# Model Order Reduction of a DC Motor System with a Buck Converter via a Gramians-Based Truncation Algorithm

**Phung Thi Anh Vu**
Hanoi University of Industry, Hanoi, Vietnam
phung.vu@haui.edu.vn

**Nguyen Viet Anh**
Hanoi University of Industry, Hanoi, Vietnam
anhnv@haui.edu.vn

**Nguyen Dang Khang**
Hanoi University of Industry, Hanoi, Vietnam
khangnd@haui.edu.vn

**Le Thi Quynh Trang**
Thai Nguyen University of Technology, Thai Nguyen, Vietnam
lethiquynhtrang@tnut.edu.vn (corresponding author)

*Received: 22 March 2025 | Revised: 22 April 2025 and 6 May 2025 | Accepted: 10 May 2025*



## ABSTRACT

**This paper presents a study on the feasibility of reducing the model order of a DC motor system controlled by a high-order DC-DC buck converter, with the objective of minimizing the complexity of the original model while preserving its essential dynamic characteristics for precise controller design. The primary goal is to develop and apply the Gramians-Based Truncation (GBT) algorithm to reduce the original fourth-order system to third-order, second-order, and first-order models. By computing the controllability and observability Gramians through the solution of Lyapunov equations, the authors perform a system balancing procedure based on singular values to identify and eliminate states with negligible contributions. The implementation of the GBT algorithm in Matlab yielded $H_\infty$ norm reduction errors of 12.134639 and 12.135958 for the third-order and second-order models, respectively. These results demonstrate the capability of these reduced models to preserve the time-domain and frequency-domain response characteristics of the original system. In contrast, the first-order model exhibits a substantially higher error (410.183959) and fails to maintain consistency in the input–output response, particularly during the startup phase and in applications requiring accurate phase signal processing. The findings confirm the viability of the GBT method in simplifying complex dynamic models and underscore the importance of selecting an appropriate reduction order to balance model accuracy with implementation simplicity in engineering applications. These results pave the way for further research on improving reduction techniques to optimize phase information preservation and better meet the demands of modern control systems.**

***Keywords-buck converter; electro-mechanical system; model reduction; Gramians-Based Truncation (GBT); dynamic response analysis***

## I. INTRODUCTION

The high-order DC motor system, which is controlled by a DC-DC buck converter encompassing numerous state variables, represents an advanced architecture. In this architecture, the buck converter steps down the high input voltage to a level suitable for powering the motor. This mechanism not only ensures efficient energy transmission, but also facilitates precise control of parameters such as voltage, current, and motor speed. An effective voltage conversion process optimizes system performance in high-demand applications, such as electric vehicles and renewable energy systems [1, 2].

In the context of modern industrial applications, the buck converter's voltage conversion capability is highly valued for its ability to maintain optimal energy efficiency while supporting advanced control strategies. However, the presence of multiple state variables increases the complexity of system modeling and controller design. This necessitates the use of nonlinear control methods, such as adaptive fuzzy sliding mode control, to guarantee stability and a rapid response under all operating conditions [3]. Additionally, phenomena such as parasitic effects induced by leakage capacitance and self-inductance contribute to switching losses and impact system reliability [4].

From an application standpoint, this system is widely implemented in areas requiring precise speed and torque control. In electric vehicles, for instance, employing a buck converter not only enables effective voltage regulation but also facilitates motor drive control, thereby enhancing both performance and durability of the drive system [5]. Moreover, in renewable energy applications, the efficient conversion from high-voltage DC to low-voltage levels optimizes energy transformation processes, meeting the stringent requirements of modern systems [2].

