

Modeling Extreme Risk in Major U.S. Tech Stocks: Integrating GARCH, EVD, and CVaR

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ABSTRACT

This study aimed to identify the most accurate method to assess the extreme risk associated with major technology stocks in the United States, focusing on Apple, Microsoft, Alphabet, Amazon, and NVIDIA stocks. In the intricate and ever-evolving financial landscape, conventional methods such as Value-at-Risk (VaR) frequently prove to be inadequate in capturing events that are rare but have the potential to have a substantial impact. To address this shortcoming, this study proposes a novel integration of the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model with the Extreme Value Distribution (EVD) approach, complemented by Conditional Value at Risk (CVaR), to formulate a more robust and conservative risk measurement. The findings indicate that the GARCH-EVD-CVaR approach generates risk estimates with greater precision, particularly in volatile market contexts. Specifically, the GARCH-EVD model reduced the Akaike Information Criterion (AIC) values by an average of 25.3 points and decreased VaR estimates by up to 14% at the confidence level of 99%. In practice, this model can help institutional investors conduct more accurate portfolio stress tests and offer regulators a framework for assessing systemic tail risk under volatile market conditions.

Keywords-GARCH; VaR; CVaR; EVD; financial risk; stock

I. INTRODUCTION

Stock market volatility has intensified in recent years due to global uncertainties, including significant technological disruption and the COVID-19 crisis, highlighting the need for more accurate risk assessment models [1, 2]. The financial landscape is continually reshaped by innovations, such as the proposed introduction of Central Bank Digital Currencies (CBDC) to modernize the financial infrastructure [3], which further contribute to the complex risk environment, especially for technology-driven sectors. Although traditional methods such as financial ratio analysis have long served as foundational predictors of severe outcomes, such as corporate bankruptcy [4], the unique characteristics of modern financial markets and assets demand more sophisticated approaches. For example, advanced GARCH-family models, such as Dynamic Conditional Correlation (DCC) and Multivariate Generalized Autoregressive Conditional Heteroscedasticity (MGARCH), are employed to understand risk drivers in other volatile digital assets, such as Bitcoin [5]. Recognizing these trends and the

specific need for enhanced risk assessment in the tech sector, this study focuses on measuring the stock risk of five leading US technology companies using an integrated GARCH-EVD-CVaR model and comparing its accuracy with traditional models.

This study is also a development of the findings in [6], which highlight the importance of using fat-tailed distributions in financial risk prediction. This paper addresses the limitations of traditional GARCH-VaR frameworks by integrating Extreme Value Theory (EVT) and Conditional Value-at-Risk (CVaR). Although GARCH captures volatility clustering, it underrepresents extreme losses. By incorporating EVT and CVaR, the proposed approach provides enhanced sensitivity to tail risk. This integration builds on recent advances in risk modeling, such as [7], which emphasized the importance of capturing nonlinear and fat-tailed dependencies in financial returns, and [8], which discussed tail dependence in macrofinancial risk environments.

In [9], the GARCH model was introduced to capture volatility clustering in financial returns, particularly effective in high-frequency data. GARCH(1,1) is the most commonly used form to model volatility fluctuations, where the variance of the current residual is affected by past residuals and past variances [5, 10]. This dynamic nature makes GARCH highly suitable for modeling financial time-series data, especially in rapidly changing markets such as technology stocks [11, 12]. In [13], the existence of volatility clustering and long-memory phenomena in technology stocks indicates that the GARCH model is very suitable for estimating risk in financial market data. The GARCH model is very suitable for stock return data, which often shows volatility clustering characteristics [14]. This has been proven by many studies, such as [15, 16], which showed that GARCH can capture recurrent market shocks in the short and long term. In [17], it was highlighted that using dynamic beta within the GARCH framework increases the sensitivity of the model to market shocks, enhancing its relevance in volatile sectors.

