Optimal Design of a Two-Stage Helical Gear Reducer with Split Output Stage: A Multi-Objective Approach Based on NSGA-II and TOPSIS

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ABSTRACT

This study presents a novel approach for the optimal design of a two-stage helical gear reducer with a split output stage, to achieve a balance between the mechanical compactness and transmission performance. The optimization problem considers two conflicting objectives: minimizing the cross-sectional area of the gearbox structure and maximizing its mechanical efficiency. To solve this Multi-Objective Optimization Problem (MOOP), the Non-dominated Sorting Genetic Algorithm II (NSGA-II) is employed, aiming to generate a set of Pareto-optimal solutions, capturing the trade-offs between the two design goals. Subsequently, the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) is applied to rank and identify the most preferable solution among the Pareto front based on decision-makers' preferences. The proposed hybrid framework enables a systematic exploration of the design space, providing engineering insights into how the gear ratio distribution and geometric parameters influence the gearbox's performance. The results demonstrate that the integration of NSGA-II and TOPSIS effectively supports the optimal design of compact and high-efficiency gear reducers with split-stage configurations.

Keywords-two-stage helical gearbox; split output stage; multi-objective optimization; NSGA-II; TOPSIS; gear ratio distribution; cross-sectional area; efficiency

I. INTRODUCTION

The design of helical gear reducers is a critical aspect in modern mechanical systems, where the trade-offs among efficiency, compactness, vibration, and transmission error must be addressed simultaneously. Traditional design methodologies often rely on deterministic approaches that lack flexibility in handling multiple conflicting objectives. In contrast, Multi-Objective Optimization (MOO) techniques have emerged as powerful tools to enhance the gearbox performance by balancing the trade-offs between objectives, such as minimizing the volume and maximizing efficiency.

Genetic Algorithms (GAs) have been widely used in MOO problems for their ability to explore complex design spaces effectively. Authors in [1] presented a comprehensive tutorial on the application of GAs in MOO, emphasizing their robustness in handling non-linear and multi-modal problems. Building on such foundations, hybrid and heuristic algorithms such as Particle Swarm Optimization (PSO) and Simulated Annealing (SA) have also been applied to the gear train design. For instance, authors in [2] demonstrated the effectiveness of PSO and SA in minimizing the gear weight while satisfying design constraints.

The integration of tribological aspects into the gear optimization was highlighted in [3], where spur gearboxes were investigated using MOO to consider the frictional behavior, revealing significant performance trade-offs. Authors in [4] addressed optimal engineering design using mathematical programming, laying the groundwork for contemporary methods. Authors in [5] conducted a comparative study of various evolutionary algorithms for gear system optimization, identifying NSGA-II as one of the most effective strategies for balancing multiple objectives. Specifically targeting helical gear trains, authors in [6] employed NSGA-II for optimizing a two-stage configuration, revealing improvements in both size and efficiency. Similarly, authors in [7] applied GA to epicyclic gear trains, achieving enhanced torque transmission and compact design. Noise and vibration—often overlooked in classical models—were incorporated into MOO in [8], where the focus was on electric bus gearboxes, achieving reductions in acoustic emissions using multi-objective techniques. Further extending the NSGA-II framework, authors in [9] explored its application in two-stage spur gearboxes, providing a detailed trade-off analysis between the transmission error and mechanical losses. Authors in [10] combined the panel acoustic participation with the response surface methodology to optimize the gearbox performance, bridging the mechanical and acoustic domains. The use of decision-making methods in conjunction with NSGA-II was emphasized in [11], where highlighted the importance of integrating ranking mechanisms in post-Pareto optimization was hilighted.

The relevance of heuristic methods was reinforced in [12], where aeroengine accessory gearboxes were optimized using hybrid metaheuristic algorithms. Moreover, authors in [13] focused on minimizing the transmission error and power losses in gear units, emphasizing the role of geometric accuracy. Authors in [14] applied evolutionary algorithms to multi-speed gearbox design, laying the groundwork for current applications in shifting control and fuel optimization.

