APPLICATION OF THE HOR ALGORITHM FOR ORDER REDUCTION OF HIGH-ORDER POWER SYSTEMS

The power system model employed in this study is derived from a suite of benchmark systems that trace their origins to actual power grids, whose block diagram is depicted in Figure 1, with a significant number based on the Brazilian Interconnected Power System (BIPS) models. These benchmark systems, primarily developed by CEPEL, have become standard references for evaluating a wide range of numerical methods in power system analysis. Their complexity and realism make them particularly suitable for the evaluation of algorithms related to eigenvalue computation, including dominant modes, poles, zeros, stability margins, the identification of rightmost eigenvalues, and those with minimal damping ratios. Additionally, they are well-suited for sensitivity analysis of parameters and large-scale model order reduction, particularly in systems described by Differential-Algebraic Equations (DAEs).

A comprehensive description of these benchmarks can be found in [24], where their structure and characteristics are detailed. Furthermore, these models have been adopted as test cases for comparative studies on model order reduction techniques, as reported in [21]. In the context of this research, the primary objective is to rigorously assess the performance of the HOR method when applied to large-scale power system models. Specifically, this study aims to compare the reduced-order models obtained via this approach against reference reduced models. This will enable a thorough evaluation of the method's effectiveness.

The model reduction process for these high-order systems necessitates a careful balance among several critical factors: the degree of order reduction (with a preference for achieving the lowest possible order), the minimization of approximation error, and the preservation of the input-output behavior in both the time and frequency domains. To this end, extensive simulation studies were conducted across a range of reduced orders (from 1 up to 65), enabling a comprehensive exploration of the trade-offs involved. Based on these criteria and empirical results, three representative reduced orders, specifically orders 10, 11, and 12, were selected for detailed analysis and comparison with the original high-order system.

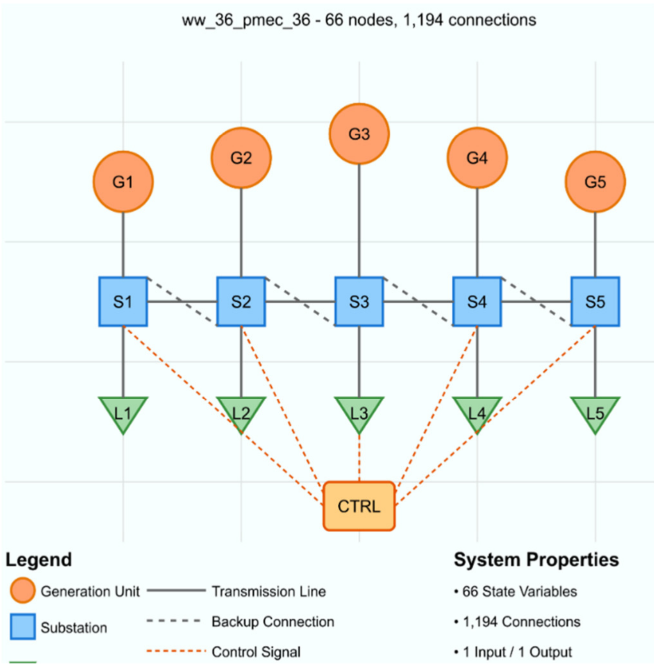

Fig. 1. Block diagram of the New England power system.

The main steps of our approach are as follows: The original 66th-order state-space model was constructed based on the standard New England system data. The HOR algorithm was set up in MATLAB. For each reduced order, the impulse and Bode responses were computed and the reduction error was quantified. All simulations were performed using MATLAB R2023a on a standard PC. The reduced models were then compared with the original system.

