A Hybrid NSGA-II and EAMR Approach for Dual-Objective Optimization of a Two-Stage Gearbox with Split Input Stage

Van Thanh Dinh
East Asia University of Technology, Trinh Van Bo Street, Hanoi City 12000, Vietnam
thanh.dinh@eaut.edu.vn

Duc Binh Vu
Viet Tri University of Industry, 09 Tien Son Street, Viet Tri City 35100, Vietnam
vubinh@vui.edu.vn

Manh Cuong Nguyen
Thai Nguyen University of Technology, 3/2 street, Tich Luong Ward, Thai Nguyen City 251750, Vietnam
nmcuong@tnut.edu.vn

Thi Thu Huong Truong
Thai Nguyen University of Technology, 3/2 street, Tich Luong Ward, Thai Nguyen City 251750, Vietnam
huongk8@tnut.edu.vn (corresponding author)

Received: 28 April 2025 | Revised: 21 June 2025 | Accepted: 28 June 2025

 DOI: https://doi.org/10.48084/etasr.11790

ABSTRACT

This study proposes a hybrid Multi-Objective Optimization (MOO) approach that integrates the Non-dominated Sorting Genetic Algorithm II (NSGA-II) and the Evaluation by an Area-based Method of Ranking (EAMR) to optimize the design of a two-stage helical gearbox featuring a split input stage. The optimization simultaneously targets two conflicting objectives: minimizing the base area of the gearbox housing and maximizing the transmission efficiency. A parametric design model was developed to capture the key geometrical and performance parameters, while ensuring compliance with the practical constraints related to gear strength and manufacturing limitations. The NSGA-II algorithm was employed to generate a set of Pareto-optimal solutions, which were subsequently ranked using the EAMR method to support decision-making. The results demonstrate the effectiveness of the hybrid approach in identifying high-performance design alternatives that offer an optimal balance between compactness and efficiency. This is the first study to apply a combination of NSGA-II and a Multi-Criteria Decision-Making (MCDM) method to the dual-objective optimization of a two-stage gearbox with a split input stage. In particular, the proposed approach successfully identifies the optimal design parameters across a wide range of transmission ratios, thereby facilitating the initial selection of appropriate gearbox transmission ratios in the early-stage design. This study also provides valuable design guidelines for engineers seeking to improve space utilization and performance in gear transmission systems.

Keywords-two-stage gearbox; split input stage; NSGA-II; EAMR; multi-objective optimization; gear design; gearbox efficiency; base area minimization; helical gears; Pareto front

I. INTRODUCTION

The increasing demand for compactness, efficiency, and durability in mechanical transmission systems has intensified the research efforts toward the optimal design of gearboxes. Among various configurations, two-stage gearboxes with split input stages have attracted attention due to their flexibility in load distribution and potential for structural and functional enhancement. However, designing such systems often entails resolving the trade-offs between conflicting objectives, such as minimizing the size or mass while maximizing the transmission efficiency. Consequently, MOO techniques, particularly hybrid approaches, have become instrumental in achieving balanced solutions. Various evolutionary algorithms and decision-making strategies have been employed to address these trade-offs. For instance, authors in [1] introduced a hybrid Sailfish Optimization Algorithm (SOA) for planetary gearbox design, demonstrating improvements in both the mass and performance-related criteria. Authors in [2] explored a three-stage wind turbine gearbox considering tribological constraints, employing MOO to ensure reliability under demanding conditions. Authors in [3] addressed uncertainty in electric vehicle gear train design, applying robust optimization under probabilistic variations. These studies underscore the emphasis on robust MOO strategies for complex gearbox configurations.

In [4], the multi-speed gearbox design was investigated using evolutionary algorithms, and thus setting the stage for numerous applications. Building on this, authors in [5] implemented heuristic algorithms to optimize aeroengine accessory gearboxes, achieving significant weight reduction without compromising the transmission characteristics. Similarly, authors in [6] optimized gear units by simultaneously minimizing the transmission error and improving efficiency, highlighting the benefits of multi-objective techniques in fine-tuning performance metrics.

