An Integrated NSGA-II and EAMR Approach for the Optimal Design of a Two-Stage Gear Transmission

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ABSTRACT

This paper presents a Multi-Objective Optimization Problem (MOOP) for the design of a two-stage gear reducer using the Non-dominated Sorting Genetic Algorithm II (NSGA-II). The optimization simultaneously targets the minimization of the gearbox volume and the maximization of the mechanical efficiency. Three key design variables are considered: the gear ratio of the high-speed stage (u_1), the Face Width Coefficient (FWC) of the high-speed stage (X_{ba1}), and that of the low-speed stage (X_{ba2}). A physics-based model is developed to compute the gearbox volume and efficiency from these design parameters. NSGA-II was employed to generate Pareto-optimal solutions, and the Evaluation based on Distance from Average Solution Ranking (EAMR) method was deployed to identify the most balanced design for each transmission ratio (u_h). The results showed that as u_h increases from 5 to 35, the gearbox volume rises from 237.08 dm^3 to 272.72 dm^3, while the efficiency decreases from 95.84% to 90.82%. A strong linear relationship (R^2 = 0.9941) between u_1 and u_h was discovered, enabling deriving a general design rule for the gear ratio allocation in preliminary gearbox design.

Keywords-two-stage gearbox; multi-objective optimization; NSGA-II; gear ratio; face width coefficient; efficiency; volume; design trade-off

I. INTRODUCTION

Gear transmission systems play a significant role in various industrial applications due to their ability to transfer power efficiently and reliably. The design of such systems often requires balancing multiple conflicting objectives, such as minimizing the structural volume while maximizing the mechanical efficiency. This trade-off has motivated extensive research into Multi-Objective Optimization (MOO) techniques applied to the gearbox design.

Early efforts employed evolutionary algorithms to automate the preliminary design phase of the gear drives, significantly reducing manual computations and design time [1]. Later studies integrated metaheuristic approaches, such as Particle Swarm Optimization (PSO) and Simulated Annealing (SA) to optimize the gear train weight while satisfying performance constraints [2]. Authors in [3] applied the Taguchi method and Grey Relational Analysis (GRA) to optimize a two-stage helical gearbox, demonstrating improvements in both design quality and decision-making clarity. Among the existing evolutionary algorithms for MOO, the NSGA-II [4] has become one of the most widely used due to its effectiveness, fast non-dominated sorting, and crowding distance mechanisms. NSGA-II has proven effective across various engineering domains [5], particularly in gear system design [6], supported by a strong theoretical foundation and implementation flexibility [7]. Authors in [8] compared several evolutionary algorithms for gear system design and concluded that NSGA-II provides a well-balanced compromise between convergence and diversity.

The first applications of genetic algorithms in gear design were encountered in [9], where an optimization method for gear trains was proposed. Authors in [10] applied NSGA-II specifically to two-stage helical gear trains and confirmed its effectiveness in exploring design trade-offs. Similarly in [11], multi-objective evolutionary algorithms were utilized to address the complexity of multi-speed gearbox optimization. To enhance decision-making in multi-criteria problems, various MCDM methods have also been integrated with optimization algorithms. Authors in [12] employed the Measurement of Alternatives and Ranking according to Compromise Solution (MARCOS) method to identify optimal designs for a two-stage gearbox, while another study by the same group [13] combined MCDM techniques with engineering constraints to develop more practical design solutions. Several other researchers have explored NSGA-II-based approaches to gear design. Authors in [14] optimized a two-stage spur gearbox, showing substantial improvements in both mass and performance. An aerospace sector study [15] used a heuristic algorithm for aeroengine gearbox optimization, emphasizing the importance of compact design and transmission reliability. Authors in [16] focused on reducing the transmission error and improving efficiency through multi-objective gear unit optimization, while authors in [17] addressed stochastic behavior in synchronizer and selector mechanisms. Authors in [18] reduced the height of a two-stage helical gearbox while increasing efficiency. Authors in [19] employed NSGA-II combined with decision-making techniques for spur gear optimization. Authors in [20] investigated the design trade-offs in spur gear systems. Authors in [21] focused on the volume-efficiency trade-off, aligning with the objectives of the present paper. Robust design aspects have been also considered, as seen in [22], where gear microgeometry under uncertainty was optimized. Authors in [23] extended the optimization frameworks to planetary gear systems, while authors in [24] used a combination of optimization and experimental validation for gearbox compactness improvement.

