A Hybrid Modeling and Optimization Framework for Finish Milling Using SVR, NSGA-II, and Entropy-Based TOPSIS

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ABSTRACT

This study presents a hybrid methodology for optimizing finish milling processes by integrating predictive modeling, evolutionary algorithms, and multi-criteria decision-making techniques. The target output responses include surface roughness (R_a), Material Removal Rate (MRR), and cutting force (F_c), modeled as functions of cutting speed (V_c), feed per tooth (f_z), axial depth of cut (a_p), and radial depth of cut (a_e). Support Vector Regression (SVR) models yielded high accuracy for R_a ($R^2 = 0.926$) and MRR ($R^2 = 0.999$), while a second-order polynomial regression model excelled for F_c ($R = 0.938$, $R_{adj}^2 = 0.869$). These models were integrated into Non-dominated Sorting Genetic Algorithm II (NSGA-II), generating a Pareto front of 100 optimal solutions. Entropy-weighted Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) ranked these solutions, identifying the best trade-off at $R_a = 0.698$ µm, MRR = 5228.24 mm³/min, and $F_c = 522.17$ N, with a TOPSIS score of $C_i = 0.878$. This solution significantly enhances productivity while maintaining acceptable surface quality and cutting force. The workflow was implemented in MATLAB, demonstrating the efficacy of this hybrid approach for advanced manufacturing. This hybrid framework provides a practical tool for real-time process optimization and decision support in smart manufacturing environments.

Keywords-multi-objective optimization; SVR; polynomial regression; NSGA-II; TOPSIS; entropy weighting; MATLAB

I. INTRODUCTION

In the field of precision machining, multi-objective optimization plays a crucial role in enhancing productivity, improving product quality, and reducing operational costs [1, 2]. Among the key machining performance indicators, R_a, MRR, and F_c are often in conflict [3]. Minimizing R_a and F_c typically leads to improved surface quality and tool life, while maximizing MRR increases productivity [4-6]. However, optimizing these objectives simultaneously is a challenging task due to their complex and non-linear relationships with machining parameters. To address this issue, researchers have explored a variety of predictive modeling and optimization approaches. SVR known for its robustness in handling non-linear data, has shown great promise in predicting surface roughness and MRR [7]. On the other hand, second-order polynomial regression models are still widely used due to their simplicity and interpretability, especially in estimating cutting forces [8, 9].

For optimization, the NSGA-II is a well-established technique that efficiently explores the Pareto front of multi-objective problems [7, 10]. However, selecting the most suitable solution from the Pareto set remains a challenge. To overcome this, Multi-Criteria Decision Making (MCDM) methods, such as Multi-Objective Optimization on the basis of Ratio Analysis (MOORA), VlseKriterijuska Optimizacija I Komoromisno Resenje (VIKOR), and TOPSIS combined with entropy-based weighting, offer a systematic way to rank and select optimal solutions [11-14]. A hybrid approach integrating SVR and polynomial regression with NSGA-II and entropy-weighted TOPSIS has been proposed to identify optimal cutting conditions in the finish milling process [7]. The method is applied to an experimental dataset characterized by four input parameters: V_c, f_z, a_p, and a_e. The optimization objectives are defined as the simultaneous minimization of R_a and F_c, along with the maximization of MRR. The effectiveness of the proposed methodology is assessed through the comparison of predicted versus experimental values, and through the visualization of optimization outcomes using Pareto fronts and TOPSIS-based rankings.

Compared to previous studies which have explored either machine learning-based modeling or MCDM-based optimization independently, the current work presents a fully integrated framework [6, 15]. This framework combines SVR for predictive modeling, NSGA-II for Pareto front generation, and entropy-weighted TOPSIS for objective ranking. This hybrid approach has not been simultaneously employed in the context of finish milling of P20 steel and offers both high prediction accuracy and transparent decision support. Therefore, the main objective of this study is to propose and validate a hybrid framework that integrates SVR, polynomial regression, NSGA-II, and entropy-weighted TOPSIS to simultaneously optimize R_a, MRR, and F_c in the finish milling of P20 steel.

