Optimal Design of a Two-Stage Helical Gearbox Using NSGA-II and MAIRCA: A Trade-Off between Performance and Axial Length

Van Thanh Dinh
East Asia University of Technology, Trinh Van Bo Street, Hanoi City 12000, Vietnam
thanh.dinh@eaut.edu.vn

Duc Binh Vu
Viet Tri University of Industry, 09 Tien Son Street, Viet Tri City 35100, Vietnam
vubinh@vui.edu.vn

Ngoc Pi Vu
Thai Nguyen University of Technology, 3/2 Street, Tich Luong ward, Thai Nguyen City 251750, Vietnam
vungocpi@tnut.edu.vn

Manh Cuong Nguyen
Thai Nguyen University of Technology, 3/2 Street, Tich Luong ward, Thai Nguyen City 251750, Vietnam
nmcuong@tnut.edu.vn (corresponding author)

Received: 27 April 2025 | Revised: 21 June 2025 | Accepted: 28 June 2025

 DOI: https://doi.org/10.48084/etasr.11758

ABSTRACT

This study presents a Multi-Objective Optimization (MOO) approach for the design of a two-stage helical gearbox, aiming to simultaneously maximize the transmission efficiency and minimize the overall axial length. The optimization framework integrates the Non-dominated Sorting Genetic Algorithm II (NSGA-II) to generate a diverse Pareto front of optimal solutions, and the Multi-Attribute Ideal-Real Comparative Analysis (MAIRCA) method to support decision-making by ranking the Pareto-optimal alternatives. A physics-based mathematical model of gearbox geometry, load capacity, and efficiency was established to evaluate the performance of each candidate design. The design variables include the gear ratio distribution between the stages and the face width coefficients, subject to practical constraints, such as the gear strength and contact ratios. The results show a clear trade-off between performance and compactness, and demonstrate the effectiveness of combining NSGA-II with MAIRCA for identifying optimal gearbox designs under conflicting objectives. This approach offers valuable insights for gearbox designers aiming to balance performance and spatial constraints in modern mechanical systems. This is the first study to apply a combined NSGA-II and MAIRCA technique for the bi-objective optimization of a two-stage helical gearbox with the goals of maximizing efficiency and minimizing the length. The proposed method successfully determines the optimal design parameters across a range of transmission ratios, thereby facilitating the early-stage selection of suitable gear ratios in preliminary gearbox design.

Keywords-two-stage helical gearbox; multi-objective optimization; NSGA-II; MAIRCA; gear ratio; face width coefficient; efficiency; gearbox length; MCDM

I. INTRODUCTION

Helical gearboxes play a vital role in various mechanical transmission systems due to their superior load-carrying capacity, smooth operation, and high efficiency. However, the optimal design of multi-stage helical gearboxes remains a complex task, particularly when addressing multiple conflicting objectives, such as performance, volume, and structural compactness. Researchers have increasingly employed MOO algorithms to tackle these challenges and identify the optimal trade-offs in gearbox design.

The foundation of many optimization approaches lies in the NSGA-II algorithm, which was proposed in [1]. NSGA-II is widely regarded for its elitism and fast non-dominated sorting capability, making it suitable for solving complex engineering problems with multiple objectives. Authors in [2] presented a method for the optimal design of spur gear sets using mathematical modeling techniques. Authors in [3] demonstrated the effectiveness of genetic algorithms in automating the preliminary design of gear drives. Similarly, authors in [4] compared the particle swarm optimization and simulated annealing for optimal weight design in gear trains, highlighting the flexibility of the metaheuristic techniques.

Research has also focused on incorporating tribological and acoustic constraints into the optimization process. Authors in [5] investigated the optimization of the spur gearbox by considering tribological aspects. In addition authors in [6] presented a comprehensive study combining design parameters with mechanical performance indices. Authors in [7] extended the optimization to a 3-stage wind turbine gearbox and highlighted the importance of tribological constraints in large-scale systems.

Multi-Objective Evolutionary Algorithms (MOEAs) have proven particularly useful in these contexts. Authors in [8] provided an overview of the evolutionary algorithms for solving the MOO problems. In the specific context of helical gear trains, authors in [9] optimized a two-stage gearbox using NSGA-II, achieving significant improvements in weight and volume. Similarly, authors in [10] applied heuristic algorithms to optimize an aeroengine accessory gearbox with respect to the power transmission efficiency and dynamic characteristics.

