An Experimental Investigation of the Relative Length of Hydraulic Jump in a Rectangular Channel with a Composite Section, Featuring a Rough Major Bed and Positive Slope

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Received: 21 April 2025 | Revised: 12 May 2025, 4 June 2025, and 9 June 2025 | Accepted: 14 June 2025

ABSTRACT

Hydraulic jumps occur when the flow shifts from supercritical to subcritical, forming a standing wave with a sudden increase in the water surface elevation in the channel. This transition gives rise to a turbulent zone, where surface rollers are generated, leading to vigorous water mixing, air entrainment, and significant dissipation of water energy. The present study deals with hydraulic jumps in a rectangular channel, having a composite section characterized by a rough major bed (made of uniform plastic granules) and a positive slope under various flow circumstances. The hydraulic characteristics were assessed for varying bed roughness and positive slopes, focusing on the relative length of the hydraulic jump (L_j/h_1) and flow rate. The experimental data analysis revealed that the roughness of the main bed and the channel slope affect the L_j/h_1 ratio. The extensive data on the hydraulic jumps in rough channel beds have facilitated the development of mathematical models to represent the L_j/h_1 ratio as a function of roughness and slope factors.

Keywords-hydraulic jump; relative length of the hydraulic jump; slope; compound rectangular channel; rough major bed

I. INTRODUCTION

The hydraulic jump, initially noted by Leonardo da Vinci in the 16th century, is employed to develop energy-efficient structures at spillway bases and other diverse applications [1-3]. In 1828, hydraulic jumps were linked to the momentum principle, a connection validated through experimental studies [4].

A hydraulic jump is a complex flow phenomenon and surface discontinuity that occurs during the shift from supercritical to subcritical flow in open-channel conditions. Its occurrence is influenced by many hydraulic structures, including thresholds, weirs, sluice gates, piers, or stilling basins. These structures increase the flow depth and enable the flow transition through the hydraulic jump. This transition involves a highly turbulent flow, marked by large-scale turbulence, surface waves, spray, energy dissipation, and air entrainment, with significant dissipative characteristics [5].

Hydraulic jumps on sloped surfaces are used to dissipate the excess energy downstream of hydraulic structures, like regulators, weirs, dams, and spillways. The properties of hydraulic jumps on inclined beds have been thoroughly examined [6-13].

Authors in [14] explored hydraulic jumps with positive slopes in triangular channels. Authors in [15] expanded the study of hydraulic jumps with positive slopes to include triangular and U-shaped channels. Subsequently, authors in [16] investigated hydraulic jumps in trapezoidal channels. Hydraulic jumps in rectangular channels with varying positive slopes were investigated in [17-21], which are highly relevant to the hydraulic jumps on rough, sloping surfaces and their environmental sustainability implications.

This study experimentally quantifies the combined effects of the major bed roughness and positive channel slope on the relative length of hydraulic jumps (L_j/h_1) in a rectangular channel with a compound section. The configuration utilized corresponds to the type D jump as classified in [7]. A dimensionless empirical relationship was established to describe the variation of the relative jump length (L_j/h_1) as a function of the incident flow's Froude number, relative roughness, and channel slope. The outcomes of this research will have practical implications for the hydraulic structure design. By uncovering the impact of relationship between the major bed's roughness and the channel's slope on the relative length, engineers and designers can make informed decisions to enhance the performance and stability of the hydraulic systems.

II. MATERIALS AND METHODS

At the EVRNZA Laboratory of Ouargla University, experiments were conducted in a rectangular channel with a composite section, featuring a rough main bed and a positive slope to study the hydraulic jumps. Figure 4 shows that all experiments were carried out in a free-surface channel. The measurements of this channel are 10 m in length, 0.5 m in height, and the plexiglass makes up its side walls. The measurements of the horizontal experimental channel are 4 m in length, 15.5 cm in height, with a minor bed (b) of 14.4 cm height, a major bed (B) of 25 cm height, and a compound rectangular section. The channel's bottom is completely horizontal.