Among these techniques, NSGA-II remains one of the most widely adopted algorithms. Authors in [7] applied NSGA-II to a two-stage spur gearbox, effectively identifying Pareto-optimal designs. Authors in [8] combined teaching-learning-based optimization with MCDM methods to further enhance the solution quality in spur gear design. Authors in [9] extended the approach to planetary gear systems by integrating dynamic analysis into the optimization framework, thereby ensuring practical applicability. Authors in [10] demonstrated the utility of NSGA-III in high-speed bearing design for aircraft gearboxes, reaffirming the adaptability of evolutionary algorithms in aerospace applications.

Alternative metaheuristic strategies have also shown promise. Authors in [11] employed particle swarm optimization and simulated annealing for gear train weight minimization, while authors in [12] incorporated tribological considerations into the spur gearbox optimization. Authors in [13] conducted a comparative study of evolutionary algorithms for gear design, confirming the robustness of NSGA-II in solving the constrained MOO problems. Authors in [14] applied genetic algorithms to optimize epicyclic gear trains, reinforcing the method's flexibility across gear configurations.

Authors in [15] further validated the effectiveness of NSGA-II in optimizing two-stage helical gear trains, highlighting its capability to resolve the trade-offs between volume and efficiency. Authors in [16] introduced the Extended Additive Mean Ratio (EAMR) method in optimizing two-stage helical gearboxes with dual gear sets in the first stage. Their approach successfully reduced the gearbox volume while maintaining high efficiency. Authors in [17] employed the MARCOS method for a similar design and achieved favorable results in balancing geometric and performance criteria. Authors in [18] combined NSGA-II with a decision-making step to identify the compromise solutions in spur gear design, further supporting the integration of MCDM techniques with evolutionary algorithms.

Despite these valuable contributions, the combination of NSGA-II and EAMR for gearboxes with split input stages remains underexplored. The unique configuration of a split input stage offers improved torque balance and spatial arrangement flexibility but introduces additional design complexities. This study proposes a hybrid optimization framework integrating NSGA-II and EAMR to address this gap, with dual-objective functions focused on minimizing the base area and maximizing the transmission efficiency. This is the first study to apply a combined NSGA-II and MCDM approach to the dual-objective optimization of a two-stage helical gearbox featuring a split input stage. While prior studies, such as [4, 5], employed NSGA-II in gearbox design optimization, their scopes were limited to standard configurations and did not consider the split input stage architecture. Moreover, the objectives in those studies were not focused on achieving optimal performance across a wide range of transmission ratios (u_h), which is a central goal of the present work. In contrast, authors in [1, 7, 9] explored gearbox designs over broad transmission ratio ranges, similarly to this study, but relied solely on traditional techniques, such as Taguchi and Grey Relational Analysis [1, 9], or used only MCDM methods without incorporating evolutionary algorithms [15-17].

By integrating NSGA-II with the EAMR method, the present study not only addresses the structural and performance complexities of the split input stage, but also provides a robust optimization framework that successfully determines the optimal design parameters over a wide transmission ratio range. This enhances the capability for early-stage gearbox configuration and supports more flexible, high-performance design solutions. The outcomes provide practical guidance for the design of space-efficient and high-performance gear transmission systems.

II. OPTIMIZATION PROBLEM FORMULATION

A. *Calculating Gearbox Bottom Area*

As illustrated in Figure 1, for a two-stage helical gearbox with two gears in the input stage, the bottom area A_b is determined by:

$$A_b = (L \times B) \tag{1}$$

where L and B are calculated by:

$$L = d_{w11} + d_{w21}/2 + d_{w12}/2 + d_{w22} + 4\delta \tag{2}$$

$$B = 2b_{w1} + b_{w2} + 7\delta \tag{3}$$

where $\delta = 7 \div 10$ (mm) [18], d_{w1i} and d_{w2i} ($i = 1 \div 2$) are the pitch diameter of the pinion and the gear of stage i, which can be calculated by [19]:

$$d_{w1i} = 2a_{wi}/(u_i + 1) \tag{4}$$

$$d_{w2i} = 2a_{wi} \times u_i/(u_i + 1) \tag{5}$$

where a_{wi} (i=1÷2) is the center distance of stage i. a_{wi} can be calculated by [19]:

$$a_{wi} = k_a \cdot (u_i + 1) \cdot \sqrt[3]{T_{1i} \cdot \frac{k_{H\beta}}{[AS_i]^2 \cdot u_i \cdot X_{bai}}} \tag{6}$$