MOO frameworks for two-stage gearboxes have been further improved by using MCDM techniques. For instance, authors in [25] proposed a novel design approach for a two-stage helical gearbox with double gear sets in the first stage, using the EAMR method. Building on these prior efforts, this study aims to optimize a two-stage gear reducer using NSGA-II with the objectives of minimizing the gearbox volume and maximizing efficiency. The design parameters include the gear ratio of the high-speed stage (u_1) and the FWCs of the high-speed and low-speed stages (X_{ba1} and X_{ba2}, respectively). The study contributes a physics-based design model and an exploration of the Pareto frontier to provide designers with practical and efficient gearbox configurations.

II. THE OPTIMIZATION PROBLEM

A. *Calculating Gearbox Volume*

The gearbox volume V_{gb} of a two-stage helical gearbox is determined with the help of (1) and Figure 1:

$$V_{gb} = L \times B \times H \tag{1}$$

In which, *L, B,* and *H* are calculated by:

$$L = d_{w11} + d_{w21}/2 + d_{w12}/2 + d_{w22} + 2 \cdot \delta \tag{2}$$

$$B = b_{w1} + b_{w2} + 6 \cdot \delta \tag{3}$$

$$H = max(d_{w21}, d_{w22}) + 6.5 \cdot \delta \tag{4}$$

where δ is a clearance parameter that typically ranges from 7 to 10 mm. The terms b_{wi}, d_{w1i}, and d_{w2i} (where i = 1 or 2 for each gear stage) represent the face width of the gear, the pitch diameter of the pinion, and the pitch diameter of the gear, respectively. These values can be calculated using:

$$b_{wi} = X_{bai} \times a_{wi} \tag{5}$$

$$d_{w1i} = 2 \times \frac{a_{wi}}{(u_i+1)} \tag{6}$$

$$d_{w2i} = 2u_i \times \frac{a_{wi}}{(u_i+1)} \tag{7}$$

where X_{bai} and a_{wi} (i =1÷2) are the FWC and the center distance of stage I, respectively. The center distance a_{wi} is computed by:

$$a_{wi} = k_a \times (u_i + 1)\sqrt[3]{\frac{T_{1i} \times k_{H\beta}}{(AS_i)^2 \times u_i \times X_{bai}}} \tag{8}$$

where T_{1i} (i=1÷2) denotes the pinion torque of stage i and it is calculated using:

$$T_{1i} = \frac{T_r}{\prod_{j=i}^{3}(u_i \cdot \eta_{hg}^{3-i} \cdot \eta_{be}^{4-i})} \tag{9}$$

B. *Calculating Gearbox Efficiency*

The gearbox efficiency (%) can be calculated by:

$$\eta_{gb} = 100 - \frac{100 \cdot P_l}{P_{in}} \tag{10}$$

where P_l represents the total gearbox power loss found by:

$$P_l = P_{lg} + P_{lb} + P_{ls} + P_{Z0} \tag{11}$$

where P_{lg}, P_{lb}, P_{ls}, and P_{Z0} are the power loss in the gears, in bearings, seals, and in the idle motion. These parameters can be computed by:

C. *Objective Functions*

In this work, the optimization is framed as a bi-objective minimization problem, addressing two fundamental performance aspects of a two-stage helical gearbox:

- Minimizing the gearbox volume:

$$minf_1(X) = A_b \tag{12}$$

- Maximizing the gearbox efficiency:

$$P_l = P_{lg} + P_{lb} + P_{ls} + P_{Z0} \tag{13}$$

The design variable vector X represents the key geometric and performance-related parameters of the gearbox, while five parameters u_1, X_{ba1}, X_{ba2}, AS_1, and AS_2 are typically used to define the gearbox geometry. It has been shown that AS_1 and AS_2 usually reach their upper bounds in optimal configurations [8]. Therefore, only the three most influential and adjustable parameters - u_1, X_{ba1}, and X_{ba2} - are selected as decision variables, yielding:

$$X = \{u_1, Xba_1, Xba_2\} \tag{14}$$

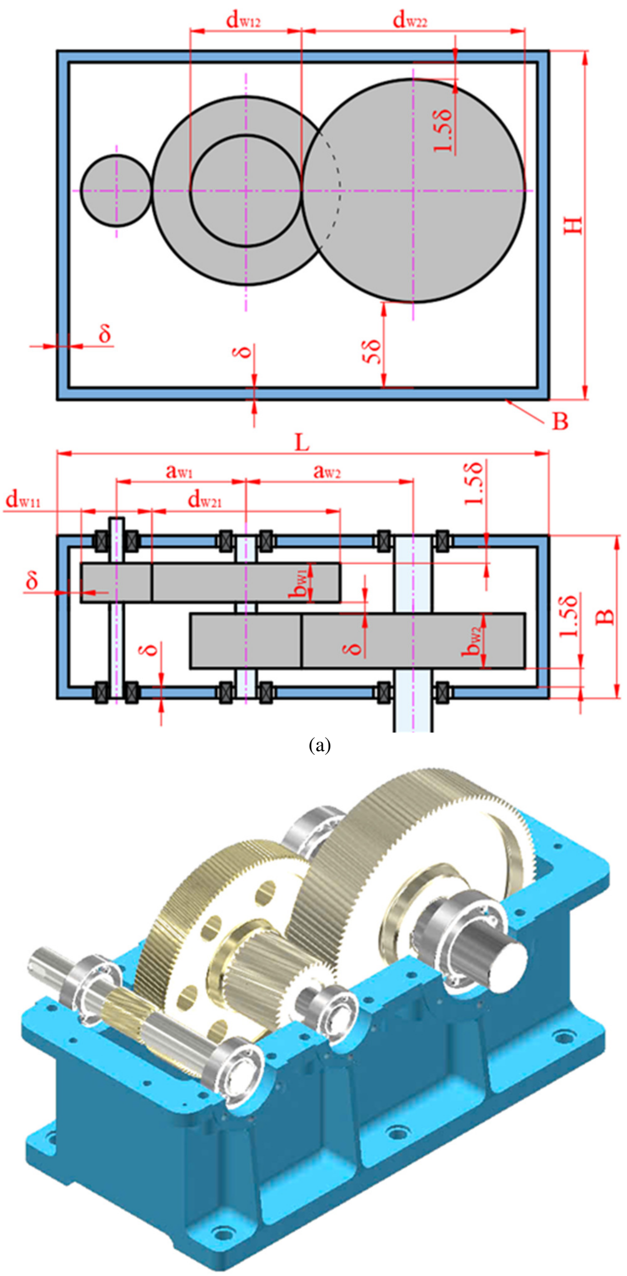

Fig. 1. Two-stage helical gearbox: (a) schema for the determination of gearbox volume, (b) 3D kinematic model.

D. Constrains

For a two-stage helical gearbox, u_i = 1÷9; X_{bai} =0.25÷0.4 (i = 1÷2). Consequently, the MOOP involves the following limitations:

$$1 \le u_i \le 9 \tag{15}$$

$$0.25 \le Xba_i \le 0.4 \tag{16}$$

III. OPTIMIZATION METHODOLOGY

A. NSGA II Method

To solve the bi-objective optimization problem involving gearbox volume minimization and efficiency maximization, the NSGA-II is adopted. NSGA-II is a widely used evolutionary algorithm designed for solving multi-objective problems by simultaneously exploring the trade-off frontier while maintaining solution diversity [4-5].

NSGA-II introduces three key mechanisms to enhance performance over earlier genetic algorithms:

- Fast non-dominated sorting to classify individuals based on Pareto dominance.
- Crowding distance assignment to ensure solution diversity.
- Elitism to preserve the best individuals across generations [4].