Various studies have also considered different gearbox configurations, such as epicyclic and planetary gear systems. For instance, authors in [11] used a genetic algorithm for optimizing an epicyclical gear train, while authors in [12] focused on the dynamic characteristics of planetary gear transmissions. These works demonstrate the growing interest in the MOO applications across different gearbox architectures.

Several authors have specifically targeted the optimization of two-stage gearboxes. Authors in [13] applied NSGA-II to minimize the volume and maximize the efficiency in a two-stage spur gearbox. Authors in [14] used acoustic performance metrics and response surface methodology to optimize the gearbox design for noise control. In [15], NSGA-II was coupled with decision-making methods to achieve well-balanced trade-offs in a two-stage spur gearbox.

Noise and vibration reduction has also become an emerging objective in gearbox design. Authors in [16] investigated the noise and vibration behavior of electric bus gearboxes and applied MOO to improve the acoustic performance. Authors in [17] explored the use of evolutionary algorithms for multi-speed gearbox design, showing that MOEAs can effectively manage the multiple speed configurations and objectives.

Multiple optimization algorithms have been compared for their applicability in gearbox design to be assessed. Authors in [18] evaluated several algorithms and confirmed the robustness of NSGA-II in handling competing objectives. Authors in [19] combined NSGA-II with decision-making techniques to enhance the final solution selection. Authors in [20] addressed the balance between the gearbox volume and efficiency using a hybrid MOO approach.

Applications in specialized contexts have also been explored. Authors in [21] proposed an optimization framework for planetary gear reducers used in mining equipment, while authors in [22] focused on optimizing the planet carrier in wind turbine gearboxes. Authors in [23] applied NSGA-III to optimize the angular contact ball bearings for aircraft gearboxes, showcasing the evolution of MOEAs toward more advanced versions.

Stochastic methods have also found use in gearbox component optimization. Authors in [24] performed a stochastic MOO of synchronizers and selectors, emphasizing the uncertainty management in the design. Finally, authors in [25] employed the MARCOS method to optimize a two-stage helical gearbox with double gears in the first stage, underlining the value of hybrid techniques combining NSGA-II with decision-making methods.

While many of the above studies concentrated on minimizing the volume or weight and maximizing efficiency, fewer have directly addressed the axial length—a crucial factor in compact gear design for constrained installations. Moreover, only a limited number of studies have employed Multi-Attributive Ideal-Real Comparative Analysis (MAIRCA) as a post-optimization decision-making tool. This paper aims to fill these gaps by proposing a hybrid NSGA-II–MAIRCA approach to simultaneously optimize the transmission efficiency and axial length in the design of a two-stage helical gearbox. The proposed method provides a structured trade-off analysis and identifies the compromise solutions suited for practical applications.

II. OPTIMIZATION PROBLEM

A. Determining the gearbox length

The length L_{gb} of a two-stage helical gearbox can be determined as (Figure 1):

$$L_{gb} = d_{w11} + d_{w21}/2 + d_{w12}/2 + d_{w22} + 2 \cdot \delta \quad (1)$$

where δ=7÷10 (mm) [26], b_{wi}, d_{w1i}, d_{w2i} (i = 1÷2) denote the width of gear, the pitch diameter of the pinion, and the gear of stage i, which are calculated by:

$$b_{wi} = X_{bai} \cdot a_{wi} \quad (2)$$

$$d_{w1i} = 2 \cdot a_{wi}/(u_i + 1) \quad (3)$$

$$d_{w2i} = 2 \cdot a_{wi} \cdot u_i \cdot/(u_i + 1) \quad (4)$$

where $Xbai$ and a_{wi} (i = 1÷2) are the face width coefficient and the center distance of stage i, respectively and a_{wi} is determined by [26]:

$$a_{wi} =$$

$$k_a \cdot (u_i + 1) \cdot \sqrt[3]{T_{1i} \cdot k_{H\beta}/([AS_i]^2 \cdot u_i \cdot X_{bai})} \qquad (5)$$

where T_{1i} (i = 1÷2) is the pinion torque of stage i, which can be computed by:

$$T_{1i} = \frac{T_r}{\prod_{j=i}^{3}(u_i \cdot \eta_{hg}^{3-i} \cdot \eta_{be}^{4-i})} \qquad (6)$$

