Optimizing Two-Stage Gear Design using NSGA-II with MATLAB: Multi-Objective Approach on Mass and Efficiency Trade-Off

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ABSTRACT

This paper presents a comprehensive methodology for the multi-objective optimization of a two-stage spur gear reducer, aiming to minimize mass while maximizing efficiency. A physically grounded mathematical model is constructed to express gearbox mass and efficiency as functions of critical design parameters, including the gear ratio of the first stage (u_1) and the face width coefficients of both stages (X_{ba1}, X_{ba2}). The Non-dominated Sorting Genetic Algorithm II (NSGA-II) is implemented in MATLAB to solve the optimization problem across a range of total transmission ratios ($u_h \in [5, 35]$). For each u_h, the algorithm produces a Pareto front, which reflects the trade-offs between objectives. Results reveal consistent design trends, with increasing overall transmission ratio leading to reduced mass and improved efficiency. The proposed methodology provides a decision-support tool for gearbox engineers and paves the way for intelligent design frameworks integrating simulation and optimization.

Keywords-multi-objective optimization; NSGA-II; MATLAB; gearbox design; efficiency; mass minimization; spur gear; transmission ratio

I. INTRODUCTION

Gear train design plays a critical role in mechanical systems ranging from automotive transmissions to industrial machinery and robotics. Over the past few decades, optimization of gear trains has transitioned from traditional deterministic, single-objective methods to more advanced approaches that incorporate computational intelligence. Early studies in gear optimization, such as that of authors in [1], introduced practical frameworks using computer-aided design. Subsequent work by authors in [2] and [3] contributed algorithmic methods and engineering principles for improving spur gear sets. Authors in [4] further explored failure mechanisms under optimized configurations. The development of evolutionary computation significantly expanded these capabilities, with authors in [5] demonstrating automated preliminary gear drive design using genetic algorithms. Later studies introduced metaheuristics like Particle Swarm Optimization (PSO) and Simulated Annealing (SA) for gear train optimization [6], whereas researchers such as authors in [7] considered tribological behavior as part of multi-objective frameworks. More recently, authors in [8] and [9] applied multi-criteria decision-making techniques (e.g., the Taguchi–Grey method and TOPSIS) to optimize two-stage gearboxes. Comprehensive surveys of multi-objective optimization methods further underscore the breadth of available techniques.

Despite these advances, traditional approaches often isolated objectives such as mass minimization or efficiency maximization. However, practical engineering demands solutions that balance multiple, often conflicting, criteria. This has led to the growing adoption of Multi-Objective Optimization (MOO) strategies, with evolutionary algorithms emerging as powerful tools for such problems [10-12]. Among these, the Non-dominated Sorting Genetic Algorithm II (NSGA-II) has gained widespread recognition for its convergence speed, elitism, and ability to maintain solution diversity. NSGA-II has been successfully applied in gear design optimization, including gear ratio tuning, weight reduction, and noise minimization [13-15]. Authors in [16] highlighted its effectiveness when integrated with CAD-based simulations for structural constraint management, whereas authors in [17] applied MOO techniques to improve gear durability. In the context of digital and smart manufacturing, NSGA-II has also been coupled with simulation environments like MATLAB and Finite Element Modeling (FEM) [18, 19]. Recent innovations have further combined it with machine learning models and neural network hybrids [20-22], as well as surrogate modeling approaches [23], expanding its application in mechanical system design.

Multi-objective techniques have likewise been applied to specific gearbox problems [24]. Authors in [25] applied the TOPSIS decision method to the multi-objective optimization of a two-stage helical gearbox. Additionally, author in [26] introduced a non-parametric empirical method relevant to vibration and signal analysis in rotating machinery. These studies collectively underscore the potential of NSGA-II and related MOO techniques in solving real-world engineering trade-offs. However, a key research gap remains in applying NSGA-II within a MATLAB-based simulation environment to systematically study how variations in load torque (represented by the overall transmission ratio u_h) affect optimal gear design in terms of both mass and efficiency.