where X_{bai} is the wheel face width coefficient of stage i_{th}, T_{1i} (i=1÷2) denotes the pinion torque of stage i and it can be determined by:

$$T_{11} = \frac{T_r}{2 \cdot u_{gb} \cdot \eta_{hg}^2 \cdot \eta_{be}^3} \quad (7)$$

$$T_{12} = \frac{T_r}{u_2 \cdot \eta_{hg} \cdot \eta_{be}^2} \quad (8)$$

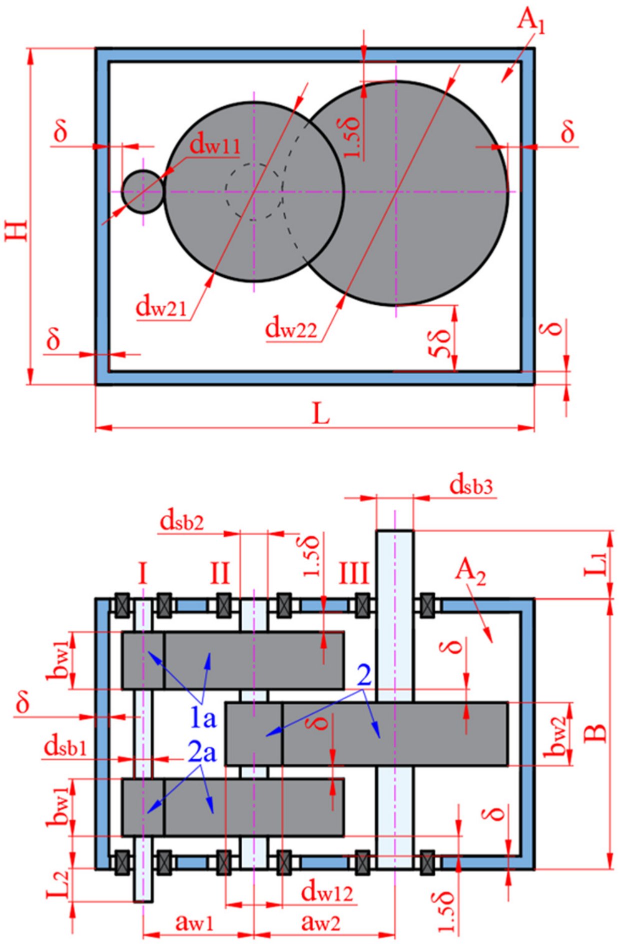

Fig. 1. Scheme for calculation of gearbox bottom area.

B. Calculating Gearbox Efficiency

The gearbox efficiency (%) can be calculated by:

$$\eta_{gb} = 100 - \frac{100 \cdot P_l}{P_{in}} \quad (9)$$

where P_l represents the total gearbox power loss, which is determined by [20]:

$$P_l = P_{lg} + P_{lb} + P_{ls} + P_{Z0} \quad (10)$$

where P_{lg}, P_{lb}, P_{ls}, and P_{Z0} denote the power losses attributed to the gears, bearings, seals, and idle motion, respectively. The computation of these loss components is conducted in accordance with the procedure detailed in [20].

C. Objective Functions

In this study, the optimization problem is formulated as a bi-objective minimization model, focusing on two critical performance metrics in the design of a two-stage helical gearbox featuring a split input stage.

- Minimizing the gearbox bottom area:

$$minf_1(X) = A_c \quad (11)$$

- Maximizing the gearbox efficiency:

$$minf_2(X) = \eta_{gb} \quad (12)$$

The design variable vector X includes the key geometrical and performance-related parameters that characterize the gearbox configuration. Conventionally, five parameters—u_1, Xba_1, Xba_2, A_{s1}, and A_{s2} —are used to define the gearbox geometry [19]. However, prior research [21] has shown that the optimal values of A_{s1}, and A_{s2} commonly approach their upper bounds. Therefore, the three most influential and adjustable parameters—u_1, Xba_1, and Xba_2—are selected as decision variables in the optimization model. The problem formulation is presented by:

$$X = \{u_1, Xba_1, Xba_2\} \quad (13)$$

D. Constrains

For the gearbox design, the gear ratio of each stage u_i is constrained within the interval (1, 9), while the face width coefficient Xba_i is limited to the range (0.25, 0.4) for i = 1, 2 [19]. Accordingly, the MOO problem is formulated under the following set of constraints:

$$1 \le u_i \le 9 \quad (14)$$

$$0.25 \le Xba_i \le 0.4 \quad (15)$$

III. METHODOLOGY

This research employs a hybrid methodological framework that combines the NSGA-II with the EAMR method to address a bi-objective optimization problem in the design of a two-stage helical gearbox with a split input stage. The two optimization objectives are: (1) minimizing the bottom area of the gearbox housing, and (2) maximizing the transmission efficiency.

A. NSGA-II for Generating Pareto-Optimal Solutions

The NSGA-II algorithm is used to solve the MOO problem and generate a set of Pareto-optimal solutions. The algorithm efficiently balances exploration and exploitation by applying genetic operations, such as selection, crossover, and mutation, guided by non-dominated sorting and crowding distance mechanisms.

Each candidate solution (individual) in the population represents a specific gearbox design defined by the decision variable vector X= [u_1, Xba_1, Xba_2], where u_1 is the gear ratio of the first stage, and Xba_1 and Xba_2 are the face width coefficients of the first and second stage, respectively. The search is conducted under the design constraints specified above. NSGA-II was configured with a population size of 100, 200 generations, a crossover probability of 0.9, and a mutation probability of 0.1. After evolution, the final population represents a Pareto front comprising trade-off solutions for both objectives.

B. EAMR for Decision-Making among Pareto Solutions

To identify the most balanced design among the Pareto-optimal solutions obtained from NSGA-II, the EAMR method is employed. This MCDM technique ranks the alternatives by comparing each solution's proximity to the average solution across all criteria.

The procedure includes the following key steps [22]:

- Constructing the decision matrix and computing the average value of each criterion across all decision makers.
- Calculating the normalized values by dividing each criterion's average by its maximum observed value.
- Determining the weighted normalized scores by using the average weights assigned to each criterion.
- Aggregating the scores separately for the objectives to be maximized (e.g., efficiency) and minimized (e.g., base area).
- Computing a final score $S_i = \frac{RV(G_i^+)}{RV(G_i^-)}$, which represents a ranking value based on the aggregated scores.
- The optimal design is selected as the one with the highest S_i value.

This approach allows for a rational, mathematically grounded selection of the most desirable design from a set of Pareto-efficient options.

IV. RESULTS AND DISCUSSIONS

A. Trend of Average Gearbox Performance with Respect to Transmission Ratio

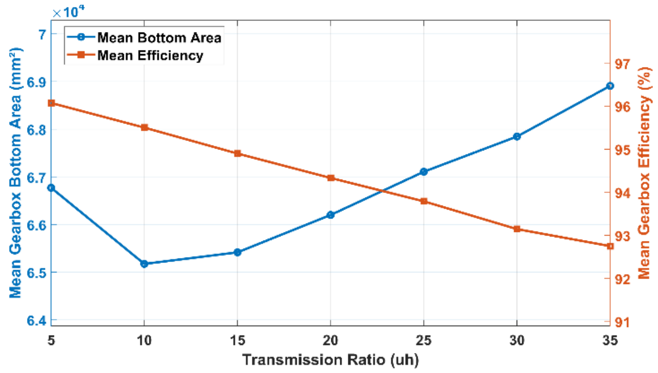

Fig. 2. Variation of mean gearbox bottom area and transmission efficiency with respect to transmission ratio (u_h).

B. Pareto Fronts across Transmission Ratios

Figure 3 presents the Pareto fronts obtained from NSGA-II for each value of u_h. The trend shows that as u_h increases, the Pareto front shifts rightward and downward, indicating a trade-off, where achieving higher transmission ratios typically results in larger gearbox areas and lower efficiencies. For u_h = 5 and 10, the solutions are clustered near the upper-left corner (i.e., small area and high efficiency), which is ideal. However, as u_h approaches 35, this balance is deteriorated.