The upstream section of the channel is mounted on a screw-nut mechanism, allowing for elevation adjustments via a metal arm that turns the screw for altering the channel's slope angle: $\tan(\alpha) = 0, 0.005, 0.01, 0.015$). It is linked to the supply basin by a circular pipe with a diameter of 150 mm, which terminates in a sealed metal box. An opening is placed in a flat metal wall of a specified width, creating an entry into the channel. This wall is designed to generate a torrential flow with a variable output section to provide a torrential flow. The height of this flow corresponds to the initial ledge height h_1 (h_1 = 2.5 cm, 3 cm, 3.5 cm, and 4 cm). The h_1 values are chosen to achieve practical Froude number ranges $Fr \approx 2{-}8$ similar to real-world scenarios. The valve can be adjusted to manage the volume flow rates. With a steady 55.55 l/s flow rate, supplied by an axial pump. Experiments were conducted using four roughness values (ε = 6 mm, 8 mm, 10 mm, 12 mm) to mimic industrial and natural settings, ensuring flow homogeneity. Plastic pellets with central holes (Figure 1) were fixed as roughness elements along the channel bed using a fishing line and a strong adhesive to ensure durability against the torrential flow. These were chosen over materials like paper, glass, sheet metal, or gravel for their rigidity and resistance to high-flow conditions.

The flow rate was measured using a rectangular weir positioned at the terminus; the specific flow rate is derived by substituting the height h into [22]:

$$Q = 0{,}3794B\sqrt{2g}\beta(1 + 0{,}16496\beta^{2{,}0716})^{3/2}\, h_{dev}^{3/2} \quad (1)$$

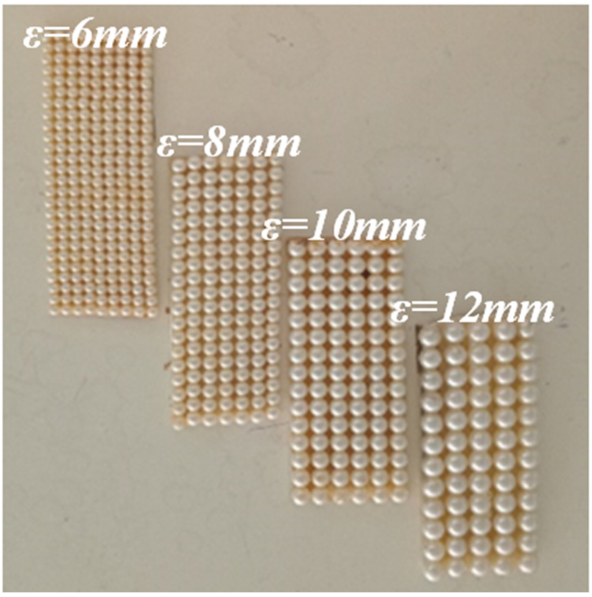

Fig. 1. Mats for different diameters with known roughness.

Fig. 2. Upstream view of the hydraulic jump.

Fig. 3. Composite section channel with rough major bed.

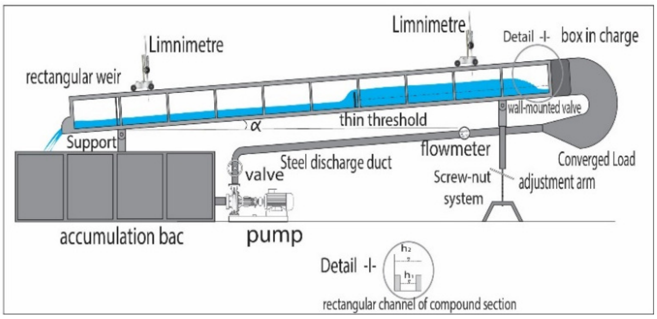

Fig. 4. Simplified schema of the compound rectangular section measuring channel with variable positive slope, used for the experiment.

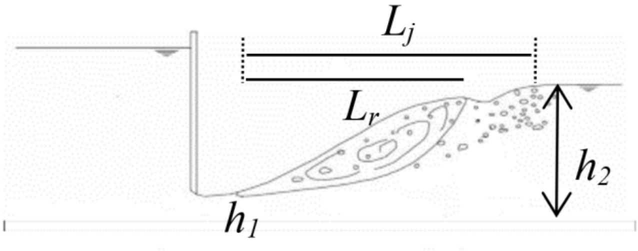

Fig. 5. Conceptual illustration for hydraulic jump.