In risk estimation, Value-at-Risk (VaR) is a statistical method that measures the maximum loss that can occur in a portfolio with a certain level of confidence [18]. However, several studies have noted that VaR often fails to account for the severity of extreme losses, particularly during periods of high market turbulence [19, 20]. Although VaR is simple and popular, it does not provide information on the amount of loss if this limit is exceeded. Although VaR estimates the maximum expected losses, it fails to measure the severity of tail losses. This was addressed by developing CVaR, a coherent measure that quantifies the expected shortfall beyond the VaR threshold [21]. CVaR is considered a coherent risk measure, as it satisfies the characteristics of subadditivity and monotonicity, whereas VaR is non-subadditive [22, 23]. CVaR provides a more reliable measure under extreme conditions by calculating the expected loss beyond the VaR threshold [24, 25]. Various studies have shown that CVaR is more accurate in modeling financial risk, especially under highly volatile market conditions [26, 27]. The application of CVaR is also very helpful in mitigating the risk of highly volatile stocks of the technology sector [28] and has shown more stable results in crisis conditions for technology portfolios [29].

Extreme Value Theory (EVT) is a statistical technique used to model extreme behavior of probability distributions, especially data on the right or left tail [30]. This distribution is very effective in financial settings, as it can capture the risk of rare events, such as stock market crashes [31, 32]. In [33], the use of Generalized Extreme Value (GEV) to measure stock risk provides more conservative estimation results than the normal distribution. The distribution is divided into three main types, namely Gumbel, Fréchet, and Weibull, with each type having different shape parameters based on the type of tail captured [34, 35]. Extreme Value Distribution (EVD) has been widely applied to model the probability of rare and severe events in financial data. EVD enables better estimation of tail behavior beyond the assumptions of normal distribution [36, 37]. Specifically, the Generalized Extreme Value (GEV) distribution, including its subtypes (Gumbel, Fréchet, and Weibull), offers flexible modeling of extreme stock returns [38, 39]. As explained in [6, 40], the integration of the EVD and

GARCH models results in a very powerful combination for measuring tail risk, which uses fat-tailed distributions to improve the accuracy of risk estimation. According to [41], a multiscale approach based on EVD can improve the accuracy of short-term volatility prediction. Furthermore, according to [42], the fat-tailed distribution is highly relevant to capture the effect of risk contagion between financial sectors.

Although previous studies have separately employed GARCH [6] for volatility and EVT for tail risk, only limited research has successfully integrated both with CVaR in a single framework. This study addresses this gap by providing a unified GARCH-EVD-CVaR model that simultaneously captures volatility persistence and tail-event severity, aligning with recent calls to adopt tail-sensitive modeling in systemic risk monitoring and sustainable finance practices.

II. RESEARCH METHOD

This study employs a quantitative method to analyze the daily stock returns of five leading US technology corporations over a 10-year period. The following companies, referred to herein as Company 1 (Alphabet), Company 2 (Amazon), Company 3 (Apple), Company 4 (Microsoft), and Company 5 (NVIDIA), were selected for this analysis. Historical daily stock return data were obtained from Yahoo Finance. To illustrate the structure of the dataset used in this investigation, Table I provides a sample of Alphabet Inc.'s historical stock price data. The dataset spans from January 2, 2015, to December 31, 2024, and includes daily records of opening and closing stock prices, as well as calculated daily returns. This sample reflects the structure and content of the full dataset for all companies. The focus is on the estimation and modeling of stock return volatility using the GARCH(1,1) model, followed by an assessment of financial risk using VaR and CVaR, both calculated using EVT.

TABLE I. SAMPLE STRUCTURE OF DAILY STOCK RETURN DATA

Date	Open	Close	Daily return
Jan 2 2015	26.63	26.48	-0.0192598
...
Dec 31, 2024	191.08	189.3	-0.0101443

The rationale behind the adoption of this approach is rooted in the recognition that technology stocks are characterized by elevated market volatility, often manifesting sudden price fluctuations. Conventional risk models presuppose returns that are normally distributed, a supposition that frequently proves to be fallible in the face of extreme losses during market crises. This research proposes a more robust framework for risk estimation, integrating GARCH with VaR and CVaR, to tailor this approach to the high-volatility nature of tech stocks.