However, integrating multiple state variables in a control system introduces significant challenges. A major concern is the parasitic effect, wherein auxiliary components such as leakage capacitance and self-inductance may increase switching losses, leading to ripples in current and torque and consequently degrading control quality [4]. Furthermore, the inherent complexity of multi-state control demands sophisticated strategies such as sliding mode control and model-based control approaches to minimize delays, saturation during duty cycles, and issues related to state trajectory convergence [6, 7]. Some studies have proposed the use of cascaded converter architectures to mitigate ripples and electromagnetic interference, thereby enabling smoother motor operation under the influence of nonlinear state variables [8]. Moreover, the system's robustness against disturbances and load variations poses a significant challenge for control implementation. Advanced control strategies, including those based on adaptive fuzzy sliding mode techniques, have been explored to ensure system stability in complex operating environments. The integration of traditional and modern control models not only improves control performance but also opens new research directions for optimizing switching processes and managing the system's dynamic behavior [9, 10]. Thus, the high-order DC motor system controlled by a DC-DC buck converter with multiple state variables offers clear advantages in energy efficiency and precise control; however, it also presents considerable challenges in design and control. Contemporary studies indicate that, to fully exploit the system's potential, it is essential to further develop advanced control strategies while simultaneously improving system reliability and interference rejection under all operating conditions.

Reduced-Order Models (ROMs) play a crucial role in simplifying high-order DC motor systems controlled by DC-DC buck converters with numerous state variables, thereby facilitating analysis, simulation, and more effective controller design. In modern systems with large numbers of state variables, such as voltage, current, and speed, ROMs help to condense the core dynamic characteristics into lower-dimensional representations. This significantly reduces computational complexity while preserving accuracy and rapid response in real-time applications [11]. The simplification of system models enables engineers to design advanced control strategies without being hindered by the complexity of the original model. Specifically, techniques such as the Second Order Arnoldi (SOAR) method with stopping criteria and reduction order selection ensure that essential system characteristics—such as dynamic response, stability, and inter-variable relationships—are fully retained [11]. Simultaneously, the application of truncation methods, such as the Linear Quadratic Gaussian (LQG) balancing algorithm, has proven effective in maintaining the system's core dynamic behavior while minimizing undesired effects from ancillary variables. This, in turn, enhances controller performance and durability [12].

Several studies [1, 2, 4, 7, 8, 13, 14] have elucidated the increasing complexity associated with a growing number of state variables and underscored the need for effective methods to simplify models, ensuring precise and stable system control. These investigations illustrate the technical context in which ROMs become indispensable tools for reducing computational load and supporting real-time control applications in fields such as electric vehicles and renewable energy. In summary, the objective of model order reduction in high-order DC motor systems controlled by DC-DC buck converters is to produce a simplified yet comprehensive representation of the system's core characteristics. This not only enhances simulation efficiency and reduces computational demands but also facilitates the design of controllers capable of rapid, stable, and effective performance under complex operating conditions. The integration of model reduction techniques such as the SOAR method with LQG balancing algorithms has demonstrated the potential of ROMs in optimizing the performance of DC motor systems, thereby opening new avenues for research and development in this field [11, 12].

The Gramians-Based Truncation (GBT) algorithm is a model order reduction technique designed to simplify complex systems while preserving essential input–output behavior characteristics. The theoretical foundation of GBT is based on solving Lyapunov equations to determine the Gramians. From these equations, dominant subspaces are extracted and states with negligible impact on overall system performance are eliminated. Through this mechanism, GBT not only guarantees the accuracy of the ROM but also significantly reduces computational costs in electrical and dynamic system applications [15]. Moreover, integrating GBT with optimization algorithms such as Moth Flame Optimization (MFO) has enhanced the reduction efficiency of complex electrical systems, ensuring minimal state errors and adherence to time–frequency domain criteria [12]. Additional studies have demonstrated the applicability of GBT in reducing the order of high-order linear systems, particularly for systems with second-order outputs, thereby improving simulation efficiency over specified time or frequency ranges [16]. Concurrently, the application of GBT in descriptor systems has been comprehensively investigated, emphasizing the role of subspace identification based on Gramians in preserving essential input–output behavior following model order reduction [13]. Finally, surveys on the quality of the GBT method have established global error bounds through the use of approximate Gramians, thereby reinforcing the reliability of large-scale dynamic models in numerical simulations [14].