The HOR algorithm was implemented in Matlab to reduce the original 66th-order model to orders 10, 11, and 12, yielding impulse and Bode plots as shown in Figure 2 and Figure 3. The corresponding transfer functions of the reduced-order systems are given as follows:

$$G_{r1}(s) = \frac{\begin{array}{c} -0.0001195s^{10}+0.01913s^9+0.02076s^8 \\ +2.466s^7+2.817s^6+97.43s^5+83.1s^4 \\ +1138s^3+494.3s^2+1848s-0.0004417 \end{array}}{\begin{array}{c} s^{10}+2.226s^9+194.6s^8+307.4s^7+1.3\times10^4 s^6 \\ +1.326\times10^4 s^5+3.424\times10^5 s^4+1.767\times10^5 s^3 \\ +2.692\times10^6 s^2+1.761\times10^5 s-15.12 \end{array}} \quad (10)$$

$$G_{r2}(s) = \frac{\begin{array}{c} 0.0001073s^{11}+0.01775s^{10}+0.1894s^9+2.53s^8 \\ +21.93s^7+113.1s^6+805.7s^5+1652s^4+8800s^3 \\ +5025s^2+1.277\times10^4 s+0.01474 \end{array}}{\begin{array}{c} s^{11}+8.758s^{10}+209.4s^9+1584s^8+1.507\times10^4 s^7 \\ +9.894\times10^4 s^6+4.327\times10^5 s^5+2.443\times10^6 s^4 \\ +3.924\times10^6 s^3+1.813\times10^7 s^2 \\ +1.237\times10^6 s-106.1 \end{array}} \quad (11)$$

$$G_{r3}(s) = \frac{\begin{array}{c}8.398\times10^{-5}s^{12}+0.01915s^{11}+0.06664s^{10}+3.075s^{9}\\+7.649s^{8}+175.2s^{7}+321s^{6}+4192s^{5}+5255s^{4}\\+3.741\times10^{4}s^{3}+2.493\times10^{4}s^{2}+5.767\times10^{4}s+0.01265\end{array}}{\begin{array}{c}s^{12}+2.608s^{11}+225.7s^{10}+451.4s^{9}+1.901\times10^{4}s^{8}\\+2.784\times10^{4}s^{7}+7.409\times10^{5}s^{6}+7.225\times10^{5}s^{5}\\+1.311\times10^{7}s^{4}+6.756\times10^{6}s^{3}+8.112\times10^{7}s^{2}\\+5.526\times10^{6}s-474.3\end{array}} \quad (12)$$

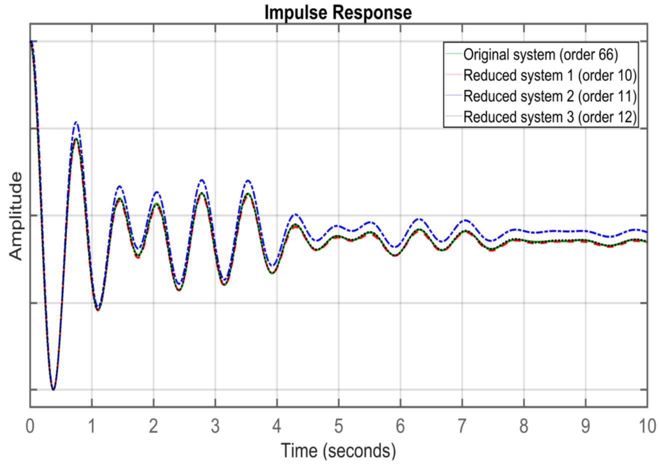

Fig. 2. Impulse responses of the original and reduced-order systems.

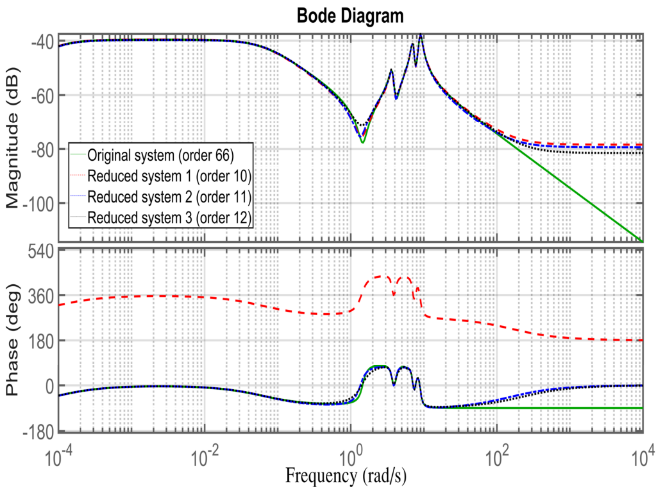

Fig. 3. Frequency responses of the original and reduced-order systems.