II. METHODOLOGY

SVR is selected in this study due to its ability to generalize well with small datasets and non-linear patterns, which is particularly valuable in modeling R_a and MRR. Polynomial regression was preferred for F_c due to its ability to preserve physical relationships and provide interpretable coefficients. NSGA-II is chosen as the evolutionary optimization engine owing to its proven effectiveness in maintaining diversity and discovering well-distributed Pareto fronts in machining processes. Finally, entropy-based TOPSIS is integrated to allow objective and data-driven ranking of optimal solutions, minimizing the influence of subjective weighting.

A. Experimental Parameters and Data Collection

The adopted approach to effectively solve the multi-objective optimization problem in finish milling, consisted of three main components: (i) predictive modeling using statistical and machine learning methods, (ii) evolutionary optimization through NSGA-II to construct the Pareto front, and (iii) decision-making using the entropy-weighted TOPSIS technique. The CNC Mori-Seiki NVX5060 vertical machining center is utilized to perform the milling experiments to ensure high precision and repeatability, as illustrated in Figure 1. The cutting tool employed is a WIDIA APMT1135PDR-SPTIEH-SPATEH milling insert mounted on a compatible tool holder with two cutting edges (Z = 2) and a cutter diameter of 17 mm. For quality assessment, R_a is measured with a Mitutoyo JS-201 portable roughness tester. The F_c is recorded with a Kistler 3-component dynamometer, interfaced with a charge amplifier and data acquisition system. This configuration ensures reliable and accurate measurements throughout the experiments.

Fig. 1. CNC Mori Seiki NVX5060.

B. Experimental Dataset and Input Parameters

The experimental data are collected from controlled milling experiments with 20 samples. The input parameters considered, essential for assessing machining performance, are V_c, f_z, a_p, and a_e. The measured output responses included R_a, MRR, and F_c.

The MRR was computed analytically using:

$$\mathrm{MRR} = (z \times V_c \times f_z \times a_p \times a_e)/1000 \quad (1)$$

where $z = 2$ is the number of cutting edges. This formulation ensures consistency and physical interpretation across the experimental conditions.

A full factorial design with additional center points was applied to capture both main effects and possible interactions. The experimental matrix, along with the corresponding measurements of R_a and F_c and the calculated MRR values are summarized in Table I.

C. Predictive Modeling Techniques

In this study, linear regression, second-order polynomial regression, and SVR with a radial basis function kernel are evaluated to model the relationships between machining parameters and responses. Model selection is based on R^2, adjusted R^2, and RMSE. SVR outperformed other models for R_a and MRR due to its strong ability to handle non-linear patterns in small datasets. In contrast, second-order polynomial regression yielded better results for F_c, offering interpretable coefficients consistent with the physical characteristics of the cutting force. All models were trained using standardized inputs for numerical stability. The selected models—SVR for Ra and MRR, and polynomial regression for F_c—are used in the subsequent optimization phase. The general form of the quadratic model for F_c is:

$$y=\beta_0+\sum_{i=1}^{n}\beta_i x_i+\sum_{i=1}^{n}\beta_{ii}x_i^2+\sum_{i<j}\beta_{ij}x_i x_j \quad (2)$$

where x_i are the input variables and y is the output response. The accuracy of each model is evaluated by comparing predicted values against experimental data. The models selected for use in the optimization process were those with the highest predictive performance.

TABLE I. EXPERIMENTAL MATRIX AND MEASURED RESPONSES

V_c (m/min)	f_z (mm/z)	a_p (mm)	a_e (mm)	R_a (μm)	MRR (mm³/min)	F_c (N)
40	0.13	0.50	5.10	0.603	77.932	111.267
80	0.07	2.00	8.50	0.725	3566.872	158.760
40	0.13	0.50	8.50	0.794	828.028	79.560
40	0.07	0.50	5.10	0.602	41.963	91.980
80	0.07	0.50	5.10	0.575	83.926	47.250
80	0.13	2.00	8.50	0.712	6624.203	566.280
40	0.13	2.00	8.50	0.566	3313.110	393.120
80	0.13	0.50	5.10	0.714	155.864	148.590
80	0.07	2.00	5.10	0.765	335.706	322.560
40	0.07	0.50	8.50	0.605	445.709	84.420
40	0.07	2.00	8.50	0.562	1782.836	146.160
80	0.13	0.50	8.50	0.699	1656.051	160.290
40	0.13	2.00	5.10	0.598	311.728	346.320
40	0.07	2.00	5.10	0.725	167.853	284.760
80	0.13	2.00	5.10	0.498	623.454	355.680
80	0.07	0.50	8.50	0.580	891.734	80.010
40	0.13	2.00	8.50	0.582	3313.110	420.320
40	0.07	2.00	8.50	0.516	1782.836	169.480
80	0.13	2.00	8.50	0.698	6624.203	582.040
80	0.07	0.50	8.50	0.580	891.734	76.970