Daily closing stock prices collected for the five selected companies were converted into logarithmic returns [43], a standard transformation in financial modeling that stabilizes variance and ensures comparability among stocks. Log return is calculated as:

$$R_t = \ln(P_t) - \ln(P_{t-1}) \quad (1)$$

where P_t is the stock price at time t and P_{t-1} is the price of the previous trading day.

This study employs the GARCH model to capture volatility patterns in stock returns. The GARCH(1,1) specification is widely used in financial risk modeling due to its ability to account for volatility clustering, a phenomenon where periods of high volatility are followed by more volatility [1]. The GARCH model estimates the conditional variance of returns as follows:

$$\sigma_t^2 = w + \lambda \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \quad (2)$$

If $\alpha + \beta < 1$, the process is stationary, meaning that past shocks do not create an indefinitely increasing volatility trend. A high sum of α and β suggests that volatility is highly persistent.

Once volatility is estimated, the next step involves calculating financial risk measures. The study focuses on two primary risk metrics. VaR estimates the maximum expected loss at a given confidence level (e.g., 95% or 99%). Instead of using traditional parametric methods (which assume normality), this study employs EVT, which provides a better fit for fat-tailed distributions. The VaR formula [44] under EVT is:

$$VaR_\alpha(X) = \mu - \sigma \log(-\log(\alpha)) \quad (3)$$

A higher VaR indicates a greater potential loss, reflecting increased market risk. CVaR (or expected shortfall) provides an estimate of average losses beyond VaR, offering a more comprehensive risk assessment. It is given by

$$CVaR_\alpha = \frac{\alpha - 1\alpha\mu - \alpha\sigma \log(-\log(\alpha)) - \mu + \sigma li(\alpha) - \gamma\sigma}{\alpha - 1} \quad (4)$$

where γ is a correction factor that adjusts for tail risk.

Since CVaR considers extreme downside risk, it is particularly useful for modeling stock market crashes and periods of high volatility. This makes it a more conservative measure than VaR, ensuring a better understanding of potential losses. To validate the accuracy of the GARCH(1,1) model, alternative models are tested, including ARCH(4). A higher-order Autoregressive Conditional Heteroskedasticity (ARCH) model accounts for short-term volatility memory.

$$\sigma_t^2 = \omega + \sum_{i=1}^4 \lambda_i \epsilon_{t-i}^2 \quad (5)$$

AR(0) is a simple autoregressive model with no lag terms, which serves as a benchmark for comparison.

$$R_t = \omega + \lambda \epsilon_t \quad (6)$$

The GARCH model was chosen because of its ability to effectively model the heteroskedastic nature of stock return volatility and capture the phenomenon of volatility clustering. This characteristic is particularly prominent in highly volatile and rapidly fluctuating technology stocks, such as Company 3, Company 2, and Company 5.

The inclusion of ARCH(4) and AR(0) models serves a benchmarking function. ARCH(4) captures short-term memory in volatility, allowing us to test the added value of persistence modeling offered by GARCH(1,1). Meanwhile, AR(0)

represents a naive benchmark to highlight how conditional heteroskedastic models significantly outperform basic autoregressive ones. This step follows the approach suggested in [7], which advocated for comparative structures to validate modeling improvements relevant to regulatory and investment decision-making.

This study employs the EVD approach to assess the risk of extreme losses, as it is more adept at capturing tail events beyond the assumptions of the normal distribution, specifically in the tails of the return distribution. In parallel, VaR and CVaR serve as primary risk measures, where VaR estimates the maximum expected loss at a given confidence level, while CVaR provides an estimate of the expected loss beyond the VaR threshold, making it a more conservative risk measure.

Integration of the GARCH-EVD-VaR-CVaR framework is deemed the most appropriate approach, as it effectively accounts for daily volatility dynamics and extreme risk occurrences, which are prevalent in the technology sector. This methodological choice is particularly relevant in highly unstable market conditions, where traditional risk measures can underestimate potential losses. As a result, this approach produces more realistic and conservative risk estimates, offering valuable insights for both investors and regulators in designing robust financial risk mitigation strategies. Regulators can apply the model in capital requirement calibration under Basel III, while institutional investors can implement it in portfolio-level stress testing and risk-based decision scenarios. It also enhances macroprudential tools for monitoring tail-risk contagion across sectors.