B. EAMR Method

The EAMR method was applied in this study as a Multi-Criteria Decision-Making (MCDM) technique to rank the Pareto-optimal solutions generated by the NSGA-II algorithm. To properly implement this approach, the following steps are strictly followed [1]:

- Step 1: Construction of decision-making matrix:

$$X_d = \begin{bmatrix} x_{11}^d & \cdots & x_{1n}^d \\ x_{21}^d & \cdots & x_{21}^d \\ \vdots & \cdots & \vdots \\ x_{m1}^d & \cdots & x_{mn}^d \end{bmatrix} \tag{17}$$

where $1 \le d \le k$, d is the decision maker's number, k denotes the total number of decision makers, and n is the number of the evaluation criteria.

- Step 2: Calculation of the average score of each alternative for each criterion:

$$\bar{x}_{ij} = \frac{1}{k}\left(x_{ij}^1 + x_{ij}^2 + \cdots + x_{ij}^k\right) \tag{18}$$

- Step 3: Creation of the weight matrix and calculation of the average weights:

$$\bar{w}_j = \frac{1}{k}\left(w_j^1 + w_j^2 + \cdots + w_j^k\right) \tag{19}$$

- Step 4: Normalization of the decision matrix, where each normalized value n_{ij} is calculated as:

$$n_{ij} = \frac{\bar{x}_{ij}}{e_j} \tag{20}$$

where:

$$e_j = max_{i\in\{1,\ldots,m\}}(\bar{x}_{ij}) \quad (21)$$

- Step 5: Calculation of the weighted normalized matrix:

$$v_{ij} = n_{ij} \cdot \bar{w}_j \quad (22)$$

- Step 6: Computation of the overall score for each alternative.

For the efficiency objective (benefit criterion):

$$G_i^+ = v_{i1}^+ + v_{i2}^+ + \cdots + v_{im}^+ \quad (23)$$

For the the gearbox bottom area (cost criterion):

$$G_i^- = v_{i1}^- + v_{i2}^- + \cdots + v_{im}^{-+} \quad (24)$$

- Step 7: Calculation of the Ranking Value (*RV*) and the overall evaluation score S_i.

The ranking score S_i for each alternative is computed as:

$$S_i = \frac{RV(G_i^+)}{RV(G_i^-)} \quad (25)$$

where $RV(\cdot)$ denotes the relative ranking value.

- Step 8: Selection of an optimal alternative.

The best alternative is the one with the highest S_i value, indicating the most balanced trade-off between efficiency and minimal gearbox size.

C. Optimization Procedure

The NSGA-II procedure begins with an initial population of candidate solutions, where each individual represents a unique combination of the decision variables $X = \{u_1, Xba_1, Xba_2\}$. The steps are:

- Initialization: Generation of an initial population of size N with randomly selected values of u_1, X_{ba1}, and X_{ba2} within their feasible ranges.
- Fitness Evaluation: Computing the two objective functions for each individual:

  $f_1(X)$: Gearbox volume to be minimized.

  $f_2(X)$: Transmission efficiency to be maximized.
- Non-dominated Sorting: Sorting of the population into different Pareto fronts based on dominance ranking.
- Crowding Distance Calculation: Assigning crowding distance values to maintain the solution diversity within each front.
- Selection: Application of the binary tournament selection using dominance rank and crowding distance.
- Genetic Operations: Generation of offspring using crossover and mutation.
- Population Update: Combining parent and offspring populations, sorting again, and selecting the top *NNN* individuals for the next generation using the elitist strategy.
- Termination: Repeating steps 2÷7 until a predefined number of generations is reached or convergence is observed.

The final output is a set of Pareto-optimal solutions, which represent the best trade-offs between gearbox compactness and efficiency.

D. Parameter Settings

In this study, the NSGA-II algorithm is implemented in MATLAB. The key parameters are:

- Population size: 100.
- Number of generations: 200.
- Crossover probability: 0.9.
- Mutation probability: 0.1.
- Distribution indices: 20 for crossover and 20 for mutation.

These settings ensure a good balance between convergence speed and exploration of the design space.