In the domain of spur gear design, authors in [15] proposed a combined NSGA-II and a decision-making approach, yielding compact, yet efficient designs. Authors in [16] addressed the volume-efficiency optimization in spur gear pairs, offering insights into material utilization. A comparative analysis of different algorithms was conducted in [17], suggesting that no single algorithm dominates across all design objectives. Beyond the traditional gear systems, authors in [18] applied GAs to planetary gear trains, while authors in [19] employed teaching-learning-based optimization for spur gears. Additionally, authors in [20] combined the shifting control optimization with the gearbox design to reduce the fuel consumption and emissions. Stochastic MOO also offers valuable alternatives under uncertainty [21]. Large-scale planetary gear reducers were addressed in [22], where a scalable MOO design strategy was proposed. Likewise, authors in [23] investigated the optimization of the planet carrier in wind turbine gearboxes. The integration of TOPSIS as a decision-making tool for selecting the best compromise solutions in helical gearbox design was effectively demonstrated in [24], underscoring the practicality of combining NSGA-II with TOPSIS. Furthermore, authors in [25] explored the use of MARCOS method in gearboxes with double gears in the first stage, demonstrating the impact of advanced Multi-Criteria Decision-Making (MCDM) techniques in industrial applications.

Despite these advancements, few studies have investigated the two-stage helical gearboxes with a split output stage, where the second stage drives dual outputs. This configuration is beneficial in specific industrial applications requiring simultaneous torque transmission to multiple shafts. Moreover, there is a lack of research addressing the trade-off between the cross-sectional compactness and transmission efficiency using a hybrid NSGA-II–TOPSIS approach in such split-stage gearboxes.

These types of two-stage helical gearboxes are widely used in various industrial applications requiring moderate to high torque transmission with a compact size and high efficiency. Typical use cases include conveyor systems, machine tools, elevators, automated manufacturing equipment, and material handling systems, where the gear reducer plays a critical role in ensuring a smooth, efficient, and reliable motion control under variable load conditions. The optimization of such gearboxes contributes directly to enhancing the performance, energy savings, and durability of these industrial systems.

Therefore, this study proposes a novel framework for the MOO of a two-stage helical gear reducer with a split-second stage, targeting the minimization of the cross-sectional area and the maximization of efficiency. NSGA-II is employed to generate the Pareto front, while TOPSIS is used to select the most preferred solution. The proposed method offers engineering insights and serves as a practical design tool for next-generation compact and efficient gear systems.

II. OPTIMIZATION PROBLEM FORMULATION

A. Calculation of Gearbox Cross-Sectional Area

For a two-stage helical gearbox with dual gears in the second stage, the cross-sectional area A_c can be calculated with the help of Figure 1.

$$A_c = (L \times H) \tag{1}$$

where L and H are calculated by:

$$L = d_{w11} + \frac{d_{w21}}{2} + \frac{d_{w12}}{2} + d_{w22} + 2\delta \tag{2}$$

$$H = \max(d_{w21}, d_{w22}) + 6.5\delta \tag{3}$$

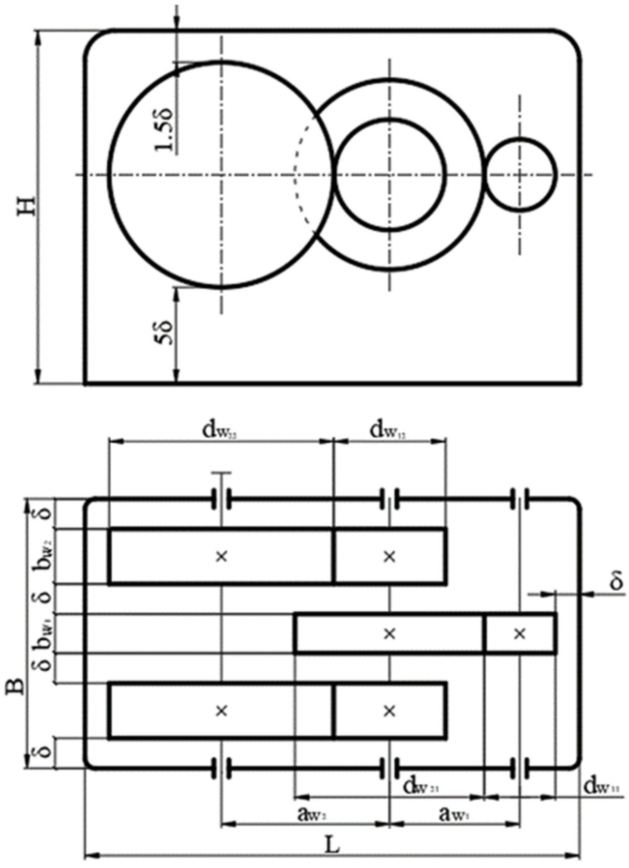

Fig. 1. Schematic for the determination of A_c.