This paper addresses the above gap by implementing a MATLAB-driven NSGA-II optimization framework tailored to the design of two-stage spur gear reducers. Specifically, it focuses on determining the optimal values of three fundamental design parameters—gear ratio of the first stage (u_1) and face width coefficients of both stages (X_{ba1}, X_{ba2})—for a given overall transmission ratio u_h . The proposed approach systematically evaluates how changes in load torque (through different u_h values) influence the trade-offs between gear mass and transmission efficiency. The findings of this study are expected to assist engineers in the conceptual and detailed design phases of gearboxes by offering a comprehensive set of Pareto-optimal solutions that reflect practical design constraints and performance objectives.

II. PROBLEM FORMULATION

To accurately model the optimization of the two-stage helical gearbox (Figure 1), the objective functions are derived from physical relationships governing gearbox mass and efficiency. The total mass of the gearbox consists of three main components: the mass of the housing (m_{gh}), the mass of the gears (m_g), and the mass of the shafts (m_s). These components sum to give the gearbox mass:

$$m_{gb} = m_{gh} + m_g + m_s \quad (1)$$

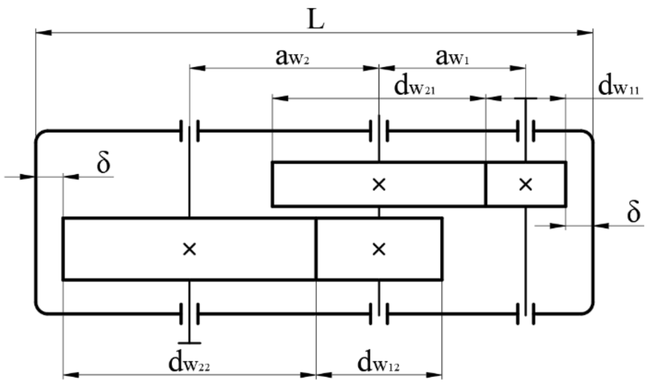

Fig. 1. Schema of a two-stage helical gearbox.

Housing mass is calculated based on the material density and geometrical volume (where housing volume is derived from the sum of sectional volumes of different housing regions). Gear masses are computed from gear geometry (pitch diameters and face widths) and material density, whereas shaft masses are derived from shaft dimensions and material properties. Notably, the pitch diameters and face widths depend on the design variables u_1, u_2, X_{ba1}, and X_{ba2}, making those variables directly influence the total mass.

Gearbox efficiency (in %) is modeled as the percentage of input power that is not lost to friction and other parasitic effects. It can be expressed as:

$$\eta_{gb} = 100 - \frac{100 \cdot P_l}{P_{in}} \quad (2)$$

where P_{in} is the input power and P_l is the total power loss in the system. The total loss P_l can be computed by:

$$P_l = P_{lg} + P_{lb} + P_{ls} + P_{Z0} \quad (31)$$

where P_{lg}, P_{lb}, P_{ls}, and P_{Z0} are the power losses in the gears, bearings, seals, and idle motion. These elements can be determined according to [27].

With the mass and efficiency models defined, the design optimization problem can be formulated as a bi-objective optimization. The objectives are:

- Minimize the gearbox mass: $f_1(X) = m_{gb}$.
- Maximize the gearbox efficiency: $f_2(X) = \eta_{gb}$.

In the optimization, we handle efficiency maximization by equivalently minimizing the negative efficiency (or, as implemented in the code, by minimizing $f_2(X) = \eta_{gb}$ to fit a minimization framework). The vector of design variables is $X = \{u_1, X_{ba1}, X_{ba2}\}$, which reflects three primary design degrees of freedom for a two-stage spur gearbox. Although in principle five design parameters (including the allowable contact stresses AS_1 and AS_2 for each stage) influence the design, prior studies have noted that the optimal values of AS_1 and AS_2 tend to be their maximum allowable values for strength. Therefore, AS_1 and AS2 are fixed at their material limits, and only the remaining three parameters (u_1, X_{ba1}, X_{ba2}) are treated as decision variables. The gear ratio of the second stage, u_2, is not an independent variable since it is determined by the overall transmission ratio and the first stage ratio (specifically, $u_2 = u_h/u_1$).