III. RESULTS AND DISCUSSION

A. *Variation of the Relative Length of the Hydraulic Jump (Lj/h1) with Froude Number (F_1)*

Figures 6-10 illustrate the variations of the relative length of the jump as a function of the Froude number for four different angles of inclination: tan(α) = 0, 0.005, 0.01, 0.015, and absolute roughness: (ε = mm) = 0, 06, 08, 10, and 12.

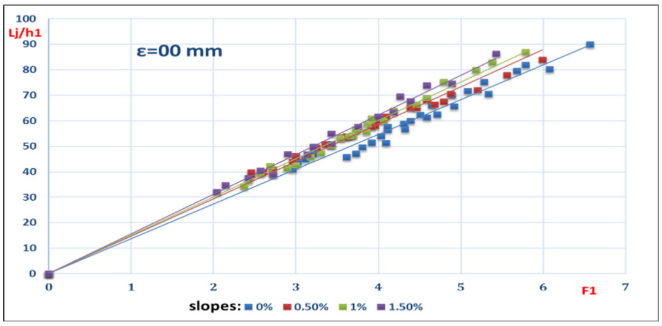

Fig. 6. Variation in the relative length of the jump with the Froude number for roughness ε = 0 mm at various slopes.

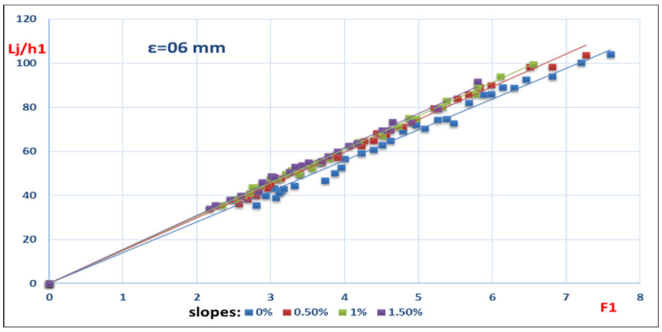

Fig. 7. Variation in the relative length of the jump with the Froude number for roughness ε = 6 mm at various slopes.

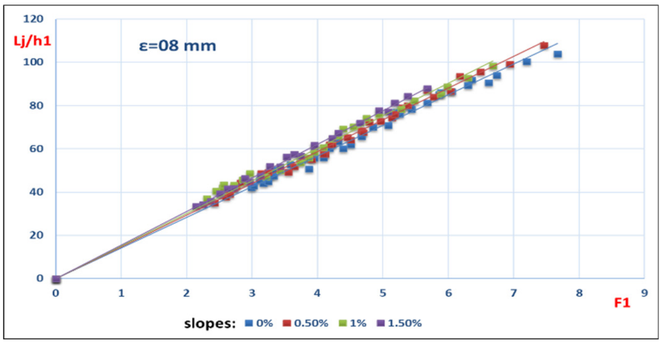

Fig. 8. Variation in the relative length of the jump with the Froude number for roughness ε = 8 mm at various slopes.

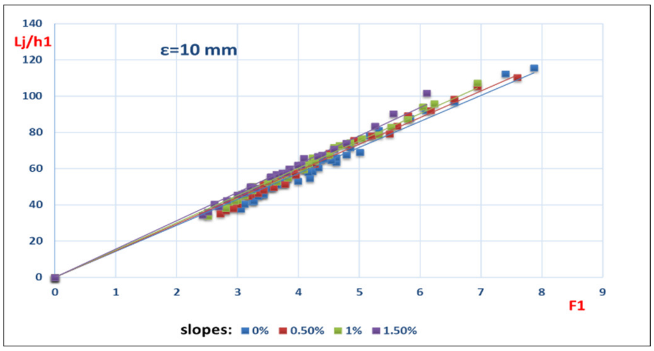

Fig. 9. Variation in the relative length of the jump with the Froude number for roughness ε = 10 mm at various slopes.

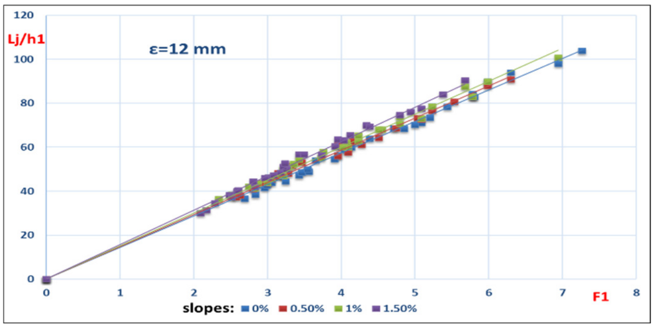

Fig. 10. Variation in the relative length of the jump with the Froude number for roughness ε = 12 mm at various slopes.