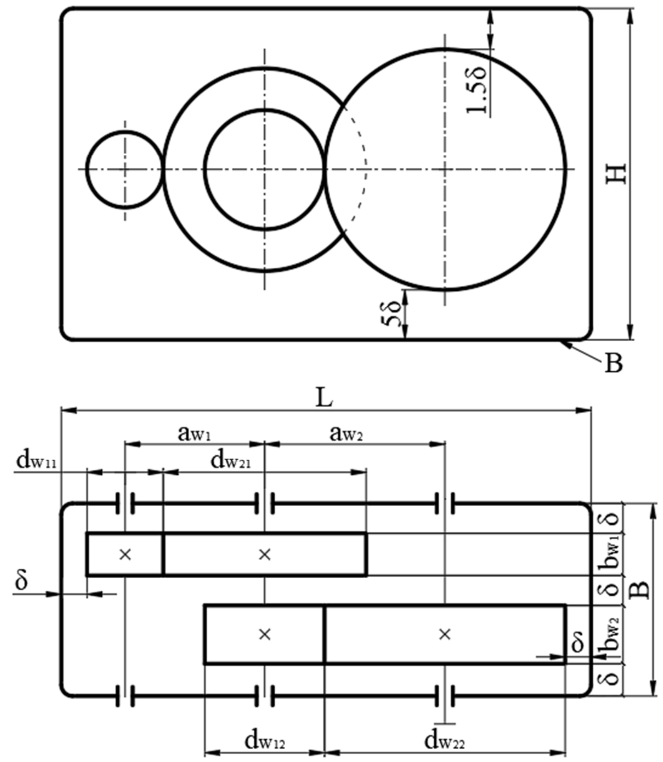

Fig. 1. Schematics for determination of gearbox volume.

B. *Determining the Gearbox Efficiency*

The gearbox efficiency (%) is determined by:

$$\eta_{gb} = 100 - \frac{100 \cdot P_l}{P_{in}} \qquad (7)$$

where P_l denotes the total gearbox power loss, which is calculated by [27]:

$$P_l = P_{lg} + P_{lb} + P_{ls} + P_{Z0} \qquad (8)$$

Specifically, P_{lg}, P_{lb}, P_{ls}, and P_{Z0} correspond to the power losses due to gear meshing, bearing friction, seal resistance, and idle motion, respectively. The computation of these losses is performed based on the methodology presented in [27].

Two essential performance features of a two-stage helical gearbox are addressed in this optimization technique, which are structured as a bi-objective minimization problem:

Minimizing the gearbox length:

$$minf_1(X) = L_{gb} \qquad (9)$$

Maximizing the gearbox efficiency:

$$minf_2(X) = \eta_{gb} \qquad (10)$$

The design variable vector X encapsulates the principal geometric and performance-related parameters governing the gearbox configuration. Although a complete geometric definition generally involves five parameters-namely, the gear ratio of the first stage (u_1), the face width coefficients of both stages (X_{ba1}, X_{ba2}), and the aspect ratios of the first and second stages (AS_1, AS_2) [28]- previous research has demonstrated that AS_1 and AS_2 tend to reach their upper bounds in most optimal solutions [28]. As a result, the present study focuses on the three most influential and practically adjustable design variables: u_1, X_{ba1}, and X_{ba2}, leading to:

$$X = \{u_1, Xba_1, Xba_2\} \qquad (11)$$

C. *Constrains*

For a two-stage helical gearbox, the gear ratio for each stage u_i typically ranges from 1 to 9, while the face width coefficient X_{bai} varies between 0.25 and 0.4 for both stages (i =1, 2) [27]. Consequently, MOOP is subject to the following constraints:

$$1 \leq u_i \leq 9 \qquad (12)$$

$$0.25 \leq Xba_i \leq 0.4 \qquad (13)$$

III. OPTIMIZATION METHODOLOGY

This study adopts a two-step optimization framework to determine the optimal design of a two-stage helical gearbox, targeting a trade-off between the transmission efficiency and axial length. The proposed methodology integrates the NSGA-II for generating a Pareto-optimal solution set and the MAIRCA method to rank and select the most balanced design alternatives. The basic contents of the two methods NSGA II and MAIRCA are:

A. *Optimization Using NSGA-II*

NSGA-II is employed to explore the Pareto front of optimal solutions based on two conflicting objectives. The key features of NSGA-II [1] include:

- Fast non-dominated sorting to classify solutions into Pareto fronts.
- Crowding distance assignment to maintain diversity.
- Elitist selection to retain high-quality solutions over generations.