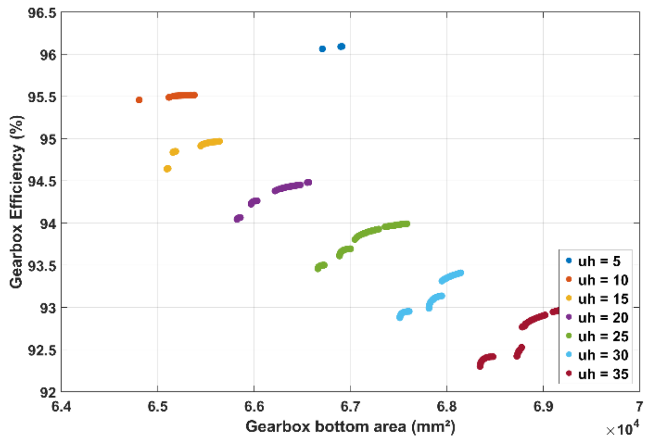

Fig. 3. Pareto fronts of gearbox designs corresponding to different transmission ratios (u_h).

C. Comparison of EAMR-Selected Solutions with Mean-Based Trends

To evaluate the quality of the solutions selected by the EAMR method, Table II compares them with the corresponding mean values of the bottom area and efficiency for each transmission ratio u_h [17]. This comparison provides insights into whether EAMR delivers superior trade-offs relative to the average performance across the Pareto front.

As shown in Table II, the EAMR-selected designs consistently outperform the average in terms of compactness. At all values of u_h, the bottom area of the EAMR solution is lower than the mean bottom area, with reductions ranging from approximately 100 mm^2 to over 1,200 mm². For example, at u_h = 35, the EAMR bottom area is 68,367.81 mm² compared to the mean of 69,538.17 mm²—a meaningful reduction for high-load, space-constrained systems. The efficiency of the EAMR-selected solutions also remains very close to the average, with only marginal differences observed (e.g., a deviation of 0.01% to 0.07% in most cases). This indicates that the EAMR method is able to preserve performance while reducing size, providing designs that are balanced and practically favorable. These findings reinforce the academic value of the EAMR approach in multi-objective engineering design. Rather than selecting extreme solutions that may over-optimize a single objective at the expense of the other, EAMR targets solutions that are proximal to the average ideal. This leads to consistent reductions in the bottom area without incurring a significant losses in efficiency.

In practical gearbox design, such balanced configurations are preferable, as they simultaneously address constraints related to the housing space, weight, and thermal performance while maintaining acceptable mechanical efficiency. As such, the use of EAMR supports robust and implementable optimization decisions in multi-criteria settings.

TABLE I. COMPARISON OF EAMR-SELECTED SOLUTIONS AND AVERAGE PERFORMANCE AT EACH u_h

u_h	EAMR bottom area (mm²)	EAMR efficiency (%)	Mean bottom area (mm²)	Mean efficiency (%)
5	66708.00	96.06	66912.45	96.09
10	64807.40	95.46	65380.30	95.52
15	65159.78	94.84	65640.99	94.97
20	65826.43	94.05	66567.82	94.48
25	66667.23	93.48	67585.30	93.99
30	67526.13	92.92	68143.39	93.41
35	68367.81	92.38	69538.17	93.04

D. Regression Analysis of Stage I Gear Ratio versus Overall Transmission Ratio

To understand the relationship between the gear ratio of the first stag (u_1) and the overall transmission ratio (u_h), a linear regression analysis was performed using two datasets:

- The mean-based values, representing the average gear ratio observed across the Pareto-optimal solutions for each u_h.
- The EAMR-based values, corresponding to the gear ratio selected from the best-compromise solution, as determined by the EAMR method.

As illustrated in Figures 4 and 5, both datasets reveal a strong linear correlation between u_1 and u_h, with coefficient of determination values R^2 exceeding 0.98 in both cases. The fitted regression models are:

- Mean-based regression, as shown in Figure 4:

$$u_1 = 0.1400 \times u_h + 1.3241 \quad (16)$$

$$u_1 = 0.1251 \times u_h + 1.2900 \quad (17)$$

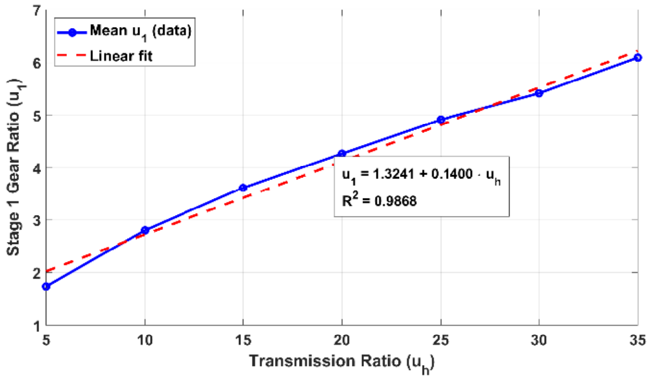

Fig. 4. Linear Regression between first-stage gear ratio (u_1) and overall transmission ratio (u_h) based on the mean values of Pareto-optimal solutions.

The regression model based on the mean values of Pareto-optimal solutions, shows a strong linear relationship to guide the gear ratio distribution. Also, a regression line is fitted to the u_1 values of the EAMR-ranked optimal designs, providing a practical guideline for assigning the transmission load between the stages.

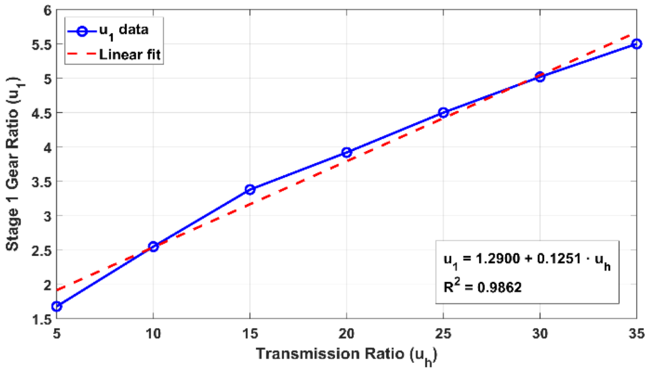

Fig. 5. Linear regression between first-stage gear ratio (u_1) and overall transmission ratio (u_h) based on the EAMR-selected solutions.

E. Benchmarking with MARCOS-Based Optimization Results

To further assess the effectiveness of the proposed NSGA-II and EAMR hybrid approach, a benchmarking analysis was conducted using the results reported in [17]. This study was selected for comparison because it addresses the same optimization problem as the present work: the dual-objective design of a two-stage helical gearbox with a split (dual-gear) input stage, targeting the minimization of gearbox bottom area and the maximization of transmission efficiency. These shared objectives and design configurations make [17] a suitable and relevant benchmark.

In [17], the optimization process was implemented in two stages. First, a single-objective problem was solved to reduce the gap between the upper and lower bounds of the design constraints. Then, the MARCOS method was applied to solve the resulting MOO problem and identify balanced design solutions.

Table II presents a comparison between the optimal solutions obtained in this study and those in [17] for the same transmission ratio u_h values. The results indicate that the NSGA-II/EAMR framework consistently yields better performance in both objective functions:

- The gearbox bottom area A_b was reduced by 3.46% -3.67%, demonstrating improved compactness.
- The gearbox efficiency η_{gb} was improved by 1.03%-2.58%, indicating enhanced transmission performance.

These findings validate the effectiveness of the proposed approach in generating well-balanced and high-quality designs.

TABLE II. COMPARISON OF OPTIMAL RESULTS BETWEEN THIS STUDY AND MARCOS-BASED METHOD [17]

Gearbox bottom area A_b			**Gearbox efficiency η_{gb}**		
Optimal solution	[17]	Difference (%)	Optimal solution	[17]	Difference (%)
6.67	6.91	3.46	96.06	95.07	1.03
6.48	6.72	3.56	95.46	94.17	1.35
6.52	6.75	3.47	94.84	93.05	1.89
6.58	6.83	3.62	94.05	92.16	2.01
6.67	6.92	3.66	93.48	91.34	2.29
6.75	7.01	3.67	92.92	90.53	2.58

V. CONCLUSION

This study proposed a hybrid approach combining the Non-dominated Sorting Genetic Algorithm II (NSGA-II), an evolutionary algorithm, and the Evaluation by an Area-based Method of Ranking (EAMR), a Multi-Criteria Decision-Making method (MCDM) to solve a bi-objective optimization problem for a two-stage helical gearbox with a split input stage. The two conflicting objectives—minimizing the gearbox bottom area and maximizing the transmission efficiency—were simultaneously addressed.