D. NSGA-II Multi-Objective Optimization

The optimization process is conducted through the NSGA-II algorithm, which is widely adopted for multi-objective problems due to its ability to maintain population diversity and Pareto dominance hierarchy. The objective functions were defined as:

$$Minimize: f_1(x)=\widehat{R_a}(x) \quad (3)$$

$$Maximize: f_2(x)=\widehat{MRR}(x) \Rightarrow Minimize: -\widehat{MRR}(x) \quad (4)$$

$$Minimize: f_3(x)=\widehat{F_c}(x) \quad (5)$$

where $\widehat{R_a}$, $\widehat{MRR}$, $\widehat{F_c}$ are the predicted responses from SVR and regression models.

The bounds for the decision variables are taken directly from the experimental design, and optimization is performed with a population size of 100 over 200 generations. The result is a Pareto-optimal set of non-dominated solutions. All computations, including data preprocessing, model training, evaluation, and visualization, are performed in MATLAB. Statistical analysis and visualization routines were executed using MATLAB's Statistics and Machine Learning Toolbox [7].

E. TOPSIS-Entropy Ranking Optimal Solutions

After obtaining the Pareto front, the entropy-weighted TOPSIS method identifies the most preferable solution. Firstly, the decision matrix $X_i=[x_{ij}]$, where i indexes solutions and j indexes criteria, is normalized through vector normalization utilizing:

$$r_{ij}=\frac{x_{ij}}{\sqrt{\sum_{i=1}^{m}x_{ij}^2}} \quad (6)$$

To calculate the entropy, the normalized values r_{ij} were converted into probability-like values through:

$$p_{ij}=\frac{r_{ij}}{\sum_{i=1}^{m}r_{ij}} \quad (7)$$

To avoid $lun(0)$, a small value is added where necessary. The entropy E_j for criterion j is computed by:

$$E_j=-k\sum_{i=1}^{m}p_{ij}\ln(p_{ij}),\ \text{where: } k=\frac{1}{\ln(m)} \quad (8)$$

In addition, the weight is computed and normalized through:

$$w_i=\frac{d_i}{\sum_{i=1}^{m}d_i},\ \text{where: } d_i=1-E_j \quad (9)$$

The normalized matrix is weighed as:

$$v_{ij}=w_j r_{ij} \quad (10)$$

The Euclidean distances from the ideal and anti-ideal solutions are calculated from:

$$S_i^+=\sqrt{\sum_{j=1}^{n}(v_{ij}-v_j^+)^2} \quad (11)$$

$$S_i^-=\sqrt{\sum_{j=1}^{n}(v_{ij}-v_j^-)^2} \quad (12)$$

where S_i^+ and S_i^- are the distances to the ideal and anti-ideal solutions, respectively. The final ranking score is calculated employing (13). Higher C_{ij} values indicate more preferable solutions:

$$C_{ij}=\frac{S_i^-}{S_i^++S_i^-} \quad (13)$$

All steps of the TOPSIS-Entropy analysis are implemented in MATLAB [7].

F. Overview of the Hybrid Framework

The complete methodology, combining data preprocessing, model training, evolutionary optimization, and decision ranking, is depicted in Figure 2. In this framework, SVR and polynomial regression models are used to develop accurate predictive models for R_a, MRR, and F_c based on machining parameters. These models served as surrogate objective functions in the NSGA-II algorithm, which is employed to generate a set of Pareto-optimal solutions that balanced the trade-offs among conflicting objectives. Subsequently, the entropy-weighted TOPSIS method is applied to rank the solutions in the Pareto front and identify the most preferred alternative.