Recognizing the application of the DC-DC buck power converter-driven DC motor system, its inherent complexity, and the significant roles of ROMs in general—and GBT in particular—the authors implemented the GBT algorithm in Matlab to reduce the order of the dynamic model presented in [17]. Subsequent simulations compared the responses of the original and ROMs, leading to assessments and evaluations regarding the performance of GBT in simplifying high-order models.

## II. GRAMIANS-BASED TRUNCATION MODEL ORDER REDUCTION ALGORITHM

GBT is a model order reduction technique for Linear Time-Invariant (LTI) systems that leverages the analysis of Gramian matrices to identify and remove states with low contributions to controllability and observability. The algorithm is implemented as follows [15]:

- Input: The original system is characterized by its state-space matrices $A$, $B$, $C$, and $D$, or by the transfer function $G(s)$. The controllability Gramian ($W_{\mathrm{n}}$) and observability Gramian ($W_{\mathrm{o}}$) are computed via Lyapunov equations, which quantify the ease of controlling and observing the system states.
- Output: The ROM, defined by matrices $A_{\mathrm{r}}$, $B_{\mathrm{r}}$, $C_{\mathrm{r}}$, $D_{\mathrm{r}}$, is obtained such that its output response closely approximates that of the original system. The reduction error is governed by the singular values of the product $W_{\mathrm{n}}W_{\mathrm{o}}$.

The solution to the Lyapunov equation is used to determine the controllability Gramian $W_{\mathrm{n}}$, satisfying (1).

$$AW_{\mathrm{n}} + W_{\mathrm{n}}A^{\mathrm{T}} + BB^{\mathrm{T}} = 0 \quad (1)$$

Similarly, the Lyapunov equation must be solved to obtain the observability Gramian $W_{\mathrm{o}}$, satisfying (2).

$$A^{\mathrm{T}}W_{\mathrm{o}} + W_{\mathrm{o}}A + C^{\mathrm{T}}C = 0 \quad (2)$$

These Gramians reveal the relative ease with which each state can be controlled and observed. The following are the steps for implementation:

- Step 1: Solve the Lyapunov equations to determine $W_{\mathrm{n}}$ and $W_{\mathrm{o}}$.
- Step 2: Perform a transformation to diagonalize $W_{\mathrm{n}}$ and $W_{\mathrm{o}}$ uniformly, thereby extracting the singular values.
- Step 3: Sort the singular values in descending order and select the reduction order, $r$, based on a predefined acceptable error threshold.
- Step 4: Eliminate the states corresponding to low singular values while retaining those with significant contributions.
- Step 5: Construct the ROM with matrices $A_{\mathrm{r}}$, $B_{\mathrm{r}}$, $C_{\mathrm{r}}$, $D_{\mathrm{r}}$, and compare its output response with that of the original system to ensure the desired performance.

GBT offers the advantage of achieving a low reduction error while preserving the inherent stability of the original system in the reduced model. This feature is crucial for maintaining robust performance in control applications. However, GBT's practical implementation is limited by its significant computational burden. Specifically, GBT requires solving Lyapunov equations, performing singular value decompositions, and executing multiple matrix multiplications. These operations can become computationally expensive, especially for large-scale systems.

## III. MODEL ORDER REDUCTION FOR A DC-DC BUCK POWER CONVERTER-DRIVEN DC MOTOR SYSTEM USING GBT

Consider the DC-DC buck power converter-driven DC motor system depicted in Figure 1 [17]. In this system, there are two main components: the converter, which modulates the output voltage from a DC source through PWM duty-cycle control, and the motor, which is powered via the converter and characterized by its electromechanical parameters, such as the armature current and the rotor's angular speed.

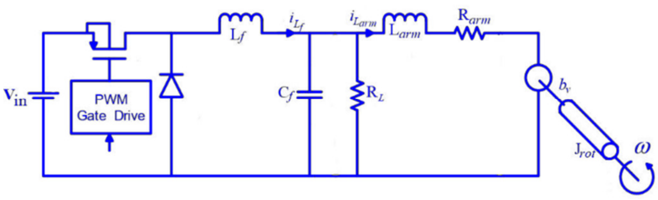


Fig. 1. Block diagram of the DC motor system with a buck DC-DC converter.