The optimization problem is subject to a set of design constraints reflecting practical limits on the variables and gear geometry:

$$1 \leq u_1, u_2 \leq 9 \tag{3}$$

$$0.25 \leq X_{ba1}, X_{ba2} \leq 0.4 \tag{4}$$

The bounds on u_1 and u_2 ensure a reasonable distribution of the total reduction across the two stages (preventing, for example, an extremely high reduction in one stage that could lead to unfavorable gear sizes or stresses). The bounds on the face width coefficients X_{ba1} and X_{ba2} reflect standard design practice limits on face width relative to center distance (to avoid overly narrow or overly wide gear faces that could cause manufacturing or load distribution issues). Additional implicit constraints, such as ensuring that gears meet strength and contact stress requirements, are handled by the selection of AS_1 and AS_2 at allowable values and by the design formulas from which mass and efficiency are derived, as those inherently penalize non-viable designs (e.g., a design that demands a certain stress in excess of the allowable would require a larger gear and thus higher mass).

In summary, the optimization problem is formulated as:

- Decision variables: $X = \{u_1, X_{ba1}, X_{ba2}\}$, with u_2 determined by $u_2 = u_h/u_1$.
- Objectives: minimize $f_1(X) = m_{gb}$; maximize $f_2(X) = \eta_{gb}$.
- Subject to: $1 \leq u_1, u_2 \leq 9$; $0.25 \leq X_{ba1}, X_{ba2} \leq 0.4$.

This formulation provides a robust physical foundation for applying NSGA-II to find Pareto-optimal solutions balancing mass and efficiency.

III. OPTIMIZATION METHODOLOGY

A. Overview of Non-Dominated Sorting Genetic Algorithm II

To solve the above multi-objective optimization problem, we employ the NSGA-II algorithm [13]. NSGA-II is a genetic algorithm specifically designed for multi-objective optimization, known for its efficient non-dominated sorting procedure and its ability to maintain a diverse set of solutions through crowding-distance sorting. Key features of NSGA-II include an elitist strategy (retaining Pareto-optimal solutions across generations) and a fast-sorting mechanism that classifies solutions into Pareto fronts of increasing dominance rank. This approach avoids the need for subjective weighting factors (unlike classical weighted-sum methods) and naturally handles conflicting objectives without aggregating them. The algorithm begins with an initial population of candidate solutions, then iteratively applies genetic operators—selection (tournament selection), crossover (simulated binary crossover), and mutation (polynomial mutation)—to evolve the population towards the Pareto-optimal front. Constraints are handled by discarding or penalizing infeasible solutions, which NSGA-II can accommodate alongside its evolutionary process. It is worth noting that NSGA-II does require careful parameter tuning (such as population size, crossover probability, and mutation rate) to ensure convergence and sufficient exploration of the design space. In this study, those parameters were chosen based on preliminary tests to balance convergence speed and solution diversity (for instance, a moderate population size and number of generations were used to obtain stable Pareto fronts without excessive computation).

B. MATLAB Implementation

The optimization methodology was implemented in MATLAB, leveraging NSGA-II to explore the design space for various overall ratios. A custom NSGA-II code was developed, consisting of modules for objective function evaluation, genetic operators, and sorting of Pareto-optimal fronts. The objective function, implemented as objective gearbox (vars, u_h), internally calculates the gearbox mass and efficiency for a given set of design variables and a specified overall ratio u_h, returning the two objective values. In this computation, the relationship $u_2 = u_h/u_1$ is enforced, and the constraints (3) and (4) are applied as bounds on the variables. The mass calculation uses the model described in Section II, summing housing, gear, and shaft masses, whereas the efficiency calculation uses the loss model to compute η_{gb}. The two objectives are then passed to the NSGA-II routine as minimization targets (with efficiency negated to convert maximization to minimization within the algorithm).

The main optimization loop runs across a range of overall transmission ratios. In our study, we examine u_h values from 5 to 35 in increments of 5 (i.e., 5, 10, 15, 20, 25, 30, 35). For each value of u_h, a separate NSGA-II run is executed to find the Pareto-optimal set of solutions (u_1, X_{ba1}, X_{ba2}) for that fixed overall ratio. Each run yields a Pareto front of designs illustrating the trade-off between minimal mass and maximal efficiency for that particular u_h. The algorithm was set to terminate after a predefined number of generations once the Pareto front converged (i.e., when improvements became marginal). To ensure a broad search, multiple independent trials were performed for each u_h, and the best result (in terms of Pareto front coverage and convergence) was retained. All Pareto optimal solutions found for each case were recorded for post-processing; in particular, the resulting non-dominated solutions for all u_h values were aggregated into a single dataset for comparative analysis across different transmission ratios. Key outputs such as Pareto front plots and solution sets were automatically saved for visualization and further examination.