The results showed that NSGA-II effectively generated diverse Pareto-optimal solutions across a wide range of transmission ratios (u_h), revealing clear trade-offs between the design compactness and efficiency. The EAMR method, then, successfully selected well-balanced solutions that outperformed the average ones in terms of compactness while maintaining competitive efficiency.

The regression analysis further revealed a strong linear relationship between the Stage I gear ratio (u_1) and the overall ratio (u_h), offering a practical design rule for distributing the gear ratios between stages. Notably, the EAMR-based regression favored allocating more transmission load to the second stage, potentially improving the load distribution and mechanical durability.

This is the first study to apply a hybrid NSGA-II and MCDM framework to the Multi-Objective Optimization (MOO) of a two-stage gearbox with a split input stage. The proposed approach not only delivers high-quality trade-off solutions, but also successfully determines the optimal design parameters across a wide range of transmission ratios—thereby assisting designers in making early-stage decisions regarding the gearbox configuration.

The integration of NSGA-II and EAMR offers a powerful and practical tool for gearbox design optimization. This approach not only enhances the quality of multi-objective solutions, but also supports rational decision-making in real-world applications that demand both high performance and compact form factor.

ACKNOWLEDGMENT

This work was supported by Thai Nguyen University of Technology (TNUT).

DATA AVAILABILITY

The simulation data supporting the findings of this study are available from the corresponding author.

REFERENCES

[1] M. Rosić, B. Rosić, and M. Sedak, "Multi-Objective Hybrid Sailfish Optimization Algorithm for Planetary Gearbox and Mechanical Engineering Design Optimization Problems," *Computer Modeling in Engineering & Sciences*, vol. 142, no. 2, pp. 2111–2145, 2025, https://doi.org/10.32604/cmes.2025.059319.

[2] A. Kumar, P. Ramkumar, and K. Shankar, "Multi-objective 3-stage wind turbine gearbox (WTG) with tribological constraint," *Mechanics Based Design of Structures and Machines*, vol. 52, no. 8, pp. 5263–5289, Aug. 2024, https://doi.org/10.1080/15397734.2023.2249987.

[3] X. Xu *et al.*, "Optimization Design for the Planetary Gear Train of an Electric Vehicle under Uncertainties," *Actuators*, vol. 11, no. 2, Feb. 2022, Art. no. 49, https://doi.org/10.3390/act11020049.

[4] K. Deb and S. Jain, "Multi-Speed Gearbox Design Using Multi-Objective Evolutionary Algorithms," *Journal of Mechanical Design*, vol. 125, no. 3, pp. 609–619, Sep. 2003, https://doi.org/10.1115/1.1596242.

[5] M. Hu, J. Zhu, L. Gong, Z. Lu, and H. Liu, "Multi-Objective Optimization of an Aeroengine Accessory Gearbox Transmission Based on a Heuristic Algorithm," *Journal of Aerospace Engineering*, vol. 38, no. 2, Mar. 2025, https://doi.org/10.1061/jaeeez.aseng-5155.

[6] E. B. Younes, C. Changenet, J. Bruyère, E. Rigaud, and J. Perret-Liaudet, "Multi-objective optimization of gear unit design to improve efficiency and transmission error," *Mechanism and Machine Theory*, vol. 167, Jan. 2022, Art. no. 104499, https://doi.org/10.1016/j.mechmachtheory.2021.104499.

[7] M. Patil, P. Ramkumar, and S. Krishnapillai, "Multi-Objective Optimization of Two Stage Spur Gearbox Using NSGA-II," in *SAE Technical Paper Series*, 400 Commonwealth Drive, Warrendale, PA, United States, vol. 1, Jul. 2017, Art. no. 1939, https://doi.org/10.4271/2017-28-1939.

[8] E. S. Maputi and R. Arora, "Multi-objective spur gear design using teaching learning-based optimization and decision-making techniques," *Cogent Engineering*, vol. 6, no. 1, Jan. 2019, https://doi.org/10.1080/23311916.2019.1665396.