The examination of the experimental data indicates that each scatter plot conforms to the shape of a singular curve. An adjustment using the linear least squares approach indicates that these curves conform to:

$$\frac{L_j}{h_1} = a \times F_{r1} \tag{2}$$

Tables I- V present the experimental results. Figures 11-15 illustrate the relationship adjustment for the four inclinations of the rectangular channel with a compound section.

TABLE I. VALUES OF PARAMETERS a_1

Roughness 0 mm		
tang(α)	**a_1**	**R^2**
0.000	13.662	0.9986
0.005	14.674	0.9988
0.010	15.036	0.9992
0.015	15.578	0.9991

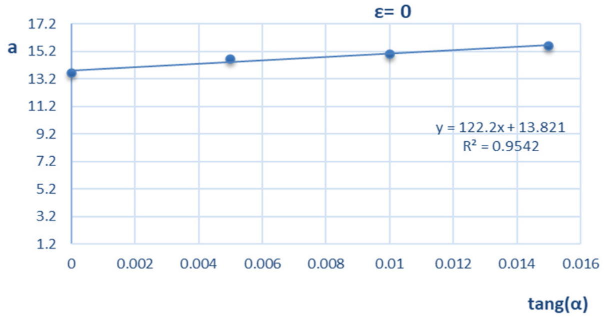

Fig. 11. Variation of the parameter a_1 with different slopes, roughness (ε = 0 mm).

TABLE II. VALUES OF THE PARAMETERS a_2

Roughness 6 mm		
tang(α)	**a_2**	**R^2**
0.000	13.950	0.9988
0.005	14.890	0.9994
0.010	15.215	0.9997
0.015	15.434	0.9995

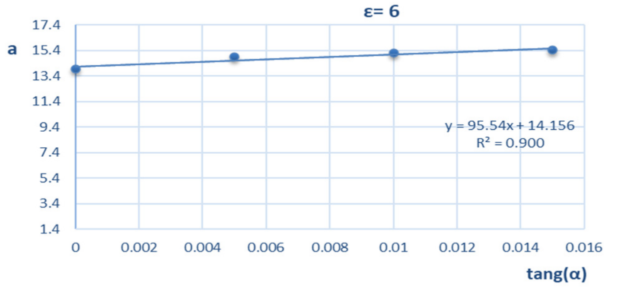

Fig. 12. Variation of the parameter a_2 with different slopes, roughness (ε = 6 mm).

TABLE III. VALUES OF PARAMETERS a_3

Roughness 8 mm		
tang(α)	**a_3**	**R^2**
0.000	14.182	0.9992
0.005	14.681	0.9991
0.010	15.066	0.9986
0.015	15.522	0.9997

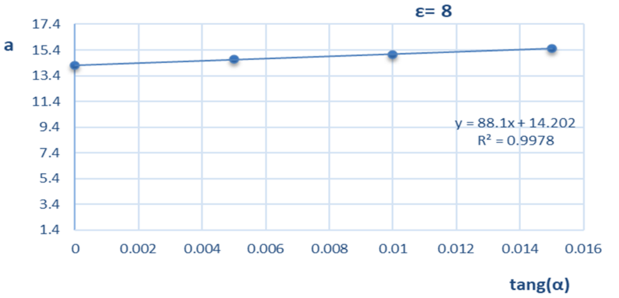

Fig. 13. Variation of the parameter a_3 with different slopes, roughness (ε = 8 mm).

TABLE IV. VALUES OF PARAMETERS a_4

Roughness 10 mm		
tang(α)	**a_4**	**R^2**
0.000	14.332	0.9975
0.005	14.688	0.9984
0.010	15.061	0.9991
0.015	15.603	0.9989

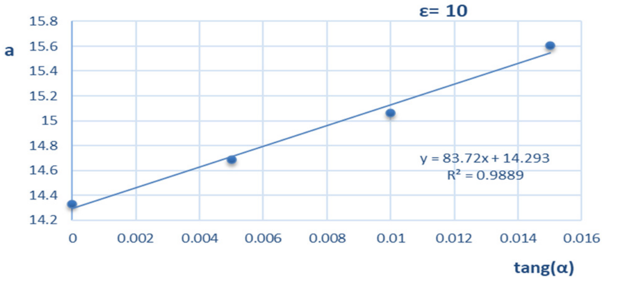

Fig. 14. Variation of the parameter a_4 with different slopes, roughness (ε = 10 mm).