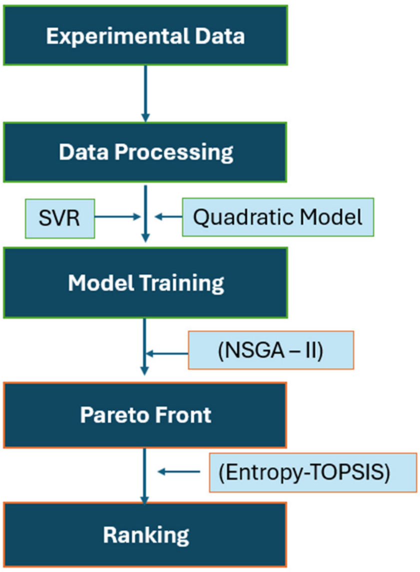

Fig. 2. Workflow of the hybrid SVR, regression, NSGA-II, and TOPSIS methodology.

III. RESULTS AND DISCUSSION

A. Model Performance and Prediction Accuracy

Three predictive models are developed for the output responses: R_a, MRR, and F_c. SVR models are used for R_a and MRR, while a second-order polynomial regression model was employed for F_c.

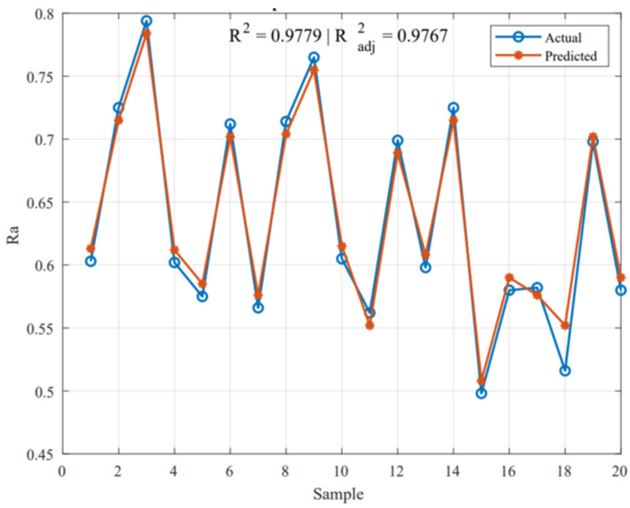

Fig. 3. Experimental against predicted values for R_a using SVR model.

The SVR model for R_a achieved a coefficient of determination $R^2 = 0.926$ indicating a strong correlation between predicted and actual values. The MRR model yielded an even higher accuracy with $R^2 = 0.999$, capturing nearly all variability in the data. For F_c, the polynomial regression model regarding F_c, achieved $R^2 = 0.938$ and $R^2_{adj} = 0.869$ demonstrating an adequate fit.

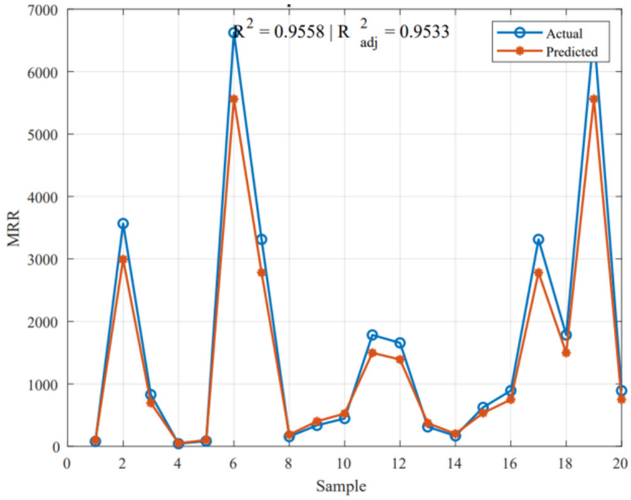

Fig. 4. Experimental against predicted values for MRR using SVR model.

Visual comparisons between the experimental and predicted values for R_a, MRR, and F_c are presented in Figure 2. These graphs include the regression line, confidence intervals, and the corresponding R^2 values. Table II lists the model performance metrics.