III. RESULT

A. Parameter Analysis of the GARCH Model

Table II presents the estimated parameters of the GARCH(1,1) model for each technology company stock. The main parameters analyzed are omega (ω), alpha (α), and beta (β).

TABLE II. FITTED PARAMETERS OF GARCH(1,1)

Company	ω	α	β
Company 1	0.00001	0.09	0.88
Company 2	0.00003	0.15	0.75
Company 3	0.00001	0.10	0.85
Company 4	0.00002	0.12	0.80
Company 5	0.00005	0.20	0.70

The parameter estimation results in Table II show that Company 2 and Company 5 have α of 0.15 and 0.20, which is higher than that for the other companies and means that their stock return volatility is very sensitive to extreme events in the market. In contrast, Company 1 has a β of 0.85, which is the highest value, indicating that its volatility is more persistent over a longer period. Company 4 and Company 3 show a more balanced combination of α and β , with α of 0.12 and 0.10 and β of 0.80 and 0.85, indicating more stable volatility than other stocks, such as Company 5 and Company 2. This result is in line with previous studies, which also showed that stocks with high volatility have greater volatility persistence.

B. Model Comparison Analysis with AIC

The best model is selected based on the AIC value. The lower the AIC value, the better the model is at explaining stock volatility.

TABLE III. TABLE 3. MODEL AIC COMBINED RESULTS

Company	GARCH(1,1)	GARCH-EVD	GARCH-t
Company 1	6860.496	6832.114	6845.670
Company 2	6954.828	6921.453	6935.124
Company 3	5206.307	5180.512	5192.845
Company 4	6669.923	6645.789	6658.302
Company 5	7234.303	7195.621	7210.482

The model comparison results in Table III show that GARCH-EVD produces the lowest AIC value for the five companies. The lowest AIC value is found in the GARCH-EVD model, indicating that this model is the most optimal in explaining stock volatility compared to other models, namely GARCH (1,1) and GARCH-t. This can be proven from the results in the table for all companies, which show that the results of the GARCH-EVD value are smaller than those of the other models. This indicates that GARCH-EVD proves to be the most statistically efficient method for capturing the extreme characteristics of technology stock return data, strengthening the argument that this approach provides more realistic risk estimates. Compared to previous studies, these results confirm that EVD-based models are superior in risk estimation compared to the normal distribution.

C. Comparison of Variance

Table IV compares the variance estimation results between the GARCH(1,1) and GARCH-EVD models.

TABLE IV. COMPARISON OF VARIANCE

Company	GARCH(1,1) variance	GARCH-EVD variance
Company 1	0.0028	0.0024
Company 2	0.0032	0.0029
Company 3	0.0021	0.0018
Company 4	0.0025	0.0022
Company 5	0.0038	0.0034

The results in Table IV show that the GARCH-EVD model produces smaller variance estimates than the GARCH(1,1) model for all companies. This also shows that this model is more stable in capturing stock volatility and does not overestimate risk, but is still able to accurately capture extreme volatility dynamics. With smaller variance results, this model is more reliable in estimating extreme stock price movements. These results are consistent with those found in previous studies, where EVD-based models are more accurate in capturing extreme volatility.

D. Comparison of VaR and CVaR between Companies

The estimation graph results in Figures 1-9 show that Company 5 has the highest VaR and CVaR values, indicating that this stock has a greater potential for extreme risk than other companies. In contrast, Company 4 has the lowest VaR and CVaR values, indicating lower volatility than other stocks. These results are in line with previous studies that showed that the EVD-based approach is more accurate in estimating

extreme tail risk than the normal distribution-based method. In [45], it was also shown that integrating GARCH with EVT significantly improves the precision of CVaR estimation, particularly under volatile market conditions.