IV. RESULTS AND DISCUSSION

A. Relationship Between u_h and Optimum First-Stage Gear Ratio u_1

Figure 2 illustrates the linear regression of the mean values of u_1 against the overall transmission ratio u_h. This trend provides a simple design rule for allocating gear ratios between the two stages in the preliminary gearbox design. A strong linear correlation is observed with a coefficient of determination R^2=0.9717, suggesting that the optimal distribution of the gear ratio between the two stages can be approximated by a linear relationship:

$$u_1 = 0.1693 \times u_h + 1.5319 \quad (26)$$

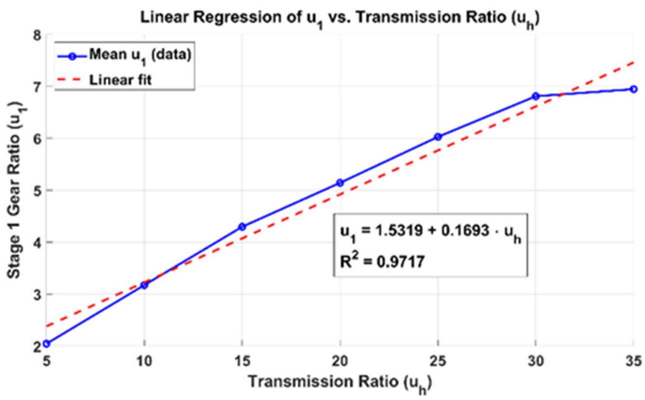

Fig. 2. Linear Regression of u_1 versus gearbox ratio (u_h).

B. Influence of u_h on Gearbox Volume and Efficiency

The impact of the overall transmission ratio u_h on the mean gearbox volume and efficiency is shown in Figure 3. As u_h increases, the gearbox volume increases nearly linearly, while efficiency generally declines. This inverse relationship highlights a typical trade-off in the gearbox design: higher reduction ratios lead to bulkier systems with greater mechanical losses.

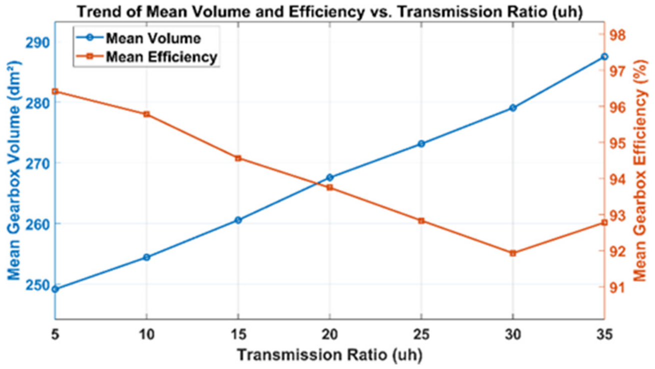

Fig. 3. Trend of mean volume and efficiency versus gearbox ratio (u_h).

For example, at u_h = 5, the mean volume is approximately 248.4 dm^3 with a high efficiency of 97.4%, whereas at u_h = 35, the volume increases to around 288.2 dm^3 and efficiency drops to 92.9%. These trends reflect the design challenges in high-reduction gearboxes where compactness and efficiency become conflicting objectives.

C. Pareto Front Distribution across Different u_h Values

Figure 4 displays the Pareto-optimal solutions obtained for each transmission ratio u_h. Each color represents a distinct u_h value, with the trade-off between gearbox volume and efficiency clearly visualized.

As u_h increases, the entire Pareto front shifts toward a higher volume and lower efficiency, confirming the trends observed in Figure 4. Moreover, the density and shape of each Pareto front reflect the sensitivity of the solution space to design variables. For lower u_h, the fronts are flatter and positioned higher on the efficiency axis, suggesting more favorable trade-offs. For higher u_h, the solutions tend to cluster near the lower-efficiency region, making design decisions more critical.

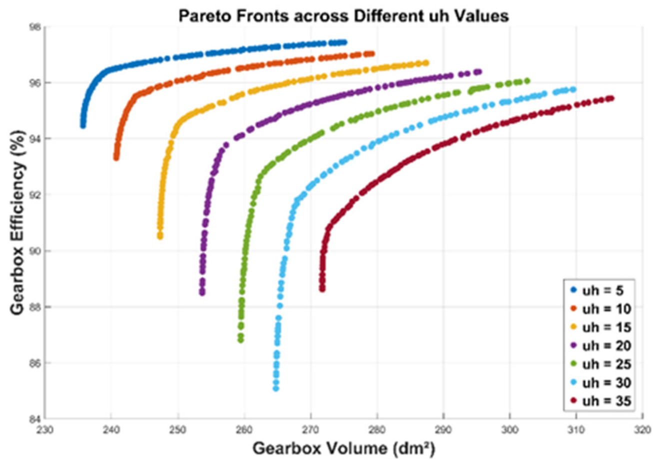

Fig. 4. Trend of mean volume and efficiency versus gearbox ratio (u_h).