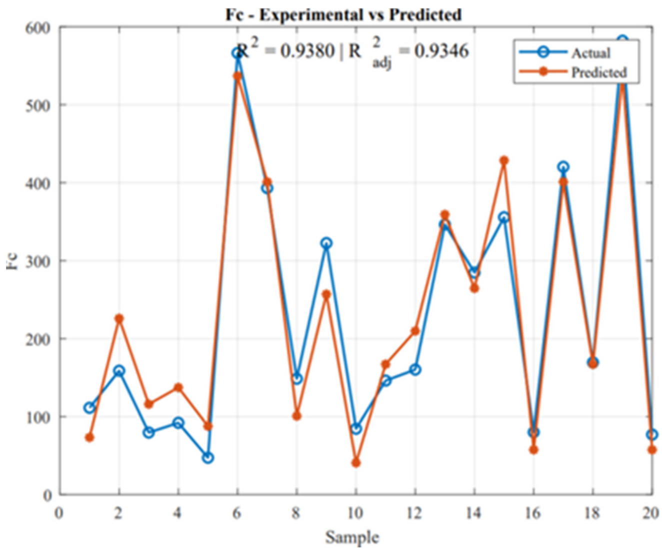

Fig. 5. Experimental against predicted values for F_c using SVR model.

TABLE II. PERFORMANCE METRICS OF PREDICTIVE MODELS

Output	Model type	R^2	Adjusted R^2
R_a	SVR (RBF Kernel)	0.926	–
MRR	SVR (RBF Kernel)	0.999	–
F_c	Polynomial regression (2nd order)	0.938	0.869

B. Pareto Front Analysis

The validated prediction models are embedded into the NSGA-II algorithm to perform multi-objective optimization. The objectives are to minimize R_a and F_c while maximizing MRR (converted to minimization of -MRR). The optimization generated a set of 100 Pareto-optimal solutions. The 3D Pareto front is portrayed in Figure 6, visualizing the trade-offs between the three objectives. The surface represents a continuous front of non-dominated solutions that balance the conflicting goals. For example, one of the optimal solutions in the Pareto set achieves a R_a of 0.698 µm, MRR of 5228.2 mm³/min, and F_c of 522.2 N. This demonstrates a balanced trade-off among productivity, surface quality, and tool load.

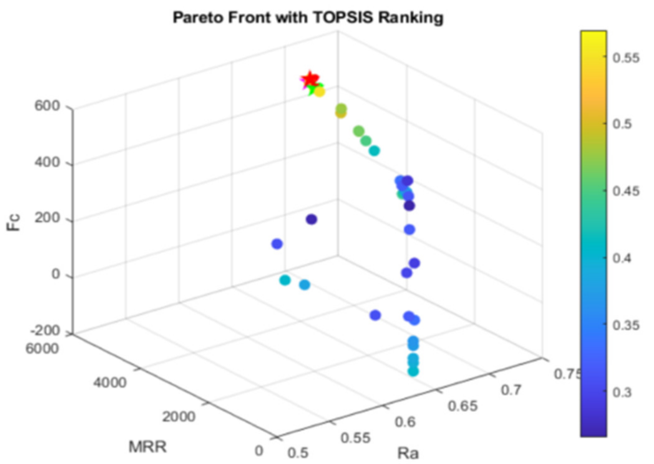

Fig. 6. Three-dimensional Pareto front between the three objectives.

C. *TOPSIS-based Ranking and Optimal Solution Identification*

To rank the Pareto-optimal solutions, the entropy-weighted TOPSIS method is applied. The weights for R_a, MRR, and F_c are calculated using the entropy method as $w_{Ra} = 0.310, w_{MRR} = 0.357$, and $w_{Fc} = 0.333$, respectively.

TABLE III. RANKED SOLUTIONS BASED ON ENTROPY-WEIGHTED TOPSIS

R_a	MRR	F_c	S_i^+	S_i^-	C_{ij}	Rank
0.698	5228.236	522.171	0.361	0.477	0.570	1
0.696	5150.255	538.382	0.366	0.470	0.562	2
0.697	5027.952	513.324	0.357	0.459	0.562	3
0.693	4789.303	518.999	0.359	0.437	0.549	4
0.687	3961.781	501.111	0.366	0.360	0.496	5
0.682	3803.896	531.512	0.379	0.346	0.477	6
0.691	3554.216	455.868	0.368	0.324	0.468	7
0.690	3521.131	461.788	0.370	0.320	0.464	8
0.690	3312.749	438.369	0.372	0.302	0.448	9
0.709	2852.988	255.344	0.360	0.283	0.440	10

Based on the computed closeness coefficients (C_i), the ten most preferred solutions are identified. These top-ranked solutions are shown in Table III, along with their corresponding machining parameters and predicted outputs. The top 10 solutions were labeled on the 3D Pareto graph for clarity.