The system parameters are set as follows: the input voltage is $V_{in} = 40\,V$, which supplies power to the entire circuit; the converter employs a filter inductor with an inductance $L_f = 10\,mH$ (i.e., $0.01\,H$) in combination with a filter capacitor $C_f = 1000\,\mu F$ (i.e., $0.001\,F$) and a load resistor $R_L = 250\,\Omega$ to ensure stable voltage filtering; the DC motor is defined by an armature with inductance $L_{arm} = 2\,mH$ (i.e., $0.002\,H$) and resistance $R_{arm} = 1.45\,\Omega$, along with other key motor parameters including the torque constant $k_t = 0.0699$ and the back-EMF constant $k_{em} = 0.069$. The rotor possesses a moment of inertia $J_{rot} = 32.5 \times 10^{-6}$ and a viscous friction coefficient $b_v = 65.12 \times 10^{-6}$. These values collectively determine the system's dynamic characteristics and directly influence its control performance.

The continuous-time dynamic model (without disturbances) is described by the state-space (1), where the state vector $x = [\xi 1, \xi 2, \xi 3, \xi 4]^T$ consists of the following: $\xi 1$ representing the current through the converter's filter inductor, $\xi 2$ denoting the converter's output voltage, $\xi 3$ corresponding to the motor's armature current, and $\xi 4$ corresponding to the rotor's angular speed. In this formulation, $u(t)$ is the control input (duty ratio) and $y(t)$ is the measured output, which is given by the rotor's angular speed $\xi 4$. The system matrices are given by:

$$A = \begin{bmatrix} 0 & -\frac{1}{L_f} & 0 & 0 \\ \frac{1}{C_f} & -\frac{1}{C_f R_L} & -\frac{1}{C_f} & 0 \\ 0 & \frac{1}{L_{arm}} & -\frac{R_{arm}}{L_{arm}} & -\frac{k_{em}}{L_{arm}} \\ 0 & 0 & \frac{k_t}{J_{rot}} & -\frac{b_v}{J_{rot}} \end{bmatrix}; B = \begin{bmatrix} \frac{V_{in}}{L_f} \\ 0 \\ 0 \\ 0 \end{bmatrix}; \quad (3)$$

$$C = [0 \quad 0 \quad 0 \quad 1]$$

Substituting the characteristic parameters into the electromechanical system yields the fourth-order transfer function of the original system [17].

$$G(s) = \frac{4.302\times10^{12}}{s^4+731s^3+6.795\times10^5 s^2+7.401\times10^7 s+7.662\times10^9} \quad (4)$$

By implementing the GBT algorithm on this electromechanical system [17] in Matlab, the system order was reduced to third-order, second-order, and first-order models, with the corresponding $H_\infty$ norm reduction errors being 12.134639, 12.135958, and 410.183959, respectively. It is observed that the reduction errors for the third-order and second-order models are nearly identical, whereas the error for the first-order model is significantly larger. The third-order, second-order, and first-order models are, respectively:

$$G_{r3}(s) = \frac{-12.13s^3+4.35\times10^5 s^2-4.1\times10^9 s+3.569\times10^{12}}{s^3+4.914\times10^5 s^2+5.425\times10^7 s+6.357\times10^9} \quad (5)$$

$$G_{r2}(s) = \frac{0.894s^2-8359s+7.265\times10^6}{s^2+110.4s+1.294\times10^4} \quad (6)$$

$$G_{r1}(s) = \frac{-397.7s+8.625\times10^4}{s+153.6} \quad (7)$$

To compare the input–output response characteristics between the original system and the ROMs, the authors constructed step response and Bode plots as illustrated in Figures 2, 3, and 4.