D. Summary of Optimal Designs

Table I summarizes the best solutions obtained from NSGA-II for each value of u_h, selected based on the optimal trade-off between compactness and efficiency. It is observed that u_1 increases progressively with u_h, aligning with the regression trend evidenced in Figure 4. The FWCs (X_{ba1} and X_{ba2}) remain close to their lower bounds, indicating their marginal impact within the feasible design space. Efficiency consistently decreases while volume increases with u_h, validating the trade-off behavior seen in the Pareto distribution.

TABLE I. BEST SOLUTIONS FOR EACH UH VALUE OBTAINED FROM NSGA-II

u_h	u_1	u_2	X_{ba1}	X_{ba2}	Volume (dm^3)	Efficiency (%)
5	1.27	3.95	0.2500	0.4000	275.03	97.43
10	2.04	4.91	0.2500	0.3999	279.16	97.03
15	2.66	5.63	0.2500	0.3990	287.54	96.70
20	3.22	6.21	0.2500	0.3999	295.32	96.38
25	3.73	6.70	0.2500	0.4000	302.56	96.06
30	4.28	7.01	0.2500	0.3999	309.71	92.44
35	4.76	7.36	0.2500	0.3999	316.42	92.89

E. Influence of Gear Geometry and Type:

The present study adopts a two-stage helical gear configuration, selected for its superior performance in terms of load-carrying capacity, transmission smoothness, and mechanical efficiency. Helical gears are widely preferred in industrial gear reducers due to their larger contact ratio and ability to transmit higher loads at moderate sizes, which contributes positively to both volume minimization and efficiency maximization.

- The design variables used in the optimization—gear ratio of the first stage (u_1) and FWCs (X_{ba1}, X_{ba2})—play a critical part in shaping the gearbox geometry.
- A higher value of u_1 shifts more reduction in the first stage, leading to smaller torque in the second stage, which typically results in reduced gear sizes and a lower total volume.
- The FWCs X_{ba1} and X_{ba2} directly influence the gear face width, and thus the gearbox width (B), as shown in (3). These parameters also indirectly influence the mechanical losses, particularly regarding bearing and gear meshing friction.

The optimization results show that X_{ba1} and X_{ba2} tend to be near their lower bounds in the optimal designs, suggesting that narrower gears may be sufficient when combined with properly distributed gear ratios. This reflects a desirable trade-off between compactness and strength made possible by the geometric advantages of helical gearing.

F. Optimal Solution Selection via the EAMR Method

While NSGA-II provides a diverse set of Pareto-optimal solutions for each transmission ratio u_h, the practical implementation of the gearbox design often requires the selection of a single best compromise solution. To address this need, the EAMR method is employed as a decision-making technique to filter the most balanced solution from each Pareto front. EAMR is an MCDM approach that evaluates alternatives based on their normalized distance from an idealized average solution, considering all objectives simultaneously. The two objectives of this study, namely the minimization of the gearbox volume and maximization of efficiency, are first normalized and transformed into benefit criteria. Then, EAMR ranks the alternatives based on their closeness to the average solution in the normalized space.

By applying the EAMR method to the Pareto solutions obtained from NSGA-II for each value of u_h, a single best solution was identified. These EAMR selected solutions are summarized in Table II and plotted in Figure IV. The results demonstrate that the selected solutions are well-balanced, offering moderate volumes and high efficiencies compared to the extreme Pareto points.