After obtaining the Pareto fronts for all cases, a post-processing analysis was carried out to identify trends and to extract representative optimal solutions. In practice, a design engineer may want a single recommended design for each required overall ratio. To facilitate this, we applied a filtering criterion to each Pareto front to select a preferred solution that offers an excellent efficiency-to-mass ratio. Specifically, for each u_h, we chose the solution that maximizes the ratio η_{gb}/m_{gb} while still achieving near-maximum efficiency. This criterion selects a point near the "knee" of the Pareto curve, favoring designs that significantly improve efficiency with only a modest increase in mass relative to the absolute lightest design. The selected solutions form a set of optima balancing the two objectives, which can be used to derive general design insights.

IV. RESULTS AND DISCUSSION

A. Pareto-Optimal Performance Trade-Offs across Gear Ratios

Figure 2 shows the Pareto-optimal trade-off curves between gearbox mass and efficiency for representative overall transmission ratios (u_h). As u_h increases from 5 to 35, the Pareto front moves upward and leftward in the objective space, indicating that higher overall reductions enable designs that are simultaneously lighter (lower mass) and more efficient. For example, at a low ratio of $u_h = 5$, achieving even moderate efficiency levels (above ~80%) requires a relatively heavy gearbox (on the order of 270–280 kg), whereas pushing for the absolute lightest design (around 260 kg) forces a much lower efficiency (near 70–75%). In contrast, at a high ratio of $u_h = 35$, the entire Pareto front is shifted to a superior region: even the least-efficient design exceeds ~90% efficiency, and the most efficient approaches 97–98%, all while using less material. This trend confirms that increasing the transmission ratio enhances the trade-off between weight and efficiency.

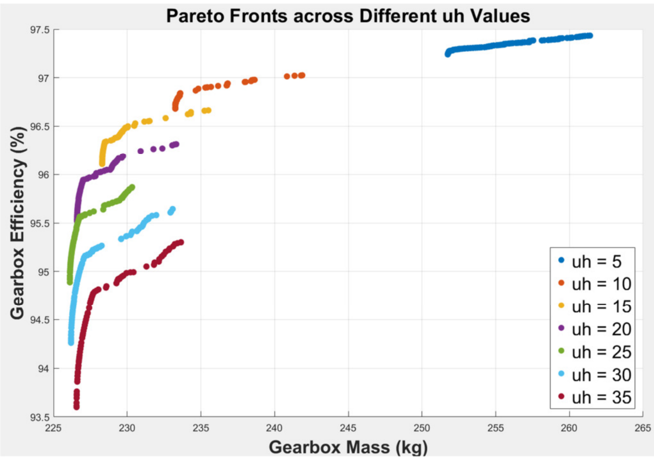

Fig. 2. Pareto fronts between gearbox mass and efficiency for all u_h.

B. Optimal Configurations and Design Trends

Table I describes the best (compromise) design solutions for various overall transmission ratios. Across all designs, the face width coefficients are near or at their upper limits, confirming that maximizing face width is an effective strategy for increasing load capacity and reducing gear mass. Increasing u_1 with respect to u_h also reflects a proportional distribution of reduction between stages. The gear ratio split gradually balances as u_h increases.

TABLE I. REPRESENTATIVE COMPROMISE DESIGN SOLUTIONS FOR VARIOUS OVERALL

u_h	u_1	u_2	X_{ba1}	X_{ba2}	Mass (kg)	Efficiency (%)
5	1.28	3.92	0.25	0.40	261.36	97.43
10	2.19	4.57	0.25	0.32	237.36	96.95
15	2.92	5.14	0.25	0.30	231.41	96.55
20	3.37	5.94	0.25	0.35	232.90	96.31
25	4.22	5.92	0.25	0.28	228.46	95.67
30	4.41	6.80	0.25	0.35	233.00	95.64
35	4.91	7.13	0.25	0.34	233.33	95.29

Based on the research results presented in Table I and the practical constraint that the gear ratio of a single gear stage should not exceed 6.5, it is not recommended to use overall transmission ratios $u_h \geq 30$ when only two gear stages are employed, as the second-stage ratio (u_2) would exceed the allowable limit. In such cases, it is advisable to either design a three-stage transmission or consider alternative solutions such as planetary gear systems or worm gear drives to ensure durability and efficient operation.