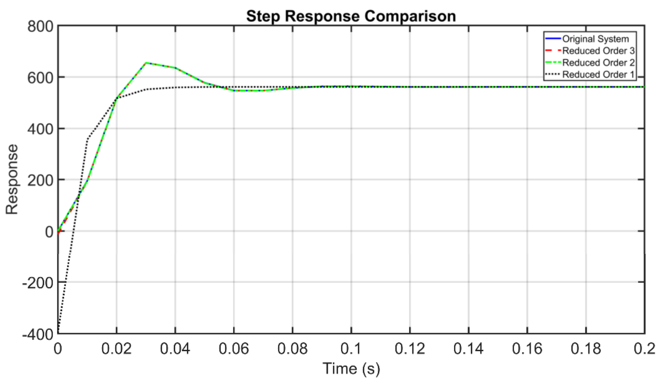


Fig. 2. Step response between the original model and the reduced models.

As illustrated in Figure 2, the step response indicates the following:

- For time intervals less than 0.08 s, the first-order model exhibits deviations from the original system. However, from 0.08 s onward, the responses converge. Hence, the first-order model can be considered for applications in the time domain beyond 0.08 s.
- Over the entire time domain, the step responses of the second-order and third-order models coincide with that of the original system, suggesting that either of these reduced models can serve as a simplified replacement for the higher-order system.

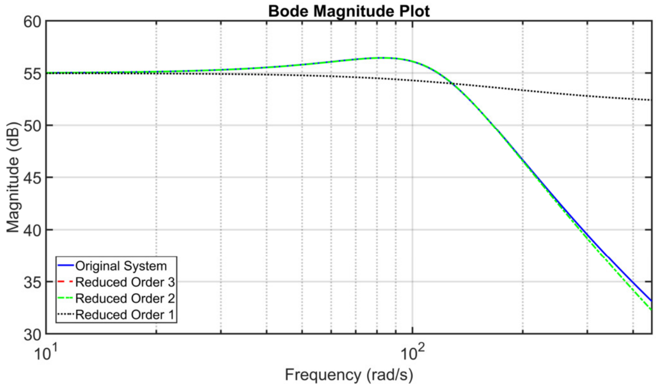


Fig. 3. Magnitude response between the original model and the reduced models.

An analysis of the magnitude response across the frequency spectrum in Figure 3 reveals that:

- At frequencies below 15 rad/s, the first-order model closely approximates the original fourth-order system. However, at frequencies above 15 rad/s, the first-order response diverges significantly, implying that the first-order model is only applicable in low-frequency scenarios.
- Within the simulated frequency range, especially below 222 rad/s, the magnitude responses of the second- and third-order models align closely with the original system, indicating that these reduced models can be employed in frequency-domain applications.

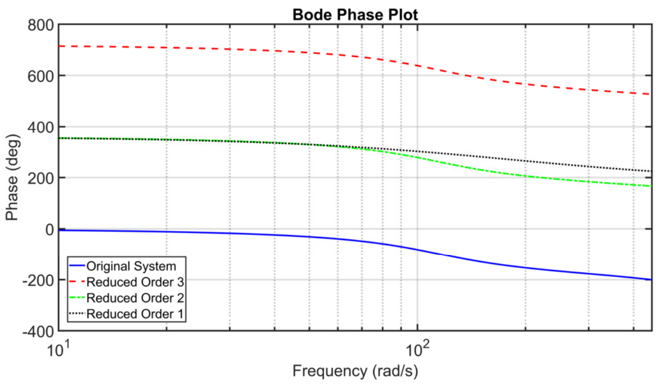


Fig. 4. Phase response between the original model and the reduced models.

The phase response in Figure 4 demonstrates substantial discrepancies between the original fourth-order system and the reduced models (first-order, second-order, and third-order), highlighting a key limitation of this reduction technique in fully replicating the phase characteristics of the original system.

The experimental findings demonstrate that reducing the system from a fourth-order model to third-order and second-order models via GBT preserves the essential dynamic characteristics, as evidenced by $H_\infty$ errors of approximately 12.13, whereas the first-order model incurs a pronounced error (410.18), indicating significant loss of phase and input–output fidelity. Consequently, the selection of the appropriate reduction order must be carefully considered to balance model accuracy with the required simplicity for engineering applications.