The main steps of the algorithm are:

1. Initialization: Generate an initial population of candidate solutions within predefined bounds.
2. Fitness Evaluation: Compute objective function values (efficiency and axial length) for each solution using the mathematical model of the gearbox.
3. Non-Dominated Sorting and Ranking: Classify solutions into different fronts based on Pareto dominance.
4. Crowding Distance Calculation: Measure the density of solutions around each individual.

5. Selection, Crossover, and Mutation: Apply genetic operators to produce offspring.
6. Environmental Selection: Form the next generation by combining parent and offspring populations and retaining the best individuals.

After a predefined number of generations, the algorithm yields a Pareto front representing a set of optimal trade-off solutions.

B. Decision-Making Using MAIRCA

The MAIRCA method is employed to rank the Pareto-optimal solutions obtained from NSGA-II based on multiple criteria. The procedure consists of the following steps [29]:

- Construct the decision matrix $X=[x_{ij}]$, where x_{ij} is the value of criterion j for alternative i:

$$X = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ x_{21} & \cdots & x_{2n} \\ \vdots & \cdots & \vdots \\ x_{mn} & \cdots & x_{mn} \end{bmatrix} \quad (14)$$

- Assign equal selection probability to each criterion using:

$$P_{A_j} = \frac{1}{m} \quad , j = 1, 2, \ldots, n \quad (15)$$

where m is the number of alternatives.

- Compute the ideal preference matrix:

$$t_{p_{ij}} = P_{A_j} \cdot w_j \quad (16)$$

where w_j is the weight of criterion j.

- Calculate the real performance matrix tr_{ij} using one of the following: For the maximization criteria, the efficiency:

$$t_{r_{ij}} = t_{p_{ij}} \cdot \left(\frac{x_{ij} - x_i^-}{x_i^+ - x_i^-}\right) \quad (17)$$

For the minimization criteria, the axial length:

$$t_{r_{ij}} = t_{p_{ij}} \cdot \left(\frac{x_{ij} - x_i^+}{x_i^- - x_i^+}\right) \quad (18)$$

where x_j^+ and x_j^- are the best and worst values of criterion j, respectively.

- Compute the deviation matrix:

$$g_{ij} = t_{p_{ij}} - t_{r_{ij}} \quad (19)$$

Calculate the overall gap for each alternative:

$$Q_i = \sum_{i=1}^{m} g_{ij} \quad (20)$$

The alternatives are ranked in ascending order of Qi. The lower the Qi is, the better is the compromise solution.

IV. RESULTS AND DISCUSSION

A. Relationship between the Stage Gear Ratio and the Overall Transmission Ratio

To investigate the distribution of the gear ratio across stages, a regression analysis was performed between the first-stage gear ratio (u_1) and the overall transmission ratio (u_h). Figure 2 illustrates the linear trend derived from the best Pareto-optimal solutions:

$$u_1 = 0.1456 \cdot u_h + 1.1758 \quad (21)$$

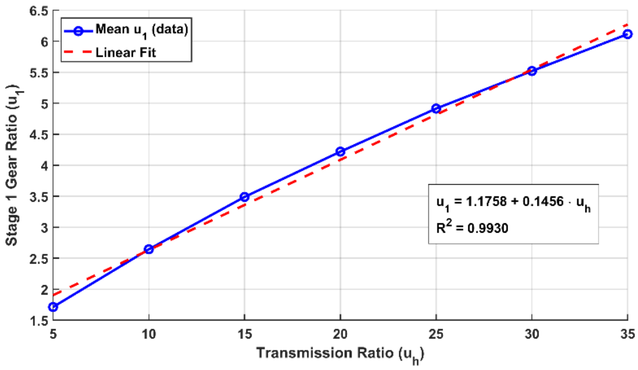

Fig. 2. Linear regression of u_1 versus u_h based on the best Pareto solutions.

This strong linear correlation (with R^2 = 0.9930) confirms that the optimal distribution of the total gear ratio tends to allocate a progressively larger portion to the first stage as u_h increases. This provides a practical design guideline, suggesting that the first-stage ratio can be estimated early in the design phase using this regression model.

B. Comparison with MAIRCA-Based Best Solutions

To assess the MAIRCA decision-making outcomes, the same regression was repeated on the best-ranked solutions from the MAIRCA method. As shown in Figure 3, a similar linear trend was observed:

$$u_1 = 0.1470 \cdot u_h + 1.1857 \quad (22)$$

The negligible difference in the slope and intercept between the two regressions implies that the MAIRCA method effectively selected solutions aligned with the overall design tendency captured from the Pareto fronts.