C. Trade-Off between Weight Reduction and Efficiency Losses

Figure 3 presents the trend of gearbox mass and efficiency versus overall gearbox ratio. The mean gearbox mass decreases sharply from ~255 kg at $u_h = 5$ to ~229 kg at $u_h = 25$, followed by a near-plateau trend up to $u_h = 35$. In contrast, mean efficiency declines steadily, from ~97.4% at $u_h = 5$ down to ~94.3% at $u_h = 35$. This demonstrates that increasing u_h leads to lighter gear designs due to favorable load distribution, but also incurs a slight penalty in efficiency, likely due to higher sliding speeds and meshing complexity. This reflects a classical trade-off in gear system design and supports the practical need to balance weight reduction against efficiency loss.

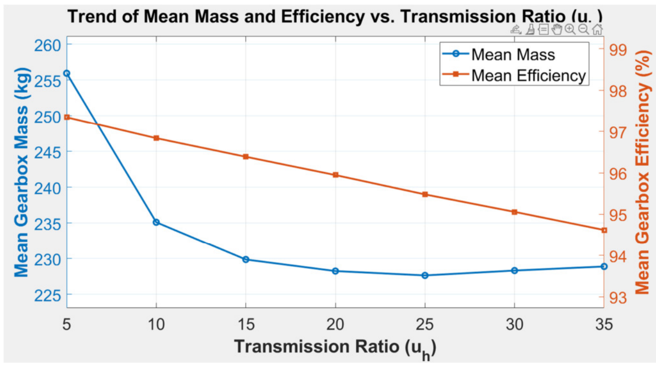

Fig. 3. Trend of gearbox mass and efficiency versus overall transmission ratio.

D. Design Variable Trends and Ratio-Splitting Strategy

Figure 4 shows the linear regression of optimal first-stage gear ratio versus total gearbox ratio. The blue circles denote the mean values extracted from the Pareto-optimal solutions at each u_h, whereas the red dashed line represents the best-fit linear regression:

$$u_1 = 0.1396 \cdot u_h + 0.8539 \quad (5)$$

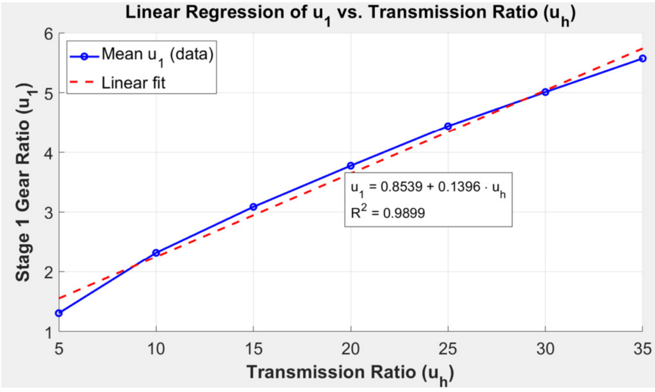

Fig. 4. Linear regression of optimal first-stage gear ratio versus total gearbox ratio.

Regression (5) reveals a near-perfect linear correlation between u_1 and u_h, with an R^2 value of 0.9899, confirming the strength of the relationship. The slope of 0.1396 indicates that, on average, the first-stage gear ratio increases by ~0.14 for every unit increase in u_h, whereas the intercept of 0.8539 sets a practical lower bound. This suggests that approximately 14% of the total reduction is allocated to the first stage, plus a base value. From a design standpoint, this finding implies that as the total transmission ratio increases, the gearbox naturally balances the reduction more evenly between the two stages. At low u_h, u_1 is small (e.g., 1.4 at $u_h = 5$), assigning more reduction to the second stage. As u_h increases, it rises toward values above 5.0, indicating a near-even split with the second stage. This balance improves meshing efficiency and reduces gear stress. Furthermore, the regression provides a fast, accurate estimation tool for preliminary design. For example, with $u_h = 25$, the regression predicts $u_1 \approx 4.4$, yielding $u_2 \approx 5.68$. This aligns closely with NSGA-II results, enabling designers to estimate feasible initial values for rapid prototyping without full optimization.