where δ = 7÷10 (mm) [18], d_{w1i}, d_{w2i} (i = 1÷2) denote the pitch diameter of the pinion and the gear of stage i, which are determined by [26]:

$$d_{w1i} = 2 \cdot a_{wi}/(u_i + 1) \tag{4}$$

$$d_{w2i} = 2 \cdot a_{wi} \cdot u_i \cdot/(u_i + 1) \tag{5}$$

where a_{wi} (i = 1÷2) represents the center distance of stage I and a_{wi} is computed by [26]:

$$a_{wi} = k_a \cdot (u_i + 1) \cdot \sqrt[3]{T_{1i} \cdot k_{H\beta}/([AS_i]^2 \cdot u_i \cdot X_{bai})} \tag{6}$$

where Xba_i is the FWC of stage i^{th}; T_{1i} (i = 1÷2) is the pinion torque of stage i, and it is determined by:

$$T_{11} = \frac{T_r}{u_{gb} \cdot \eta_{hg}^2 \cdot \eta_{be}^3} \tag{7}$$

$$T_{12} = \frac{T_r}{2 \cdot u_2 \cdot \eta_{hg} \cdot \eta_{be}^2} \tag{8}$$

B. Calculations of Gearbox Efficiency

The gearbox efficiency (%) is calculated by:

$$\eta_{gb} = 100 - \frac{100 \cdot P_l}{P_{in}} \tag{9}$$

where P_l is the total gearbox power loss, determined by [27]:

$$P_l = P_{lg} + P_{lb} + P_{ls} + P_{Z0} \tag{10}$$

where P_{lg}, P_{lb}, P_{ls}, and P_{Z0} represent the power losses in the gears, bearings, seals, and during idle motion, respectively. The calculation of these components follows the methodology outlined in [27].

C. Objective Functions

This study formulates the optimization task as a bi-objective minimization problem, targeting two key performance indicators of the design of a two-stage helical gearbox with split output stage:

- Minimizing the gearbox cross-sectional area:

$$minf_1(X) = A_c \tag{11}$$

- Maximizing the gearbox efficiency:

$$minf_2(X) = \eta_{gb} \tag{12}$$

The design variable vector X encompasses the principal geometrical and performance-related parameters of the gearbox. Typically, five parameters are employed to define the geometry: u_1, Xba_1, Xba_2, AS_1, and AS_2 [19]. However, previous studies [28] have indicated that the optimal values of AS_1, and AS_2 tend to converge toward their upper bounds. Consequently, the three most influential and tunable parameters u_l, Xba_1, and Xba_2 are selected as the decision variables for the optimization process. The formulation is established as:

$$X = \{u_1, Xba_1, Xba_2\} \tag{13}$$

D. Constrains

For the gearbox, the gear ratio of each stage u_i is constrained within the range of 1-9, and the face width coefficient Xba_i is limited to the interval of 0.25-0.4 for (i=1, 2) [26]. The MOO problem is subject to the following constraints:

$$1 \le u_i \le 9 \tag{14}$$

$$0.25 \le Xba_i \le 0.4 \tag{15}$$

III. METHODOLOGY

This study employs a two-stage optimization framework that integrates the NSGA-II and the technique for TOPSIS to solve the MOOP of a two-stage helical gearbox with a split output stage. The primary objectives are to minimize the cross-sectional area and maximize the transmission efficiency, which are typically conflicting criteria in gearbox design.