For instance, at $u_h = 5$, the EAMR solution yields a volume of 237.08 dm³ with an efficiency of 95.84%, while for u_h =35, the corresponding optimal solution has a volume of 272.72 dm³ and an efficiency of 90.82%. The selected values of u_1 across different values of u_h levels follow a near-linear trend (as analyzed above), confirming the stability of EAMR in identifying practically feasible and consistent designs. A comparison between the linear regression results of u_1 versus u_h, based on two different data sources, reveals notable distinctions in model consistency and trend clarity. The regression obtained from the EAMR-selected optimal solutions (Figure 4) demonstrates an excellent linear fit, with the equation $u_1 = 1.7957 + 0.2046 \times u_h$ and a coefficient of determination R^2=0.9941, indicating a highly consistent behavior across different transmission ratios.

TABLE II. BEST SOLUTIONS FOR EACH VALUE OBTAINED FROM NSGA-II

u_h	u_1	X_{ba1}	X_{ba2}	Volume (dm^3)	Efficiency (%)
5	2.54	0.30	0.40	237.08	95.84
10	3.92	0.27	0.40	242.65	95.16
15	5.04	0.25	0.40	250.10	94.55
20	6.07	0.25	0.40	256.60	93.62
25	6.96	0.25	0.40	262.87	92.80
30	7.86	0.25	0.40	268.21	91.85
35	8.82	0.25	0.40	272.72	90.82

In contrast, the regression based on the mean values of all Pareto-optimal solutions (Figure 1) yields a lower slope ($u_1 = 1.6496 + 0.1652 \times u_h$) and a slightly reduced fit quality (R^2 = 0.9688). The latter also exhibits a slight curvature, especially at higher u_h values, suggesting increased variability within the Pareto sets. These findings highlight that the EAMR method tends to favor solutions with higher stage-1 gear ratios, likely to enhance the transmission efficiency in the fast stage while simplifying the design of the slower stage. Moreover, the near-perfect linear trend observed in the EAMR-based regression indicates that the selected solutions are not only well-balanced in terms of performance, but also highly suitable for preliminary design modeling and rule-based automation. In contrast, the average-based model captures general trends but lacks the predictive precision and structural consistency required for practical application.

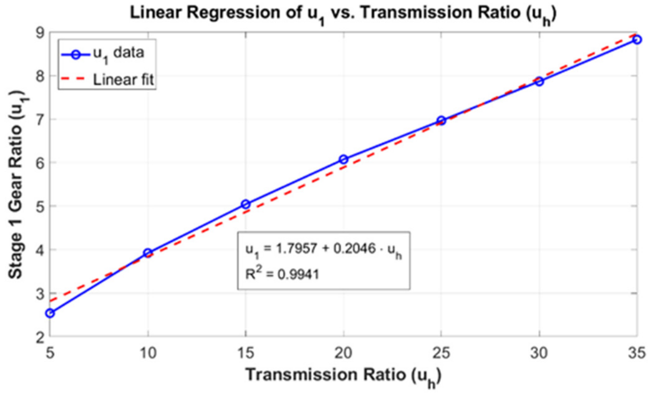

Fig. 5. Linear regression of u_1 versus gearbox ratio (u_h) using EAMR.

V. CONCLUSIONS

In this study, a Multi-Objective Optimization (MOO) framework was developed for the design of a two-stage helical gearbox using the Non-dominated Sorting Genetic Algorithm II (NSGA-II). The optimization simultaneously targeted the minimization of the gearbox volume and the maximization of the transmission efficiency, based on three key design variables: the gear ratio of the first stage (u_1), and the Face Width Coefficients (FWCs) of the first and second stages (X_{ba1} and X_{ba2}).

The results revealed a clear trade-off between compactness and efficiency, with higher overall transmission ratios (u_h) leading to larger gearbox volumes and lower efficiencies. A strong linear relationship was observed between u_1 and u_h, offering a practical rule for distributing gear ratios across stages during the preliminary design. Additionally, the Pareto fronts generated for different u_h values provided valuable insights into the design space, and supported the selection of optimal configurations under conflicting objectives.

Compared to prior research, this study offers a more focused analysis on the relationship between the input parameters and objective trends across varying transmission ratios. The integration of regression analysis and trade-off visualization strengthens the decision-making process for gearbox designers.

The novelty of this work lies in its systematic exploration of how the optimal distribution of design parameters-particularly u_1-evolves with respect to varying u_h, and how this evolution can be generalized through linear approximation to guide future designs.

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