TABLE V. VALUES OF PARAMETERS a_5

Roughness 12 mm		
tang(α)	a_5	R^2
0.000	14.328	0.9994
0.005	14.639	0.9994
0.010	14.997	0.9993
0.015	15.63	0.9993

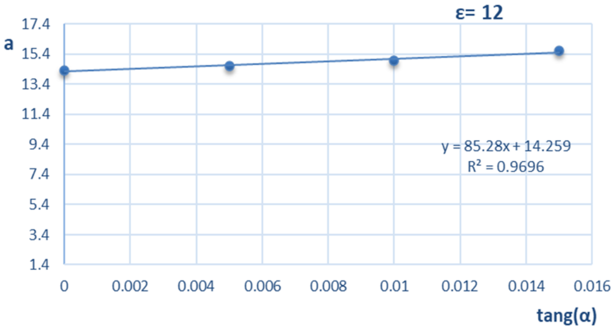

Fig. 15. Variation of the parameter a_5 with different slopes, roughness (ε = 12 mm).

Figures 11-15 illustrate the variation of the parameter a_n in relation to the slopes (tang(α)) of the channel.

The tangent of the slope angle (α) exhibits a linear distribution, which can be expressed by:

$$a_1 = 122.2\, tang(\alpha) + 13.821 \quad (3)$$

($R^2 = 0.9542$, $\varepsilon = 0$ mm)

$$a_2 = 95.54 \text{ tang}(\alpha) + 14.156 \quad (4)$$

($R^2 = 0.9000$, $\varepsilon = 6$ mm)

$$a_3 = 88.10 \text{ tang}(\alpha) + 14.202 \quad (5)$$

($R^2 = 0.9978$, $\varepsilon = 8$ mm)

$$a_4 = 83.72 \text{ tang}(\alpha) + 14.293 \quad (6)$$

($R^2 = 0.9889$, $\varepsilon = 10$ mm)

$$a_5 = 85.28 \text{ tang}(\alpha) + 14.259 \quad (7)$$

($R^2 = 0.9696$, $\varepsilon = 12$ mm)

Substituting the a parameters with their corresponding expressions in (2) yields:

$$\frac{Lj}{h_1} = (122.2 \text{ tang}(\alpha) + 13.821)F_1, (\varepsilon = 0 \text{ mm}) \quad (8)$$

$$\frac{Lj}{h_1} = (95.54 \text{ tang}(\alpha) + 14.156)F_1, (\varepsilon = 6 \text{ mm}) \quad (9)$$

$$\frac{Lj}{h_1} = (88.10 \text{ tang}(\alpha) + 14.202)F_1, (\varepsilon = 8 \text{ mm}) \quad (10)$$

$$\frac{Lj}{h_1} = (83.72 \text{ tang}(\alpha) + 14.293)F_1, (\varepsilon = 10 \text{ mm}) \quad (11)$$

$$\frac{Lj}{h_1} = (85.28 \text{ tang}(\alpha) + 14.259)F_1, (\varepsilon = 12 \text{ mm}) \quad (12)$$

TABLE VI. VALUES a AND b OF THE ROUGHNESS COEFFICIENT CORRELATED WITH THE CHANNEL'S INCLINATION ANGLE

tang(α)	**0 , 0.005, 0.01, 0.015**		
***ε /(B-b)* (mm)**	***a***	***b***	R^2
0	122.2	13.821	0.9542
6	95.54	14.156	0.9000
8	88.10	14.202	0.9978
10	83.72	14.293	0.9889
12	85.28	14.259	0.9696