TABLE I. PARAMETERS AND VARIABLES USED IN THE OPTIMIZATION PROBLEM

Symbol/Abbreviation	Description
u_1	Gear ratio of the first stage
u_2	Gear ratio of the second stage
u_h	Total gearbox ratio
Xba_1, Xba_2	Face width coefficient of the first and second stage
a_{w1}, a_{w2}	Center distances of the first and second stages
b_{w1}, b_{w2}	Face widths of gears in the first and second stages
d_{w1i}, d_{w2i}	Pitch diameters of the pinion and gear of stage i
δ	Clearance or tolerance distance
T_{1i}	Torque on pinion of stage i
V_{gb}	Gearbox volume
A_b	Gearbox bottom cross-sectional area
η_{gb}	Gearbox efficiency
P_l	Total power losses in the gearbox
P_{lg}, P_{lb}, P_{ls} and P_{Z0}	Power losses in gears, bearings, seals, and idle motion.
FWC	Face width coefficient
X	Vector of decision variables
$f_1(X)$	Objective function 1
$f_2(X)$	Objective function 2

A. NSGA-II for Pareto Front Generation

NSGA-II is utilized to generate a diverse set of Pareto-optimal solutions that reflect the trade-offs between the objectives. The algorithm operates as:

- Initialization: A random population of feasible solutions is generated within the defined design space.
- Non-dominated sorting: Individuals are ranked based on Pareto dominance.
- Crowding distance: A diversity-preserving mechanism is applied to maintain a well-spread Pareto front.
- Selection and variation: Tournament selection, crossover, and mutation operators are applied to evolve the population across generations.
- Elitism: Parent and offspring populations are combined, and the best individuals are retained for the next generation.

The result of this phase is a Pareto front P of non-dominated solutions offering various trade-offs between compactness and efficiency.

B. TOPSIS for Final Solution Selection

To identify the most appropriate solution from the Pareto front generated by NSGA-II, TOPSIS is applied. The latter is a well-established MCDM technique that evaluates and ranks alternatives based on their distance to an ideal solution and a nadir (worst-case) solution. The method is implemented through the following systematic steps [29]:

- Step 1: Construct the decision matrix

The decision matrix X of m alternatives and n criteria is constructed:

$$X = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ x_{21} & \cdots & x_{2n} \\ \vdots & \cdots & \vdots \\ x_{mn} & \cdots & x_{mn} \end{bmatrix} \tag{16}$$

where x_{ij} denotes the performance of the i^{th} alternative with respect to the j^{th} criterion.

- Step 2: Normalize the decision matrix

Each entry of the matrix is normalized to obtain dimensionless values:

$$k_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}} \tag{17}$$

- Step 3: Compute the weighted normalized decision matrix

The normalized values are multiplied by their corresponding weights w_j to obtain the weighted normalized matrix:

$$l_{ij} = w_j \times k_{ij} \tag{18}$$

- Step 4: Identify the ideal and anti-ideal solutions

The ideal solution A^+ and anti-ideal solution A^- are determined by:

$$A^+ = \{l_1^+, l_2^+, \dots, l_j^+, \dots, l_n^+\} \tag{19}$$

$$A^- = \{l_1^-, l_2^-, \dots, l_j^-, \dots, l_n^-\} \tag{20}$$

where $l_j^+ = max(l_{ij})$ for the beneficial criteria and $l_j^- = min(l_{ij})$ for the cost criteria, for $j = 1, 2, 3, \dots, n$.

- Step 5: Calculate the separation measures

The Euclidean distances of each alternative from the ideal and anti-ideal solutions are calculated by:

$$D_i^+ = \sqrt{\sum_{j=1}^{n} (l_{ij} - l_j^+)^2} \tag{21}$$

$$D_i^- = \sqrt{\sum_{j=1}^{n} (l_{ij} - l_j^-)^2} \tag{22}$$

- Step 6: Compute the closeness coefficient

The relative closeness R_i of each alternative to the ideal solution is given by:

$$R_i = \frac{D_i^-}{D_i^- + D_i^+} \tag{23}$$

- Step 7: Rank the alternatives

The alternatives are ranked in descending order based on their R_i values. The solution with the highest R_i is considered the most preferable in terms of proximity to the ideal trade-off between the objectives.

C. Implementation Details

The NSGA-II algorithm is implemented in MATLAB with a population size of 200, a crossover probability of 0.9, and a mutation rate of 0.1. The algorithm run for 200 generations. The output Pareto front was post-processed with the TOPSIS method in Excel. This hybrid NSGA-II–TOPSIS methodology provides a robust and systematic approach for the optimal design of compact, high-efficiency gearboxes, especially those with split-stage configurations requiring balanced load distribution and geometric integration.