Returns, VaR, and CVaR Over Time

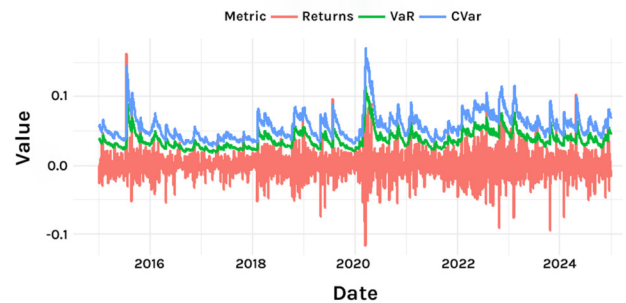


Fig. 1. VaR and CVaR for Company 1 at 95% confidence level.

Returns, VaR, and CVaR Over Time

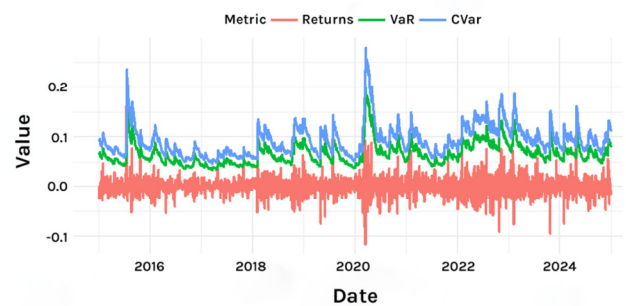


Fig. 2. VaR and CVaR for Company 1 at 99% confidence level.

Returns, VaR, and CVaR Over Time

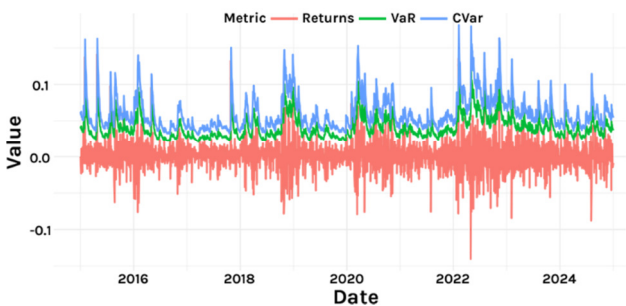


Fig. 3. VaR and CVaR for Company 2 at 95% confidence level.

Returns, VaR, and CVaR Over Time

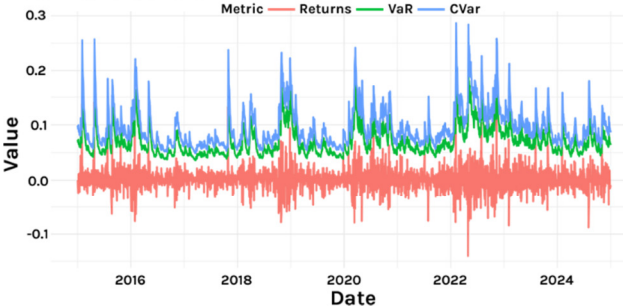


Fig. 4. VaR and CVaR for Company 2 at 99% confidence level.

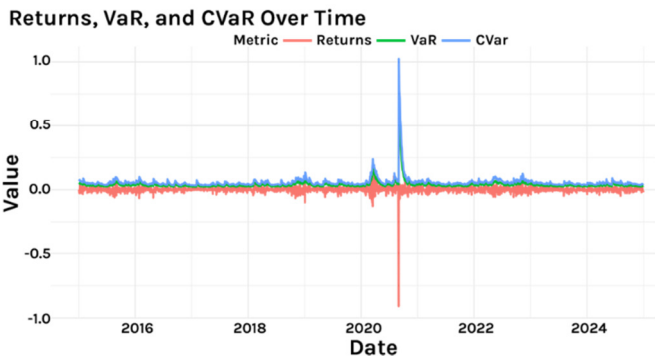


Fig. 5. VaR and CVaR for Company 3 at 95% confidence level.