C. Trade-Off Trends: Gearbox Length Versus Efficiency

Figure 4 presents the average gearbox length and mean efficiency at different transmission ratios u_h. As u_h increases, the efficiency consistently decreases, whereas the gearbox length initially decreases slightly before increasing. This exhibits a clear trade-off: compact designs with high gear ratios tend to compromise the mechanical efficiency.

This behavior is attributed to the higher losses in the second-stage gears and bearings as their gear ratio $u_2 = u_h/u_1$ becomes large, especially at higher u_h values.

D. Pareto Fronts and Impact of u_h

The complete distribution of Pareto fronts across different u_h values is portrayed in Figure 5. Each front represents the trade-off surface between the gearbox efficiency and axial length. Lower u_h values (e.g., 5–15) offer solutions with both high efficiency and short gearbox length. Conversely, higher u_h values (e.g., 30–35) exhibit a shift toward longer gearboxes with reduced efficiency. This emphasizes the importance of appropriately choosing u_h to ensure a balanced gearbox design.

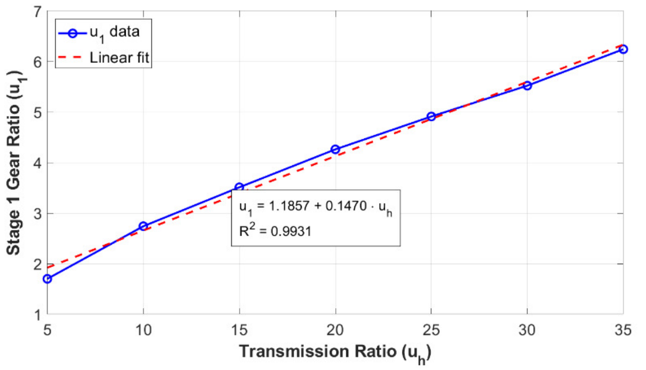

Fig. 3. Linear regression of u_1 versus u_h based on MAIRCA-selected designs.

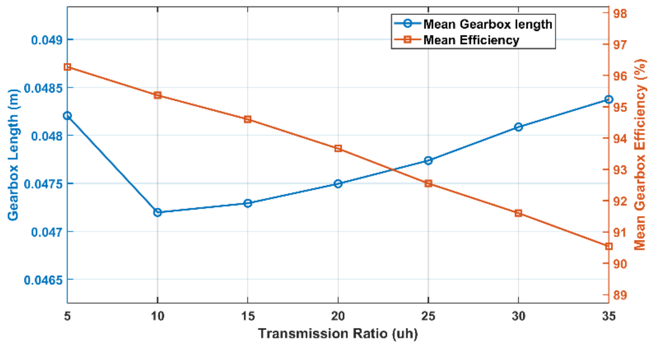

Fig. 4. Mean gearbox length and efficiency versus transmission ratio (u_h).

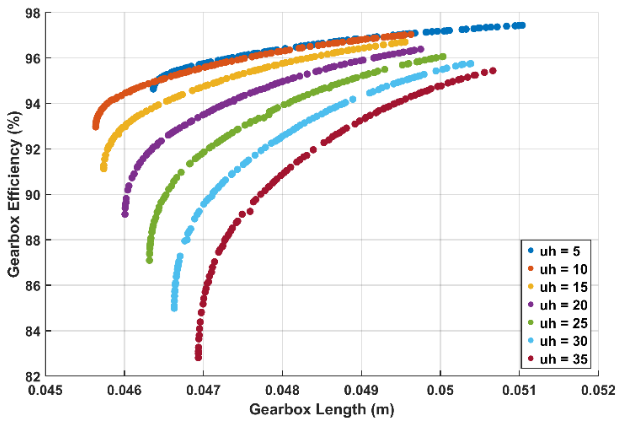

Fig. 5. Pareto fronts for various overall transmission ratios u_h.

E. Final Designs Selected by MAIRCA

Table I summarizes the gearbox configurations selected by the MAIRCA method for different u_h values.

F. Comparison with the Taguchi–GRA Based Study [30]

To further validate the effectiveness of the current optimization framework (NSGA-II and MAIRCA), a comparison is made with the optimal design results reported in [30], where the same two-stage helical gearbox structure was optimized using the Taguchi method and Grey Relational Analysis (GRA). Both studies share the same design objectives—namely, maximizing the gearbox efficiency while minimizing the axial length—under equivalent design constraints and physical assumptions.