An examination of the impulse response plots in Figure 2 reveals the following observations:

- For time intervals less than 1 s, all three reduced-order models (orders 10, 11, and 12) closely match the original 66th-order system. However, for times exceeding 1 s, the 11th-order model diverges from the original, even at steady state.
- Over the entire simulation period, the impulse responses of the 10th-order and 12th-order models exhibited strong alignment with the original system, with the 12th-order model demonstrating a more precise match. Therefore, the 10th- or 12th-order reduced models may be considered viable substitutes for the more complex 66th-order model in time-domain applications.

As illustrated in Figure 3, the Bode plots reveal the following observations:

- In the magnitude response for frequencies below 10^2 rad/s, all three reduced-order models (orders 10, 11, and 12) closely follow the original system, with slight deviations in the range from 0.5 rad/s to 2.2 rad/s. For frequencies above 10^2 rad/s, the response of all three reduced models deviates from that of the original 66th-order system.
- In the phase response, the 10th-order model exhibits complete phase discrepancy relative to the original across the entire frequency range. For frequencies below 20 rad/s, the 11th- and 12th-order models approximate the original system; however, for frequencies above 20 rad/s, their phase responses deviate entirely from that of the original.
- Therefore, the reduced models of orders 10, 11, and 12 can be considered as alternatives to the original model in frequency-domain applications where the responses are approximately similar, thereby providing a simpler computational model compared to the high-order system.

The evaluation of the reduced-order models is based not only on direct responses (Bode, impulse) but also on the error relative to the original model. The error plots in both the frequency domain (gain, phase) and the time domain (impulse response error) are analyzed in Figures 4, 5, and 6 to provide a comprehensive assessment of performance.

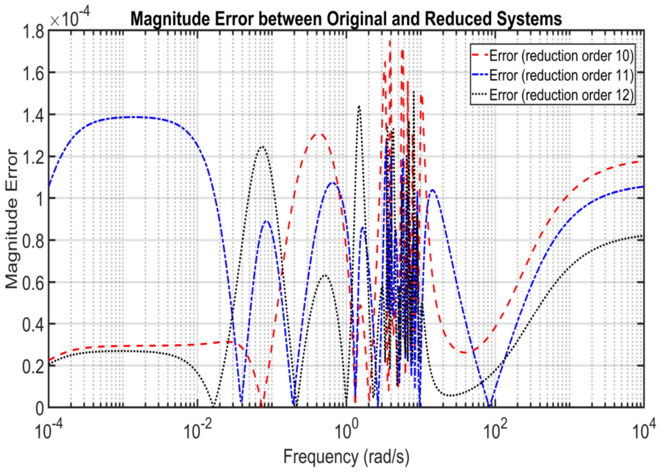

Fig. 4. Magnitude error plots between the original and reduced-order systems.

As illustrated in Figure 4, the magnitude error of the 10th-order model oscillates at a higher level compared to those of the 11th- and 12th-order models. This finding suggests that the 10th-order model has omitted some critical dynamic components, particularly in frequency ranges exhibiting resonance or high-order oscillations. The 11th- and 12th-order models show lower magnitude errors across most of the frequency band, with the 12th-order model exhibiting the lowest error.