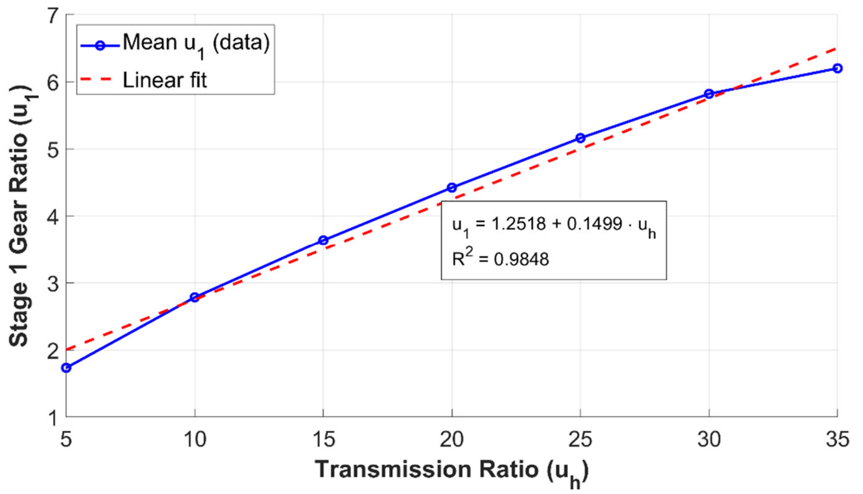

Fig. 2. Linear regression of u_1 versus u_h from Pareto front.

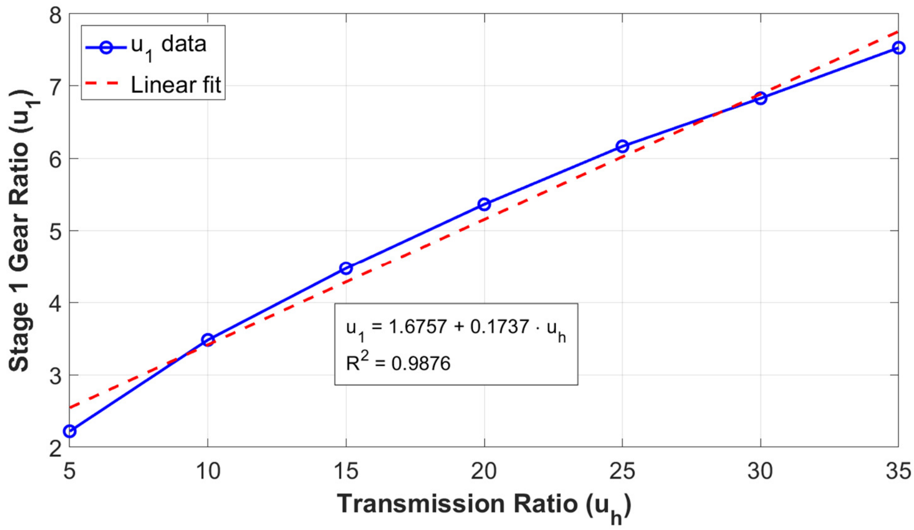

Fig. 3. Linear regression of u_1 versus u_h from TOPSIS solutions.

For the Pareto front, as shown in Figure 2, the regression line (with $R^2 = 0.9848$) is:

$$u_1 = 0.1499 \cdot u_h + 1.2518 \quad (24)$$

For the TOPSIS results, as displayed in Figure 3, the regression yields (with $R^2 = 0.9876$):

$$u_1 = 0.1737 \cdot u_h + 1.6737 \quad (25)$$

IV. RESULTS AND DISCUSSION

This section presents and analyzes the outcomes obtained from the NSGA-II–TOPSIS optimization framework, focusing on the influence of the overall transmission ratio u_h on the gearbox design, including the gear ratio distribution, cross-sectional area, and mechanical efficiency.

A. Relationship between Stage 1 Gear Ratio and Total Transmission Ratio

Figures 2 and 3 illustrate the linear regression results of the first-stage gear ratio u_1 with respect to the total transmission ratio u_h, derived from the Pareto-optimal solutions and from the TOPSIS-selected solutions, respectively.