TABLE I. MAIRCA-BASED OPTIMAL DESIGNS FOR VARYING *uh*

u_h	u_1	X_{ba1}	X_{ba2}	Length (m)	Efficiency (%)
5	1.70	0.4	0.4	0.47186	95.89
10	2.74	0.4	0.4	0.46211	94.75
15	3.51	0.4	0.4	0.6395	93.87
20	4.26	0.4	0.4	0.46555	92.60
25	4.91	0.4	0.4	0.46896	91.56
30	5.57	0.4	0.4	0.47185	90.54
35	6.21	0.4	0.4	0.47458	89.74

Table II presents the optimal design parameters for varying overall gear ratios (u_h), as obtained in [30]. Meanwhile, Table III provides a quantitative comparison of the gearbox length (L_{gb}) and efficiency (η_{gb}) between the present study and [30], along with their percentage deviations.

As shown in Table III, the gearbox length obtained from this study is consistently shorter than that in [30], with a maximum reduction of approximately 19.87%. In terms of efficiency, the results are largely consistent, with deviations typically below ±3%. These findings confirm that the proposed NSGA-II and MAIRCA approach can identify gearbox designs that are not only efficient, but also more compact compared to those derived from the classical Taguchi–GRA methods.

TABLE II. OPTIMAL DESIGN PARAMETERS OBTAINED BY TAGUCHI–GRA IN [30]

u_h	u_1	X_{ba1}	X_{ba2}	AS_1	AS_2
10	1.00	0.25	0.40	350	420
15	1.57	0.25	0.40	350	420
20	2.12	0.25	0.40	367.5	420
25	2.68	0.40	0.40	367.5	420
30	3.23	0.40	0.40	367.5	420
35	3.79	0.40	0.40	350	420

TABLE III. COMPARISON OF GEARBOX PERFORMANCE BETWEEN THE PROPOSED METHOD AND TAGUCHI–GRA METHODS

Gearbox length L_{gb}			Gearbox efficiency η_{gb}		
Proposed method	[30]	Difference (%)	Proposed method	[30]	Difference (%)
0.462	0.573	19.35	94.75	91.800	3.11
0.464	0.579	19.87	93.87	93.862	0.01
0.466	0.537	13.31	92.60	94.428	-1.97
0.469	0.514	8.760	91.56	91.374	0.21
0.472	0.512	7.720	90.53	91.016	-0.54
0.475	0.502	5.400	89.12	91.134	-2.26

V. CONCLUSIONS

This study proposed a comprehensive framework for the optimal design of a two-stage helical gearbox by integrating the Non-dominated Sorting Genetic Algorithm II (NSGA-II) with the Multi-Attribute Ideal-Real Comparative Analysis (MAIRCA) method. The bi-objective optimization aimed to simultaneously maximize the transmission efficiency and minimize the axial length, which are two conflicting objectives in the gearbox design.

Through NSGA-II, diverse Pareto fronts were obtained for different overall transmission ratios (u_h), revealing that lower u_h values (typically 5–15) provide more favorable trade-offs between compactness and performance. A strong linear correlation was found between the first-stage gear ratio (u_1) and the overall ratio (u_h), offering a practical guideline for gear ratio distribution in preliminary design.

The application of the MAIRCA method effectively prioritized the optimal solutions by evaluating both ideal and realistic expectations. The selected designs consistently featured the maximum allowable face width coefficients, emphasizing their influence on the load-carrying capacity and efficiency.

Overall, the integration of NSGA-II and MAIRCA has proven to be a robust approach for multi-objective gearbox optimization, providing not only a rich set of optimal trade-offs, but also practical decision support for selecting the most balanced configurations. This is the first study to apply a combined NSGA-II and Multi-Criteria Decision-Making (MCDM) method to achieve the bi-objective optimization of a two-stage gearbox, explicitly targeting the maximization of the transmission efficiency and the minimization of the axial length. The proposed method successfully identified the optimal design parameters across a wide range of overall gear ratios (u_h), thereby enabling informed and convenient selection of the appropriate gearbox ratios at the early stages of design. Future work may extend this framework by incorporating additional objectives, such as gear noise or manufacturing cost, or by validating the optimal designs through experimental testing.

ACKNOWLEDGMENT

The authors gratefully acknowledge the support from Thai Nguyen University of Technology (TNUT), Vietnam.

REFERENCES

[1] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan, "A fast and elitist multiobjective genetic algorithm: NSGA-II," *IEEE Transactions on Evolutionary Computation*, vol. 6, no. 2, pp. 182–197, Apr. 2002, https://doi.org/10.1109/4235.996017.