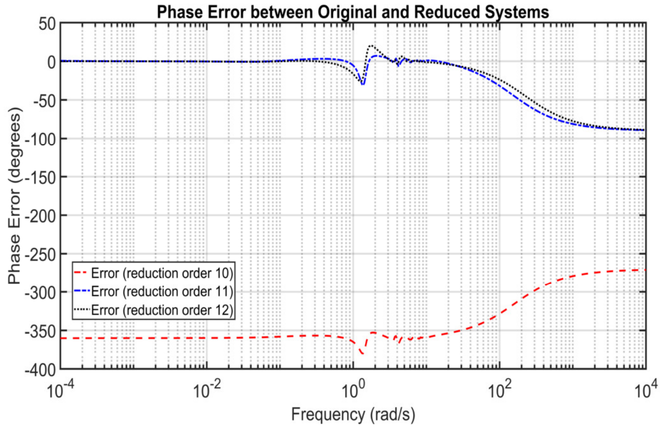

Fig. 5. Phase error plots between the original and reduced-order systems.

Figure 5 reveals that the phase discrepancies between the original and reduced-order models are more pronounced in frequency regions with strong oscillatory characteristics. The 10th-order model exhibits significant phase errors at higher frequencies, followed by the 11th-order model, whereas the 12th-order model displays the smallest phase error.

Fig. 6. Time-domain error plots between the original and reduced-order systems.

In the time domain, the impulse response error plot in Figure 6 reflects the degree of matching in oscillation amplitudes when the system is subjected to an impulse stimulus. Across the entire time interval, the 11th-order model demonstrates the highest error, whereas the 12th-order model exhibits the smallest discrepancy.

The reduction errors measured in the H_∞ norm between the original system and the reduced models for orders 10, 11, and 12 are 2.034786×10^{-4}, 1.388157×10^{-4}, and 1.726308×10^{-4}, respectively.

Overall, the evaluation results demonstrate that reducing the order of the New England power system from 66 to lower orders using the HOR algorithm effectively preserves the core dynamic characteristics of the original system while significantly reducing computational load. Despite the presence of varying degrees of approximation in different time and frequency domains, all reduced-order models exhibit potential as viable substitutes for the original, with the optimal model (particularly the 12th-order) distinguished by its low overall error and high fidelity. Thus, the application of the HOR algorithm not only simplifies the model structure but also ensures the accuracy and stability required for control and analysis in modern power grids. Through rigorous simulation and comparison, this study demonstrates that the HOR algorithm provides reduced-order models for the New England power system that closely preserve the original dynamics. The results highlight the suitability of HOR for practical power system analysis and control, offering a balance between model simplicity and accuracy.

IV. CONCLUSION

In this study, we implemented the Hankel-norm Optimal Reduction (HOR) algorithm to perform model order reduction on a 66-state variable New England power system model. Our investigation demonstrates that the HOR algorithm substantially decreases computational complexity while reliably preserving the system's essential dynamic and frequency-domain characteristics. A thorough evaluation, encompassing impulse response analysis and Bode plot comparisons, reveals that reduced-order models, particularly those of 10th and 12th order, maintain a high level of accuracy relative to the original model. Notably, although lower-order approximations offer excellent short-term fidelity, slight deviations become apparent in some intermediate-order models, underscoring the necessity of carefully balancing reduction depth with accuracy.

The practical implications of these findings are significant for the simulation, analysis, and control design of large-scale power systems. By enabling rapid computational performance without compromising critical dynamic features, the HOR algorithm not only streamlines simulation tasks but also facilitates the development of advanced control strategies. As modern power grids become increasingly complex, such scalable model reduction techniques are essential for real-time monitoring, fault diagnosis, and optimization of operational performance.

Looking ahead, future research can build on these results by integrating the HOR framework with other optimization methodologies. For instance, combining data-driven model order reduction techniques or incorporating adaptive control algorithms could further enhance both the robustness and efficiency of reduced models. Furthermore, the extension of the HOR approach to nonlinear systems or systems characterized by time-varying parameters is a promising direction that could broaden the algorithm's applicability to a wider array of real-world scenarios. These avenues hold considerable potential for advancing model reduction theory and practice, ultimately contributing to the improved stability and efficiency of modern power systems.

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