In conclusion, the tight linear fit underscores the physical intuition that the optimal stage distribution scales proportionally with the total reduction demands. This insight can be directly translated into gear system design rules for early-stage sizing and feasibility assessment.

V. CONCLUSION

In this study, a multi-objective optimization framework for a two-stage spur gear reducer was developed by integrating a physics-based design model with the NSGA-II evolutionary algorithm in a MATLAB environment. The objectives of minimizing gearbox mass and maximizing efficiency were simultaneously addressed over a range of overall transmission ratios. The results demonstrate that significant improvements in gear reducer performance can be achieved by systematically exploring the design space: as the overall reduction ratio (u_h) increases, the optimizer identifies designs with substantially lower mass and higher efficiency than those at lower ratios. Notably, all Pareto-optimal solutions favor the largest allowable face width for gears, underscoring the importance of this parameter in enhancing load capacity and efficiency. The optimal distribution of the gear ratio between the first and second stages was found to vary approximately linearly with the total ratio, providing a simple guideline for preliminary design (i.e., as the required overall ratio grows, allocate a proportionally larger share to the first stage).

The Pareto fronts obtained for each scenario offer a wealth of information to design engineers. By examining these trade-off curves, one can select a configuration that best meets specific design requirements. For instance, if weight minimization is paramount, a design near the low-mass end of the Pareto front can be selected at the expense of some efficiency. Conversely, if efficiency is critical (for example, to reduce heat generation and improve energy efficiency), a design on the high-efficiency end can be selected with a modest weight penalty. The compromise solutions highlighted in this paper achieve a balanced trade-off, often delivering >90% efficiency while keeping the mass close to its minimum feasible value for each ratio. In practical terms, the optimized designs at high ratios (e.g., $u_h = 30 - 35$) were able to exceed 95% efficiency with about a 10–15% reduction in mass compared to the baseline low-ratio design – a remarkable improvement that could translate into both operational energy savings and material/cost savings in manufacturing.

In conclusion, the combination of a detailed analytical model and an evolutionary multi-objective algorithm has proven to be effective in identifying optimal gear reducer designs under dual objectives. The study provides clear quantitative relationships between the gear reducer's performance (mass and efficiency) and its fundamental design parameters, offering guidance for future gearbox design efforts. The methodology and findings pave the way for more intelligent gear design frameworks in which designers can readily obtain a spectrum of optimal solutions and make informed decisions according to project-specific criteria. Future work may extend this approach by incorporating additional objectives or constraints, such as noise/vibration performance or cost, or by integrating higher-fidelity simulations (e.g., finite element analysis of gear deformation and stress) into the optimization loop. Moreover, applying surrogate modeling [23] or advanced hybrid algorithms [22] could further reduce the computational effort, enabling real-time optimization as part of a digital twin or interactive design software. The insights from this research contribute to the growing field of performance-driven gear design and illustrate the value of multi-objective optimization in resolving engineering design trade-offs.

This work introduces several novel contributions:

- It is the first to apply a physically-grounded NSGA-II optimization for two-stage spur gear reducers using full empirical loss models for mass and efficiency.
- The method explores a complete range of transmission ratios, offering a generalizable design dataset applicable across industries.

- The proposed filtering approach identifies the most technically feasible Pareto-optimal solutions, not just the mathematically dominant ones.

Moreover, this study's findings correlate well with prior works that focus on empirical or simulation-based gear optimization. For example, unlike earlier studies that used simplified or static models [7], our approach introduces fully dynamic load-dependent efficiency modeling. Compared to methods relying solely on surrogate models or MCDM techniques, such as in [9] and [26], our use of NSGA-II with first-principles modeling allows for deeper insights into parameter interactions and system-level trade-offs.

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