Both models confirm a strong linear correlation between u_1 and u_h, implying that as the required total gear ratio increases, a larger portion is allocated to the first stage. However, the TOPSIS-based trend suggests a higher stage-1 ratio, which potentially reduces the gear size in the second (split) stage.

B. Trends in Cross-Sectional Area and Efficiency

Figure 4 illustrates the trends of the mean gearbox cross-sectional area and mean efficiency as functions of u_1, as u_h increases. The mean cross-sectional area increases steadily due to the larger gear ratios and geometries required to accommodate higher reductions. The mean efficiency slightly decreases, possibly due to the increased meshing losses and shaft deflections at higher gear ratios. This trend emphasizes the inherent trade-off between compactness and performance in the gearbox design.

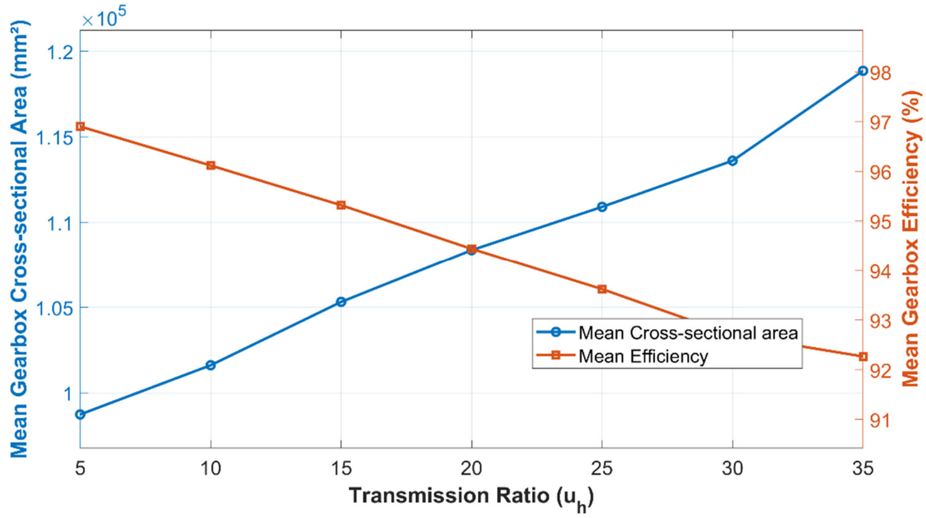

Fig. 4. Trends of mean cross-sectional area and mean efficiency versus u_h.

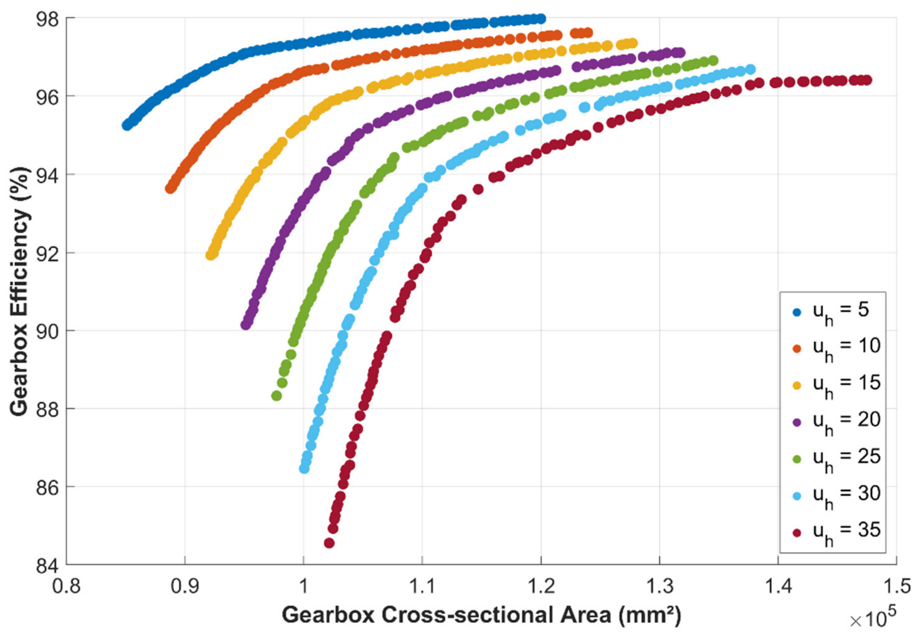

Fig. 5. Pareto fronts for different values of u_h.