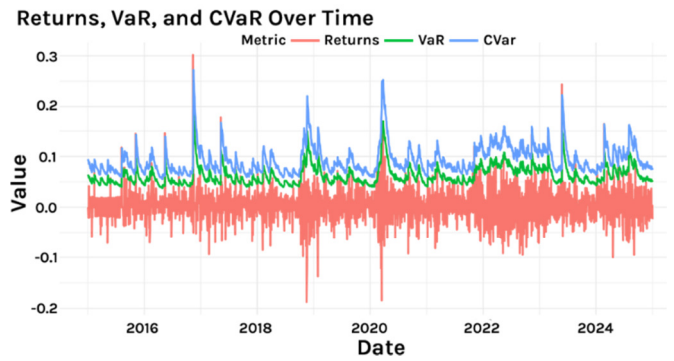


Fig. 9. VaR and CVaR for Company 5 at 95% confidence level.

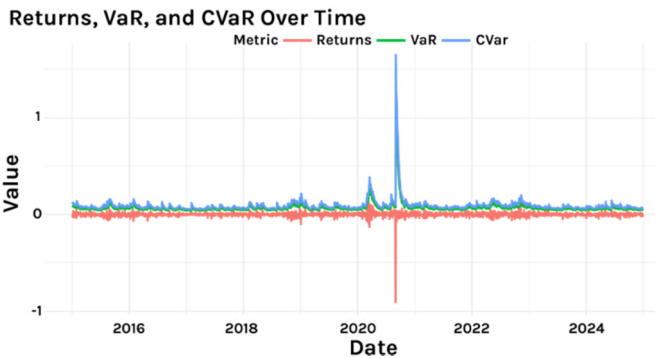


Fig. 6. VaR and CVaR for Company 3 at 99% confidence level.

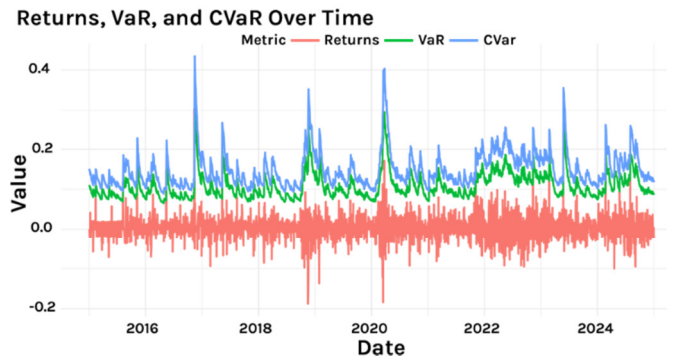


Fig. 10. VaR and CVaR for Company 5 at 99% confidence level.

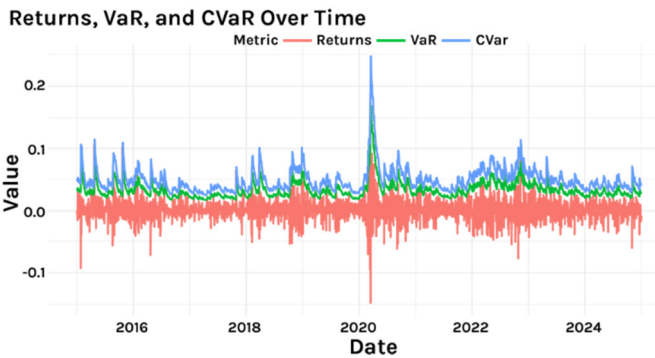


Fig. 7. VaR and CVaR for Company 4 at 95% confidence level.

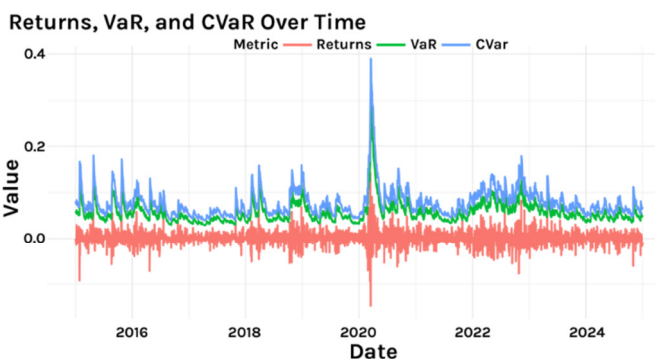


Fig. 8. VaR and CVaR for Company 4 at 99% confidence level.