[2] H. Wang and H. P. Wang, "Optimal engineering design of spur gear sets," *Mechanism and Machine Theory*, vol. 29, no. 7, pp. 1071–1080, Oct. 1994, https://doi.org/10.1016/0094-114x(94)90074-4.

[3] C. Gologlu and M. Zeyveli, "A genetic approach to automate preliminary design of gear drives," *Computers & Industrial Engineering*, vol. 57, no. 3, pp. 1043–1051, Oct. 2009, https://doi.org/10.1016/j.cie.2009.04.006.

[4] V. Savsani, R. V. Rao, and D. P. Vakharia, "Optimal weight design of a gear train using particle swarm optimization and simulated annealing algorithms," *Mechanism and Machine Theory*, vol. 45, no. 3, pp. 531–541, Mar. 2010, https://doi.org/10.1016/j.mechmachtheory.2009.10.010.

[5] M. Patil, P. Ramkumar, and K. Shankar, "Multi-Objective Optimization of Spur Gearbox with Inclusion of Tribological Aspects," *Journal of Friction and Wear*, vol. 38, no. 6, pp. 430–436, Nov. 2017, https://doi.org/10.3103/s1068366617060101.

[6] J. Alam and S. Panda, "A comprehensive study on multi-objective design optimization of spur gear," *Mechanics Based Design of Structures and Machines*, vol. 51, no. 9, pp. 5272–5298, Sep. 2023, https://doi.org/10.1080/15397734.2021.1996246.

[7] A. Kumar, P. Ramkumar, and K. Shankar, "Multi-objective 3-stage wind turbine gearbox (WTG) with tribological constraint," *Mechanics Based Design of Structures and Machines*, vol. 52, no. 8, pp. 5263–5289, Aug. 2024, https://doi.org/10.1080/15397734.2023.2249987.

[8] C. A. Coello Coello, D. A. Van Veldhuizen, and G. B. Lamont, *Evolutionary Algorithms for Solving Multi-Objective Problems*. Boston, MA: Springer US, 2007.

[9] R. C. Sanghvi, A. S. Vashi, H. P. Patolia, and R. G. Jivani, "Multi-Objective Optimization of Two-Stage Helical Gear Train Using NSGA-II," *Journal of Optimization*, vol. 2014, pp. 1–8, 2014, https://doi.org/10.1155/2014/670297.

[10] M. Hu, J. Zhu, L. Gong, Z. Lu, and H. Liu, "Multi-Objective Optimization of an Aeroengine Accessory Gearbox Transmission Based on a Heuristic Algorithm," *Journal of Aerospace Engineering*, vol. 38, no. 2, Mar. 2025, https://doi.org/10.1061/jaeeez.aseng-5155.

[11] K. Daoudi, E. M. Boudi, and M. Abdellah, "Genetic Approach for Multiobjective Optimization of Epicyclical Gear Train," *Mathematical Problems in Engineering*, vol. 2019, no. 1, Jan. 2019, Art. no. 9324903, https://doi.org/10.1155/2019/9324903.

[12] Q. Zhang, S. Xu, Y. Yuan, X. Yan, D. Huang, and J. Yu, "Multi-objective Optimization Design and Dynamic Characteristic Analysis based on Planetary Gear Transmission," *Journal of Network Intelligence*, vol. 8, no. 3, pp. 639–657, 2023.

[13] M. Patil, P. Ramkumar, and S. Krishnapillai, "Multi-Objective Optimization of Two Stage Spur Gearbox Using NSGA-II," in *SAE Technical Paper Series*, 400 Commonwealth Drive, Warrendale, PA, United States, Jul. 2017, vol. 1, Art. no. 1939, https://doi.org/10.4271/2017-28-1939.

[14] L. Qi, J. Zhou, and H. Xu, "Multi-objective optimization of gearbox based on panel acoustic participation and response surface methodology," *Journal of Low Frequency Noise, Vibration and Active Control*, vol. 41, no. 3, pp. 1108–1130, Sep. 2022, https://doi.org/10.1177/14613484221091075.

[15] E. S. Maputi and R. Arora, "Multi-objective optimization of a 2-stage spur gearbox using NSGA-II and decision-making methods," *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, vol. 42, no. 9, pp. 1–22, Sep. 2020, https://doi.org/10.1007/s40430-020-02557-2.