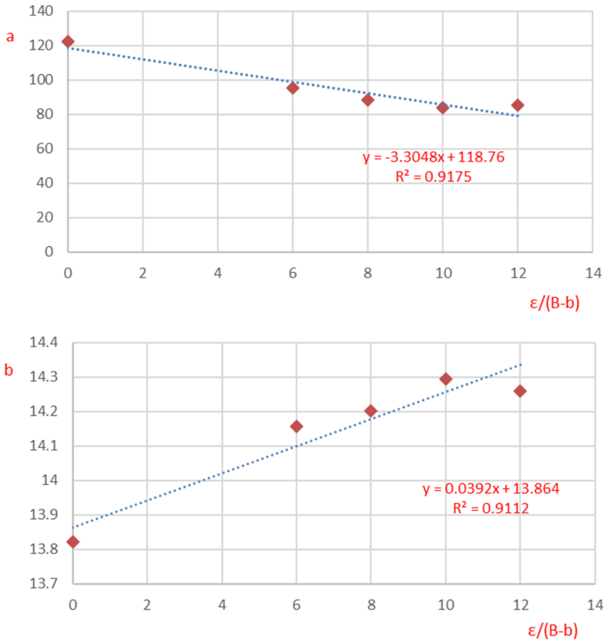

Fig. 16. Variation of parameters a and b as a function of roughness.

$$a' = -3.3048\, \varepsilon /(B-b) + 118.76, R^2 = 0.9175 \quad (13)$$

$$b' = 0.039\,2\, \varepsilon /(B-b) + 13.864, R^2 = 0.9112 \quad (14)$$

It is advantageous to consolidate the five relationships into a comprehensive equation in the following format:

$$\frac{Lj}{h_1} = \left[\left(\frac{-3.3048\varepsilon}{(B-b)} + 118.76\right) tang(\alpha) + \frac{0.0392\varepsilon}{(B-b)} + 13.864\right] F_1 \quad (15)$$

where ε /(*B-b*) is the relative roughness in the major bed and tang (*α*) is the channel's inclination angle.

The global relation (15), being explicit and dimensionless in (L_j/h_1), offers a direct and practical method for determining the relative length of the hydraulic jump (L_j/h_1), given the incident Froude number F_1, the slope of the tangential channel (α), and the absolute roughness (ε).

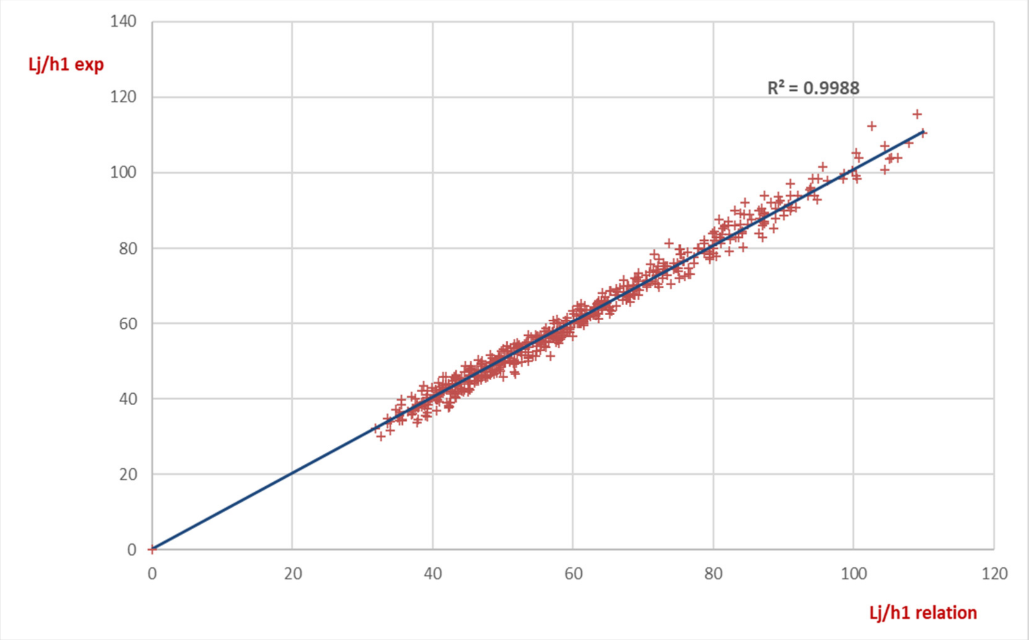

Fig. 17. Variation of the experimental relative length of the jump L_j/h_1, as a function of the theoretical relative length calculated using (15).