This paper makes a significant contribution by introducing an effective model order reduction framework for high-order DC motor systems controlled by DC-DC buck converters using the GBT algorithm. Our approach involves the rigorous computation of controllability and observability Gramians via Lyapunov equations, followed by the application of singular value-based balancing to identify and eliminate states with minimal contributions. The resulting third-order and second-order models achieve $H_\infty$ norm errors of approximately 12.13, thereby preserving the essential time- and frequency-domain dynamics of the original fourth-order system, while the first-order model exhibits unacceptable error levels. These findings underscore the critical importance of selecting an appropriate reduction order to balance model fidelity with computational simplicity and provide a robust foundation for designing precise control strategies in complex electromechanical systems.

## IV. CONCLUSION

This study addresses the feasibility of reducing the order of a high-order DC motor system controlled by a DC-DC buck converter, with the dual objective of minimizing the complexity of the original model and preserving its critical dynamic characteristics for precise controller design. The primary goal of this study is to develop and apply the Gramians-Based Truncation (GBT) algorithm to reduce a fourth-order system to third-order, second-order, and first-order models. The research methodology involves the computation of the controllability and observability Gramians by solving the Lyapunov equations. This is followed by balancing the system based on singular values to identify and eliminate states with minimal contributions. The implementation of the GBT algorithm in Matlab yielded $H_\infty$ norm reduction errors of 12.134639 and 12.135958 for the third-order and second-order models, respectively. These results demonstrate the algorithm's capacity to preserve the original system's time-domain and frequency-domain response characteristics. In contrast, the first-order model exhibited a significantly larger error (410.183959) and failed to ensure consistent input–output behavior, particularly during the startup phase and in applications that require precise phase signal processing. The findings confirm the viability of the GBT approach for simplifying complex dynamic models and underscore the importance of selecting an appropriate reduction order to balance model accuracy with the simplicity required for engineering applications. These results pave the way for further research aimed at refining reduction techniques to optimize phase information preservation and better meet the demands of modern control systems.

## ACKNOWLEDGMENT

The authors gratefully acknowledge Thai Nguyen University of Technology, Vietnam, for supporting this work. This work is independent of any grant-funded research projects.

## REFERENCES

[1] D. Montoya-Acevedo, W. Gil-González, O. D. Montoya, C. Restrepo, and C. González-Castaño, "Adaptive Speed Control for a DC Motor Using DC/DC Converters: An Inverse Optimal Control Approach," *IEEE Access*, vol. 12, pp. 154503–154513, 2024, https://doi.org/10.1109/ACCESS.2024.3482982.

[2] Z. R. Labidi, H. Schulte, and A. Mami, "A Model-Based Approach of DC-DC Converters Dedicated to Controller Design Applications for Photovoltaic Generators," *Engineering, Technology & Applied Science Research*, vol. 9, no. 4, pp. 4371–4376, Aug. 2019, https://doi.org/10.48084/etasr.2829.

[3] B. Pollet, G. Despesse, and F. Costa, "A New Non-Isolated Low-Power Inductorless Piezoelectric DC–DC Converter," *IEEE Transactions on Power Electronics*, vol. 34, no. 11, pp. 11002–11013, Nov. 2019, https://doi.org/10.1109/TPEL.2019.2900526.

[4] B. Babes, A. Boutaghane, N. Hamouda, M. Mezaache, and S. Kahla, "A Robust Adaptive Fuzzy Fast Terminal Synergetic Voltage Control Scheme for DC/DC Buck Converter," in *2019 International Conference on Advanced Electrical Engineering*, Algiers, Algeria, 2019, pp. 1–5, https://doi.org/10.1109/ICAEE47123.2019.9014717.

[5] A. Chandwani and A. Mallik, "A Reduced Stage Configuration of Three-Phase Isolated AC/DC Converter for Auxiliary Power Units," *IEEE Transactions on Vehicular Technology*, vol. 71, no. 4, pp. 3687–3703, Apr. 2022, https://doi.org/10.1109/TVT.2022.3146805.