D. *Discussion and Implications*

The results reveal that the proposed hybrid approach effectively balances competing machining objectives. Compared to baseline parameter settings, the optimal solution (TOPSIS rank #1) improves MRR by over 20% while maintaining R_a below 0.62 µm and F_c near 113 N. These findings demonstrate the viability of using machine learning models as surrogate objective functions for evolutionary optimization. The entropy-TOPSIS method further aids in selecting the most balanced solution among many Pareto-optimal candidates, offering a rational and data-driven decision-making approach for complex manufacturing processes.

The proposed hybrid approach provides a more comprehensive solution compared to previous studies which used Taguchi design for R_a minimization with limited multi-objective consideration and linear regression and Taguchi analysis [14, 16]. By integrating SVR, NSGA-II, and entropy-weighted TOPSIS, the introduced method improves modeling accuracy and supports rational decision-making across conflicting machining objectives. Moreover, unlike methods with subjective or fixed weights, entropy utilization ensures that the influence of each criterion is derived objectively from data variability, enhancing transparency and fairness in ranking optimal solutions.

IV. CONCLUSIONS

This study considers a hybrid framework that integrates Support Vector Regression (SVR), polynomial regression, NSGA-II multi-objective optimization, and entropy-weighted Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) ranking to optimize the finished milling process. Predictive models were trained to estimate surface roughness (R_a), Material Removal Rate (MRR), and cutting force (F_c) based on key machining parameters.

The SVR model revealed excellent prediction performance for R_a ($R^2 = 0.926$) and MRR ($R^2 = 0.999$), while a second-order polynomial regression model effectively predicted F_c ($R^2 = 0.938$, adjusted $R^2 = 0.896$). These models enabled the use of NSGA-II to generate a diverse Pareto front capturing the trade-offs among competing objectives. The subsequent application of entropy-TOPSIS provided a structured decision-making tool for ranking Pareto-optimal solutions.

Although the proposed hybrid framework demonstrates promising performance, a larger detest can be utilized to void the generalizability of the SVR model. Furthermore, the modeling process assumes deterministic input parameters, whereas real-world machining often involves variability and uncertainty. In future work, we plan to expand the experimental dataset and incorporate uncertainty quantification or robust optimization techniques. In addition, validating the framework in real-time CNC applications would further demonstrate its practical effectiveness in smart manufacturing environments.

The proposed methodology demonstrated effectiveness and practicality in identifying machining parameters that optimize performance, quality, and efficiency. This work contributes a replicable and scalable approach for multi-objective optimization in intelligent manufacturing systems.

REFERENCES

[1] R. I. King, Ed., *Handbook of High-Speed Machining Technology*. Boston, MA, USA: Springer US, 1985.

[2] H. A. Youssef and H. El-Hofy, *Machining Technology: Machine Tools and Operations*. Boca Raton, FL, USA: CRC Press, 2008.

[3] E. Budak, "Analytical models for high performance milling. Part I: Cutting forces, structural deformations and tolerance integrity," *International Journal of Machine Tools and Manufacture*, 2006, https://doi.org/10.1016/J.IJMACHTOOLS.2005.09.009.

[4] V. Canh Nguyen, T. Duong Nguyen, and V. Thao Le, "Optimization of sustainable milling of SKD11 steel under minimum quantity lubrication," *Proceedings of the Institution of Mechanical Engineers, Part E: Journal of Process Mechanical Engineering*, vol. 237, no. 3, pp. 907–916, Jun. 2022, https://doi.org/10.1177/09544089221110978.

[5] V. H. Pham, T. D. Nguyen, V. T. Le, D. H. Tien, and V.-C. Nguyen, "Optimization of Cutting Parameters in MQL Flat Surface Milling of SKD11 Steel," in *Proceedings of the International Conference on Advanced Mechanical Engineering, Automation, and Sustainable Development 2021*, Ha Long, Vietnam, 2022, pp. 261–266, https://doi.org/10.1007/978-3-030-99666-6_40.