Figure 17 demonstrates that most experimental measurements cluster around the initial justifying bisector, confirming its reliability. Additionally, the high correlation coefficient ($R^2 = 0.9988$) indicates a strong agreement between the experimental data and the applied model. Furthermore, the Mean Absolute Error (MAE = 1.77571) reflects the model's high accuracy, while the Mean Relative Error (RE = 0.0304) and the standard deviation of the relative error ($\sigma_{RE} = 0.0241$) highlight the consistency of the predictions across various conditions. Collectively, these metrics affirm the applicability of the general relationship for designing hydraulic structures, including damping basins.

IV. CONCLUSION

An experimental study was conducted to investigate the hydraulic jump in a rectangular channel with a compound cross-section characterized by a rough bed and a positive slope. The study analyzed the effects of the bed roughness and channel slope on the hydraulic jump's behavior. A dimensionless empirical relationship was developed to describe the variation of the relative jump length (L_j/h_1) as a function of Froude number ($Fr \approx 2-8$), relative roughness ($\varepsilon/(B-b)$), and channel slope (tan(α)).

The results demonstrated that L_j/h_1 decreases with an increasing absolute roughness and channel slope. However, the applicability of the linear model beyond the tested conditions (e.g., higher Froude numbers or different channel geometries) may be limited. For higher Froude numbers ($Fr > 8$) or non-rectangular channel geometries, the nonlinear effects may emerge due to the complex interactions between the turbulent flow, roughness, and slope, necessitating further calibration or more sophisticated models to ensure the accuracy in the hydraulic structure design.

NOTATIONS

B: Width of the major bed [m]

b: Width of the minor channel bed [m]

ε: Roughness [m]

$\varepsilon/(B-b)$: Relative roughness in the major bed [ ]

F_1: Upstream Froude number ($F_1 = \sqrt{\frac{Q^2}{gb^2h_1^3}}$) [ ]

g: Gravity acceleration [m / s^2]

Lj: Length of jump [m]

Lj/h_1: Relative length of jump [ ]

tang (α): Channel's inclination angle [ ]

h_1: Supercritical initial flow depth [m]

h_2: Subcritical flow Sequent depth [m]

Q: Volume flow [m^3/s]

β: Aspect of form. $\beta=b/B$=14.4/25= 0.576

h_{dev} : depth of the overflowing water blade measured by the limnimeter instrument [m]

REFERENCES

[1] S. Montes, *Hydraulics of open channel flow*, Baltimore, MD, USA: American Society of Civil Engineers, 1998.

[2] A. S. Kote and P. B. Nangare, "Hydraulic Model Investigation on Stepped Spillway's Plain and Slotted Roller Bucket," *Engineering, Technology & Applied Science Research*, vol. 9, no. 4, Aug. 2019 pp. 4419–4422, https://doi.org/10.48084/etasr.2837.

[3] S. M. Kori, A. A. Mahessar, M. Channa, A. A. Memon, and A. R. Kori, "Study of Flow Characteristics Over a Rounded Edge Drop Structure in Open Channel," *Engineering, Technology & Applied Science Research*,

vol. 9, no. 3, pp. 4136–4139, Jun. 2019, https://doi.org/10.48084/etasr.2584.

[4] C.F. Li, "Determining the Location of Hydraulic Jump by Lmodel Test And Hec-2 Flow Routing," M. S. thesis, College of Engineering and Technology, Ohio University., 1995.

[5] H. Chanson, *Energy Dissipation in Hydraulic Structures*, 1st ed. Boca Raton, FL, US: CRC Press, 2015.

[6] B. A. Bakhmeteff and A. E. Matzke, "The Hydraulic Jump in Sloped Channels," *Journal of Fluids Engineering*, vol. 60, no. 2, pp. 111–118, Feb. 1938, https://doi.org/10.1115/1.4020643.

[7] C. E. Kindsvater, "The Hydraulic Jump in Sloping Channels," *Transactions of the American Society of Civil Engineers*, vol. 109, no. 1, pp. 1107–1120, Jan. 1944, https://doi.org/10.1061/TACEAT.0005733.

[8] N. Rajaratnam, "Hydraulic Jumps," in *Advances in Hydroscience*, vol. 4, Elsevier, 1967, pp. 197–280.