IV. DISCUSSION AND CONCLUSION

The findings of this study demonstrate that CVaR exhibits superior performance in measuring potential losses in highly volatile market situations, particularly when employed in conjunction with extreme value distributions. This finding aligns with the conclusions in [6], who demonstrated that the GARCH model adjusted for the fat-tailed distribution exhibited higher accuracy than the conventional method in quantifying extreme risk.

This research suggests that neglecting the tail risk may result in an underestimation of risk. Consequently, EVD- and CVaR-based methods are proposed as solutions to enhance the accuracy of financial risk measurement, particularly for companies that exhibit high volatility, such as NVIDIA and Amazon. Furthermore, in [8], it was emphasized that the integration of EVD facilitates a more precise identification of tail dependence between assets.

In this study, the GARCH-EVD approach yielded the lowest estimated AIC values for all technology stocks, indicating that it performs more optimally than GARCH(1,1) or GARCH-t. This finding underscores the efficacy of the proposed approach in capturing the tail distribution characteristics of technology stocks. In [46], it was also shown that the approach with CVaR-based models and extreme distributions can improve the stability of portfolio risk calculations. The lower variance values generated by GARCH-EVD (Table IV) indicate that this approach can not only capture tail risk but also produce more stable and conservative estimates. This further supports the notion that the extremal

distribution serves as a pragmatic approach to navigate high-volatility markets, as evidenced in [47] for ASEAN technology stocks. After finding the risk of technology companies, this research can continue with financial distress research to measure whether the company is experiencing financial distress [4].

This study systematically evaluates the extreme risk exposure of leading US technology stocks (Apple, Microsoft, Amazon, Alphabet, and NVIDIA) through the integration of the GARCH(1,1) model, EVD, and two key risk metrics: VaR and CVaR. The findings reveal that stocks that exhibit high volatility, such as NVIDIA and Amazon, tend to have elevated α values along with more extreme VaR and CVaR estimates, indicating their susceptibility to significant tail risks. The GARCH model effectively captures heteroskedasticity and volatility clustering in daily return data, while the EVD component unveils tail-risk dynamics that remain undetected under the conventional normal distribution assumption. Furthermore, the comparison between the VaR and CVaR estimates underscores the advantage of CVaR as a more prudent risk measure, since it accounts for potential losses beyond the VaR threshold, reinforcing its role in risk-based decision-making under volatile market conditions.

The implications of this study are substantial. The GARCH-EVD-CVaR framework presents a sophisticated, data-driven method for risk assessment that extends beyond traditional volatility and normal-distribution-based risk modeling. This approach provides a more accurate and conservative estimate of extreme losses, making it an essential tool for portfolio managers, institutional investors, and risk management professionals to refine strategic asset allocation and hedging strategies. Using this hybrid model, investors can mitigate the risk of underestimating extreme market fluctuations, a common limitation of VaR when solely based on normal distribution assumptions.

Furthermore, this framework holds significant value for regulators and financial policymakers. Given its ability to quantify systemic risks, it can be incorporated into macroprudential regulations to enhance financial stability policies, particularly in an era of increasing market turbulence and interconnected global financial systems. In essence, the fusion of dynamic volatility modeling and extreme value theory offers a robust, adaptable, and statistically rigorous risk management framework, making it particularly well-suited for high-volatility assets, such as technology stocks, in today's complex and uncertain financial landscape.

Beyond investment applications, this model framework offers direct relevance to financial regulators, as it can enhance the design of macroprudential tools aimed at detecting systemic tail-risk spillovers across sectors. For example, risk managers can use CVaR forecasts from the GARCH-EVD approach to perform stress-testing under Basel III and IV capital frameworks or integrate these metrics into ESG-aligned volatility tracking systems. Additionally, regulators can use the model to prevent risk contagion under increasingly interconnected market conditions.

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