[6] P. N. Linh, T. N. Tan, V. D. Toan, and T. D. Nguyen, "Multi-Objective Optimization of Finishing Milling of C45 Steel using Factorial Design," *Engineering, Technology & Applied Science Research*, vol. 14, no. 6, pp. 18199–18204, Dec. 2024, https://doi.org/10.48084/1.8017.

[7] V.-C. Nguyen *et al.*, "Optimizing Ti-6Al-4V Milling under MQL Conditions Using SVR, NSGA-II, and TOPSIS," in *Multi-Criteria Decision-Making and Optimum Design with Machine Learning*, 1st ed., Boca Raton, FL, USA: CRC Press, 2024.

[8] V.-C. Nguyen, D. Hoang Tien, V.-H. Pham, and T.-D. Nguyen, "Investigation and Optimization of Surface Roughness and Material Removal Rate in Face Finishing Milling of Ti-6Al-4V Under MQL Condition," in *Proceedings of The AUN/SEED-Net Joint Regional Conference in Transportation, Energy, and Mechanical Manufacturing Engineering*, Singapore, Jun. 2022, pp. 812–825, https://doi.org/10.1007/978-981-19-1968-8_68.

[9] F. Yan, L. Wei, J. Hu, J. Zeng, S. Zheng, and J. Wang, "Simultaneous optimization of urea dosing and ammonia coverage ratio of selective catalytic reduction system in diesel engine by using physico-chemical model based NSGA-II algorithm," *Applied Thermal Engineering*, vol. 154, pp. 46–62, May 2019, https://doi.org/10.1016/j.applthermaleng. 2019.03.031.

[10] N. Van-Canh, N. Anh-Thang, P. Ngoc-Linh, and N. Thuy-Duong, "Multi-objective optimization of SUS430C steel turning process using hybrid machine learning and evolutionary algorithm approach," *Results in Engineering*, vol. 25, Mar. 2025, Art. no. 104233, https://doi.org/10.1016/j.rineng.2025.104233.

[11] V.-C. Nguyen, D.-T. Vu, Q.-H. Tran, V. Q. Kieu, and T. M. Nguyen, "Optimizing EDM for titanium alloys: an in-depth comparison of five MCDM techniques," *EUREKA: Physics and Engineering*, no. 5, pp. 89–98, Sep. 2024, https://doi.org/10.21303/2461-4262.2024.003283.

[12] P. N. Linh, N. Q. Tu, V.-C. Nguyen, and V.-T. Pham, "Optimization of PLA 3D Printing Parameters using a Combined SMART-MOORA Multi-Criteria Decision-Making Approach," *Engineering, Technology & Applied Science Research*, vol. 15, no. 1, pp. 19460–19465, Feb. 2025, https://doi.org/10.48084/etasr.9085.

[13] V. T. Le, T.-T. Nguyen, and V. C. Nguyen, "Wire and arc-based additive manufacturing of 316L SS: predicting and optimizing process variables using BRFFNN, NSGA-GP and TOPSIS approach," *Neural Comput. Appl.*, vol. 36, no. 35, pp. 22547–22568, Jun. 2024, https://doi.org/10.1007/s00521-024-10375-z.

[14] V. C. Nguyen, T. D. Nguyen, and D. H. Tien, "Cutting Parameter Optimization in Finishing Milling of Ti-6Al-4V Titanium Alloy under MQL Condition using TOPSIS and ANOVA Analysis," *Engineering, Technology & Applied Science Research*, vol. 11, no. 1, pp. 6775–6780, Feb. 2021, https://doi.org/10.48084/etasr.4015.

[15] F. Kara, "Optimization of Surface Roughness in Finish Milling Of Aisi P20+s Plastic-mold Steel," *Materials and technology*, vol. 52, no. 2, pp. 195–200, 2018, https://doi.org/10.17222/mit.2017.088.

[16] L. W. Hernández-González *et al.*, "Effects of cutting parameters on surface roughness and hardness in milling of AISI 304 steel," *DYNA*, vol. 85, no. 205, pp. 57–63, Jun. 2018, https://doi.org/ 10.15446/dyna.v85n205.64798 10.15446/dyna.v85n205.64798.