[6] A. Beato, L. Fagnano, R. N. Gulesin, G. Ippoliti, L. Moretti, and G. Orlando, "Robust Sensorless Control of a PMSM: Experimental Validation on an Appliance," in *2024 32nd Mediterranean Conference on Control and Automation*, Chania, Greece, 2024, pp. 872–877, https://doi.org/10.1109/MED61351.2024.10566238.

[7] T. Ahmed, M. H. Baloch, N. Khan, G. Mehr, B. A. Mirjat, and Y. A. Memon, "Experimental Analysis and Control of a Wind-Generator System through a DC-DC Boost Converter for Extremum Seeking," *Engineering, Technology & Applied Science Research*, vol. 11, no. 1, pp. 6714–6718, Feb. 2021, https://doi.org/10.48084/etasr.3948.

[8] B. Lingamchetty, A. Raghuwanshi, and A. Ojha, "Battery Connected Multi-level Inverter Fed PMSM for Electric Vehicle Applications," in *2023 IEEE Renewable Energy and Sustainable E-Mobility Conference*, Bhopal, India, 2023, pp. 1–6, https://doi.org/10.1109/RESEM57584.2023.10236083.

[9] J. A. Prakosa, N. Alias, Purwowibowo, and C. Astuti, "Performance comparison of applying integer and fractional order calculus to DC motor speed control experiments," in *2023 International Conference on Radar, Antenna, Microwave, Electronics, and Telecommunications*, Bandung, Indonesia, 2023, pp. 79–83, https://doi.org/10.1109/ICRAMET60171.2023.10366595.

[10] G. Nadh and A. Rahul S, "Clamping Modulation Scheme for Low-Speed Operation of Three-Level Inverter Fed Induction Motor Drive With Reduced CMV," *IEEE Transactions on Industry Applications*, vol. 58, no. 6, pp. 7336–7345, Nov. 2022, https://doi.org/10.1109/TIA.2022.3199193.

[11] C. Giamouzis, D. Garyfallou, N. Evmorfopoulos, and G. Stamoulis, "A low-rank balanced truncation approach for large-scale RLCk model order reduction based on extended Krylov subspace and a frequency-aware convergence criterion." arXiv, Nov. 12, 2024, https://doi.org/10.48550/arXiv.2411.13571.

[12] A. Goel and A. K. Manocha, "Balanced Truncation Constrained Order Reduction of Complex Power Systems using Moth Flame Optimization,"

in *2024 15th International Conference on Computing Communication and Networking Technologies*, Kamand, India, 2024, pp. 1–6, https://doi.org/10.1109/ICCCNT61001.2024.10726177.

[13] H. Kang, Q. Yuan, X. Su, T. Guo, and Y. Cong, "Modal truncation method for continuum structures based on matrix norm: modal perturbation method," *Nonlinear Dynamics*, vol. 112, no. 13, pp. 11313–11328, Jul. 2024, https://doi.org/10.1007/s11071-024-09628-2.

[14] L.-H. Zhang and R.-C. Li, "Quality of Approximate Balanced Truncation." arXiv, Jun. 09, 2024, https://doi.org/10.48550/arXiv.2406.05665.

[15] Z.-H. Xiao, Y.-X. Fang, and Y.-L. Jiang, "Laguerre-Based Low-Rank Balanced Truncation of Discrete-Time Systems," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 70, no. 8, pp. 3014–3018, Aug. 2023, https://doi.org/10.1109/TCSII.2023.3253159.

[16] J. Przybilla, I. P. Duff, P. Goyal, and P. Benner, "Balanced Truncation of Descriptor Systems with a Quadratic Output." arXiv, Feb. 22, 2024, https://doi.org/10.48550/arXiv.2402.14716.

[17] J. Yang, H. Wu, L. Hu, and S. Li, "Robust Predictive Speed Regulation of Converter-Driven DC Motors via a Discrete-Time Reduced-Order GPIO," *IEEE Transactions on Industrial Electronics*, vol. 66, no. 10, pp. 7893–7903, Oct. 2019, https://doi.org/10.1109/TIE.2018.2878119.