[9] N. Rajaratnam and V. Murahari, "Flow Characteristics of Sloping Channel Jumps," *Journal of the Hydraulics Division*, vol. 100, no. 6, pp. 731–740, Jun. 1974, https://doi.org/10.1061/JYCEAJ.0003975.

[10] M. A. Mikhalev and Hoang Ty An, "Kinematic characteristics of a hydraulic jump on a sloping apron," *Hydrotechnical Construction*, vol. 10, no. 7, pp. 686–690, Jul. 1976, https://doi.org/10.1007/BF02381815.

[11] W. H. Hager and N. V. Bretz, "Discussion of ‘ *Simplified Design of Contractions in Supercritical Flow* ’ by Terry W. Sturm (May, 1985, Vol. 111, No. 5)," *Journal of Hydraulic Engineering*, vol. 113, no. 3, pp. 422–424, Mar. 1987, https://doi.org/10.1061/(ASCE)0733-9429(1987)113:3(422).

[12] W. H. Hager and D. Li, "Sill-controlled energy dissipator," *Journal of Hydraulic Research*, vol. 30, no. 2, pp. 165–181, Mar. 1992, https://doi.org/10.1080/00221689209498932.

[13] S. A. Ead and N. Rajaratnam, "Hydraulic Jumps on Corrugated Beds," *Journal of Hydraulic Engineering*, vol. 128, no. 7, pp. 656–663, Jul. 2002, https://doi.org/10.1061/(ASCE)0733-9429(2002)128:7(656).

[14] M. Debabeche, S. Cherhabil, A. Hafnaoui, and B. Achour, "Hydraulic jump in a sloped triangular channel," *Canadian Journal of Civil Engineering*, vol. 36, no. 4, pp. 655–658, Apr. 2009, https://doi.org/10.1139/L08-136.

[15] S. Cherhabil, "Le Ressaut Hydraulique Dans Les Canaux Prismatiques à Pente Variable," PhD thesis, Department of Civil and Hydraulic Engineering, Mohamed Khider University, Biskra, Algeria, 2010.

[16] Kateb, "Etude Theorique Et Experimentale De Quelques Types De Ressauts Hydrauliques Dans Un Canal Trapezoïdal," PhD thesis, Department of Civil and Hydraulic Engineering, Mohamed Khider University, Biskra, Algeria, 2014.

[17] B. Nouacer, "Contribution A L’etude Theorique Et Experimentale De Ressaut Hydraulique Evoluant Dans Un Canal Rectangulaire Incline A Fond Rugueux," PhD thesis, Department of Civil and Hydraulic Engineering, Kasdi Merbah University Ouargla, Ouargla, Algeria, 2023.

[18] S. K. Gupta, "Evaluation of hydraulic jump characteristics in rough sloping surfaces for sustainable environment: A laboratory investigation," *Sigma Journal of Engineering and Natural Sciences – Sigma Mühendislik ve Fen Bilimleri Dergisi*, pp. 148–159, 2025, https://doi.org/10.14744/sigma.2025.00012.

[19] S. K. Gupta and V. K. Dwivedi, "Experimental investigation of hydraulic jump characteristics in sloping rough surfaces for sustainable development," *Engineering Research Express*, vol. 6, no. 2, p. 025103, Jun. 2024, https://doi.org/10.1088/2631-8695/ad3acf.

[20] S. K. Gupta and V. K. Dwivedi, "Effect of Surface Roughness and Channel Slope on Hydraulic Jump Characteristics: An Experimental Approach Towards Sustainable Environment," *Iranian Journal of Science and Technology, Transactions of Civil Engineering*, vol. 48, no. 3, pp. 1695–1713, Jun. 2024, https://doi.org/10.1007/s40996-023-01246-z.

[21] Sanjeev Kumar Gupta and Vijay Kumar Dwivedi, "Prediction of Depth Ratio, Jump Length and Energy Loss in Sloped Channel Hydraulic Jump for Environmental Sustainability," *Evergreen*, vol. 10, no. 2, pp. 942–952, Jun. 2023, https://doi.org/10.5109/6792889.

[22] H. Rachedi, "Analyse d’un Écoulement Au Travers d’une Contraction Latérale.," Master's thesis, Department of Hydraulics, University of Biskra, Biskra, Algeria, 2005.