[16] Y. Lei, L. Hou, Y. Fu, J. Hu, and W. Chen, "Research on vibration and noise reduction of electric bus gearbox based on multi-objective optimization," *Applied Acoustics*, vol. 158, Jan. 2020, Art. no. 107037, https://doi.org/10.1016/j.apacoust.2019.107037.

[17] K. Deb and S. Jain, "Multi-Speed Gearbox Design Using Multi-Objective Evolutionary Algorithms," *Journal of Mechanical Design*, vol. 125, no. 3, pp. 609–619, Sep. 2003, https://doi.org/10.1115/1.1596242.

[18] R. Ananthapadmanabhan, S. A. Babu, K. Hareendranath, C. Krishnamohan, S. Krishnapillai, and K. A, "Investigation on Multiple Algorithms for Multi-Objective Optimization of Gear Box," *IOP Conference Series: Materials Science and Engineering*, vol. 149, Sep. 2016, Art. no. 012049, https://doi.org/10.1088/1757-899x/149/1/012049.

[19] Q. Yao, "Multi-objective optimization design of spur gear based on NSGA-II and decision making," *Advances in Mechanical Engineering*, vol. 11, no. 3, Mar. 2019, https://doi.org/10.1177/1687814018824936.

[20] D. Miler, D. Žeželj, A. Lončar, and K. Vučković, "Multi-objective spur gear pair optimization focused on volume and efficiency," *Mechanism and Machine Theory*, vol. 125, pp. 185–195, Jul. 2018, https://doi.org/10.1016/j.mechmachtheory.2018.03.012.

[21] W. Xin, Y. Zhang, Y. Fu, W. Yang, and H. Zheng, "A multi-objective optimization design approach of large mining planetary gear reducer," *Scientific Reports*, vol. 13, no. 1, Oct. 2023, Art. no. 18640, https://doi.org/10.1038/s41598-023-45745-5.

[22] P. X. Yi, L. J. Dong, and Y. X. Chen, "The Multi-Objective Optimization of the Planet Carrier in Wind Turbine Gearbox," *Applied Mechanics and Materials*, vol. 184–185, pp. 565–569, Jun. 2012, https://doi.org/10.4028/www.scientific.net/amm.184-185.565.

[23] J. H. Kim, B. S. Kim, D. Im, J. H. Park, D. J. Moon, and Y. J. Park, "Optimization of high-speed angular contact ball bearing for aircraft gearbox utilizing an evolutionary multi-objective algorithm, NSGA-III," *Advances in Mechanical Engineering*, vol. 16, no. 9, Sep. 2024, https://doi.org/10.1177/16878132241272197.

[24] M. Gobbi, G. Mastinu, and M. Caudano, "Stochastic Multi-Objective Optimisation of a Gearbox Synchroniser and Selector Mechanism," in *Design Engineering, Volumes 1 and 2*, Washington, DC, USA, Jan. 2003, pp. 113–124, https://doi.org/10.1115/imece2003-43005.

[25] L. D. Bao, V. D. Binh, D. V. Thanh, K. M. Nguyen, and L. X. Hung, "Multi-Objective Optimization of a Two-stage Helical Gearbox with Double Gears in the First Stage using MARCOS," *Engineering, Technology & Applied Science Research*, vol. 14, no. 6, pp. 18245–18251, Dec. 2024, https://doi.org/10.48084/etasr.8865.

[26] T. Chat and L. Van Uyen, *Design and Calculation of Mechanical Transmissions Systems*, vol. 1. Hanoi, Vietnam: Educational Republishing House, 2007.

[27] D. Jelaska, *Gears and Gear Drives*, 1st ed., New Tork, NY: Wiley, 2012.

[28] X. H. Le and N. P. Vu, "Multi-Objective Optimization of a Two-Stage Helical Gearbox Using Taguchi Method and Grey Relational Analysis," *Applied Sciences*, vol. 13, no. 13, Jun. 2023, Art. no. 7601, https://doi.org/10.3390/app13137601.

[29] K. Sri, S. Hartati, A. Harjoko, and R. Wardoyo, *Fuzzy Multi-Attribute Decision Making (Fuzzy MADM)*, Yogyakarta, Indonesia: Graha Ilmu, 2006.

[30] D. Van Thanh, B. T. Danh, V. Le The Anh, T. H. Danh, H. X. Tu, and N. M. Cuong, "Multi-objective Optimization of a Two-Stage Helical Gearbox to Increase Gearbox Efficiency and Decrease Gearbox Length," in *Lecture Notes in Networks and Systems*, Cham: Springer Nature Switzerland, 2024, pp. 195–211.