[9] Z. Qingyong *et al.*, "Multi-objective Optimization Design and Dynamic Characteristic Analysis based on Planetary Gear Transmission," *Journal of Network Intelligence*, vol. 8, no. 3, pp. 639– 657 Aug. 2023.

[10] J. H. Kim, B. S. Kim, D. Im, J. H. Park, D. J. Moon, and Y. J. Park, "Optimization of high-speed angular contact ball bearing for aircraft gearbox utilizing an evolutionary multi-objective algorithm, NSGA-III," *Advances in Mechanical Engineering*, vol. 16, no. 9, Sep. 2024, https://doi.org/10.1177/16878132241272197.

[11] V. Savsani, R. V. Rao, and D. P. Vakharia, "Optimal weight design of a gear train using particle swarm optimization and simulated annealing algorithms," *Mechanism and Machine Theory*, vol. 45, no. 3, pp. 531–541, Mar. 2010, https://doi.org/10.1016/j.mechmachtheory.2009.10.010.

[12] M. Patil, P. Ramkumar, and K. Shankar, "Multi-Objective Optimization of Spur Gearbox with Inclusion of Tribological Aspects," *Journal of Friction and Wear*, vol. 38, no. 6, pp. 430–436, Nov. 2017, https://doi.org/10.3103/s1068366617060101.

[13] M. Mendez, D. A. Rossit, B. Gonzalez, and M. Frutos, "Proposal and Comparative Study of Evolutionary Algorithms for Optimum Design of a Gear System," *IEEE Access*, vol. 8, pp. 3482–3497, 2020, https://doi.org/10.1109/access.2019.2962906.

[14] K. Daoudi, E. M. Boudi, and M. Abdellah, "Genetic Approach for Multiobjective Optimization of Epicyclical Gear Train," *Mathematical Problems in Engineering*, vol. 2019, no. 1, Jan. 2019, https://doi.org/10.1155/2019/9324903.

[15] R. C. Sanghvi, A. S. Vashi, H. P. Patolia, and R. G. Jivani, "Multi-Objective Optimization of Two-Stage Helical Gear Train Using NSGA-II," *Journal of Optimization*, vol. 2014, pp. 1–8, 2014, https://doi.org/10.1155/2014/670297.

[16] T. Q. Hung, V. D. Binh, D. V. Thanh, L. A. Tung, and N. K. Tuan, "Multi-Objective Optimization of a Two-Stage Helical Gearbox with Two Gear Sets in First Stage to Reduce Volume and Enhance Efficiency using the EAMR Technique," *Engineering, Technology & Applied Science Research*, vol. 15, no. 1, pp. 19288–19294, Feb. 2025, https://doi.org/10.48084/etasr.9224.

[17] L. D. Bao, V. D. Binh, D. V. Thanh, K. M. Nguyen, and L. X. Hung, "Multi-Objective Optimization of a Two-stage Helical Gearbox with Double Gears in the First Stage using MARCOS," *Engineering, Technology & Applied Science Research*, vol. 14, no. 6, pp. 18245–18251, Dec. 2024, https://doi.org/10.48084/etasr.8865.

[18] Q. Yao, "Multi-objective optimization design of spur gear based on NSGA-II and decision making," *Advances in Mechanical Engineering*, vol. 11, no. 3, Mar. 2019, https://doi.org/10.1177/1687814018824936.

[19] T. Chat and L. Van Uyen, *Design and Calculation of Mechanical Transmissions Systems*, 1st ed. Hanoi, Vietnam: Educational Publishing House, 2007.

[20] D. Jelaska, *Gears and Gear Drives*, 1st ed. New York, New York State: Wiley, 2012.

[21] X. H. Le and N. P. Vu, "Multi-Objective Optimization of a Two-Stage Helical Gearbox Using Taguchi Method and Grey Relational Analysis," *Applied Sciences*, vol. 13, no. 13, Jun. 2023, Art. no. 7601, https://doi.org/10.3390/app13137601.

[22] M. Keshavarz-Ghorabaee, E. Zavadskas, M. Amiri, and J. Antucheviciene, "Evaluation by an Area-based Method of Ranking Interval Type-2 Fuzzy Sets (EAMRIT-2F) for Multi-criteria Group Decision-making," *Transformations in Business and Economics*, vol. 15, no. 3 (39), pp.76–95, Dec. 2016.