C. *Pareto Front Analysis*

The Pareto fronts in Figure 5 provide a visual comparison of the optimal trade-offs between the cross-sectional area and efficiency for different u_h values. Each front demonstrates a clear conflict between the two objectives, and the fronts shift toward the worst performance (larger area, lower efficiency) as u_h increases. The curvature of the Pareto sets also indicates that at higher ratios, the marginal gain in efficiency requires a significantly larger compromise in size.

D. *Comparison Between TOPSIS Solutions and Pareto Averages*

TABLE II. COMPARISON OF TOPSIS-SELECTED SOLUTIONS WITH PARETO FRONT AVERAGE

u_h	Mean Pareto area (mm^2)	Mean Pareto efficiency (%)	TOPSIS area (mm²)	TOPSIS efficiency (%)	Δ_{Area} (%)	$\Delta_{Efficiency}$ (%)
5	119,991.11	97.98	85,758.36	95.45	-28.53	-2.58
10	123,948.98	97.62	90,018.91	94.22	-27.37	-3.49
15	127,713.19	97.35	94,626.63	93.40	-25.91	-4.06
20	131,731.64	97.11	98,382.55	92.51	-25.32	-4.74
25	134,512.27	96.91	101,601.02	91.64	-24.47	-5.44

To evaluate the effectiveness of the TOPSIS method, its selected solutions were compared to the mean values of the Pareto fronts, as shown in Table II. The results indicate that:

- The TOPSIS solutions result in substantially smaller cross-sectional areas (by approximately 24%-29% depending on u_h), showing a clear preference toward compactness.
- However, this comes at the cost of slightly lower efficiency, with a reduction ranging from 2.6%-5.4% compared to the Pareto front average.

This trade-off highlights the decision-making bias of TOPSIS when both objectives are weighted equally or when compactness is prioritized. Nonetheless, the selected solutions still lie within the Pareto front region, indicating that they are technically optimal and feasible.

V. CONCLUSION

This study developed a comprehensive Multi-Objective Optimization (MOO) framework for the design of two-stage helical gearboxes, addressing the simultaneous goals of minimizing the gearbox cross-sectional area and maximizing the transmission efficiency. The research was motivated by the practical challenge of balancing compactness and performance—two inherently conflicting design objectives in gear reducer systems. Although several prior studies have explored the gear optimization using Non-dominated Sorting Genetic Algorithm II (NSGA-II) or Multi-Criteria Decision-Making (MCDM) methods, most have focused on fixed transmission ratios or limited problem settings, leaving a gap in generalized modeling across varying design requirements.

To bridge this gap, the present work employed the NSGA-II algorithm across a wide range of transmission ratios ($u_h \in [5, 35]$), considering three key design variables: the gear ratio of the first stage u_1, and the face width coefficients of both stages Xba_1 and Xba_2. A physics-based model was used to compute the gearbox volume and efficiency, enabling an accurate evaluation of the trade-offs. The optimization process yielded Pareto-optimal solutions for each value of u_h, revealing a consistent trend: higher transmission ratios resulted in increased gearbox volume and reduced efficiency. These findings are critical for applications requiring high reduction ratios where spatial constraints are severe.

A significant contribution of this study lies in the discovery of a strong linear relationship between u_1 and u_h. This trend—validated through the mean values of Pareto-optimal solutions (R_2 = 0.9717) and TOPSIS selected best compromises (R_2 = 0.9941)—provides a generalized rule for allocating the gear ratios between stages. The present work offers a scalable and generalizable insight that can directly support the rule-based preliminary design.

Furthermore, by integrating the TOPSIS method into the decision-making process, the study moves beyond a mere generation of Pareto fronts. It enables the practical selection of balanced solutions that offer meaningful compromises in real-world design contexts. These selected solutions were shown to align closely with the discovered linear relationship, reinforcing the stability and predictive strength of the model.

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