

# Predictive Modeling and Experimental Validation of the Penetration Depth in Abrasive Water Jet Machining

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## ABSTRACT

This paper thoroughly examines the development of a prediction model for key process parameters, along with its evaluation through rigorous experimentation, to enhance the cutting performance of Abrasive Water Jet (AWJ) machining. Numerous analytical and predictive models for forecasting penetration depth in AWJ machining have been reported by various researchers. In this work, a dimensional analysis approach is employed to develop a predictive model for penetration depth that incorporates the kerf wall drag coefficient and applies the boundary layer theorem. The study experimentally investigates several ferrous and non-ferrous workpiece materials, such as EN 8 steel and alumina ceramic. Experiments on two distinct target materials utilizing two different AWJ machine setups show good agreement with the proposed model. The industrial application of these models will increase understanding of the complex AWJ machining process and reduce the need for costly experiments in the future.

**Keywords**-EN 8 steel; Abrasive Water Jet (AWJ) cutting; Material Removal Rate (MRR); penetration depth; process innovation

## I. INTRODUCTION

Finnie's erosion model is referenced to develop a model for predicting the total penetration depth resulting from the deformation and cutting of only ductile materials. However, this erosion model is limited because it does not consider the effects of abrasive particle shape and size [1]. By expanding the Finnie model, researchers have developed an enhanced erosion model that can forecast penetration depth and incorporate the effects of abrasive particle size and shape [2]. A mathematical model based on indentation hardness theory was also proposed for the impingement erosion process, which is effective for relatively low impact speeds [3]. The moving particle forces the excised material to flow out along the cavity's walls until it is under enough strain to break off. It is assumed that the amount of material removed is correlated with the particle's penetration depth [4].

Regression analysis and Buckingham's  $\pi$  theorem have been used to examine how the Abrasive Water Jet (AWJ) machining process parameters affect penetration depth [5]. Compared with the regression model, the semi-empirical model showed a higher correlation for accurately estimating penetration depth [6]. A model for efficiently selecting AWJ cutting parameters for the required depth of cut in a given material was developed using fuzzy set theory [7]. As a result, determining the magnitudes of the most influential AWJ parameters on the depth of cut requires less experimentation. In the absence of a mathematical model for the process, the fuzzy logic methodology is particularly useful [8].

In the machining of artificial rock-like materials using AWJ cutting, the effects of several material parameters, including absorbed fracture energy, compressive strength, crack velocity, and Young's modulus, as well as test parameters like traverse rate, applied pump pressure, and abrasive mass flow rate, have been studied by the authors in [9]. The authors discovered that there are two sets of important parameters: the first set contains the lowest threshold values that must be exceeded to initiate the material destruction process, whereas the second set must not be exceeded to achieve an efficient cutting process. Additionally, it was discovered that the materials' fracture velocities had a considerable impact on the cutting process's threshold parameters, depth of cut, and specific energy [10].

In the present study, a prediction model was developed using a dimensional analysis approach to incorporate the kerf

wall drag coefficient and predict penetration depth, applying the boundary layer theorem [11]. Furthermore, a comparative evaluation of the predictive model was performed using AWJ machining experimental data for two distinct target materials, namely EN 8 steel and alumina ceramic.

## II. MATHEMATICAL MODELING

Accounting for each of the above-listed elements is challenging, and attempting to do so may result in a final equation with numerous hidden parameters, rendering the model too complex for practical use [12]. According to a thorough investigation of the kerf production process [13], particle velocity at any given time is determined by the penetration depth. The frictional drag force  $F_D$  of the kerf wall and jet broadening are responsible for the velocity's dependency on cut depth. While jet broadening contributes to this dependency, the frictional drag of the kerf wall is expressed as:

$$F_D = C_D A_W \frac{\rho V_i^2}{2} \quad (1)$$

According to Finnie's model, as shown in (2), the amount of material removed by a single particle depends on:

$$\delta V = f(\varepsilon_\omega, \sigma_f, V, m, \alpha) \quad (2)$$

Here,  $\alpha$  is the impact angle of the abrasive particle. As the water jet enters the material, the surface curvature changes, and as the particles exit the nozzle tip and strike the workpiece surface, their direction also changes [14]. Representing accurately the exact impact angle for each particle is therefore challenging. For simplification, we assume an average particle impact angle from the top to the bottom of the kerf for the mathematical model.

The average particle impact angle can be defined as a function of multiple variables. Substituting the value of  $\alpha$  into (2), we obtain:

$$\delta V = \varepsilon_\omega \frac{B_2 m V^2}{\sigma_f} \left(\frac{d}{h_s}\right)^a \left(\frac{p}{\sigma_f}\right)^b \quad (3)$$

where  $B_2$ ,  $a$ , and  $b$  are parameters expressed as:  $B_2 = AB_1^j$ ,  $a = a_1 j$ ,  $b = b_1 j$ .

The total material rate is the product of the material removed during a specific period by each abrasive particle. Careful examination of the nozzle's exit in AWJ machining

reveals that some abrasive particles in the water jet do not actively participate in the cutting process, even though they possess enough energy to cut through the workpiece material. Abrasive particles may collide with each other and fail to participate in cutting unless they are on the nozzle's periphery. Based on this, the following equation can be developed:

$$h = \beta_3 \varepsilon_\omega \frac{m_t P}{u \sigma_f d_n \rho_\omega} \left(\frac{d}{h_s}\right)^a \left(\frac{P}{\sigma_f}\right)^b \quad (4)$$

where  $\varepsilon_\omega$  is the kerf wall drag coefficient;  $\sigma_f$  is the flow stress of the work material,  $V$  is the abrasive particle velocity,  $m$  is the average mass of the particle,  $\alpha$  is the impact angle,  $u$  is the nozzle traverse speed,  $h$  is the depth of cut,  $b$  is the average kerf width,  $h_s$  is the standoff distance,  $d$  is the nozzle diameter;  $P$  is the water pressure,  $m_t$  is the total abrasive mass flow rate, and  $\beta_3 = 2\eta_e \beta_2 \eta_m^2 x^2$  is a constant that can be determined experimentally or through regression analysis.

### III. EXPERIMENTAL STUDY

Various experimental sets were conducted to investigate the impact of the suggested process factors. The trials were performed with specific parameter settings, such as abrasive mass flow rate, water pressure, and standoff distance, analyzing their effects on the depth of penetration. Table I indicates that the water pressure was controlled between 275 and 380 MPa in increments of 35 MPa, the abrasive mass flow rate was maintained between 0.0068 and 0.0135 kg/s in increments of 0.0023 kg/s, and the nozzle traverse speed was varied between 0.66 and 1.66 mm/s in increments of 0.33 mm/s. Every machining process was completed with a single cutting pass. To reduce inaccuracy, at least three measurements were made. To determine the range and values of the process parameters, a literature survey, a few preliminary experiments, and machine limitations were employed. Taguchi's Design of Experiments technique was used to determine the number of experiments for each of the four components and four levels of process parameters. A total of sixteen statically planned experiments were conducted for the range of process parameters listed in Table I. The nozzle position was kept at 90° for every test.

TABLE I. TAGUCHI DESIGN OF EXPERIMENTS

Observation	Pressure (Mpa)	Mass of abrasive (kg/s)	Traverse speed (mm/s)	Standoff distance (mm)
1	275	0.0068	0.66	2
2	310	0.0068	0.99	3
3	345	0.0068	1.33	4
4	380	0.0068	1.66	5
5	310	0.0091	1.33	2
6	275	0.0091	1.66	3
7	380	0.0091	0.66	4
8	345	0.0091	0.99	5
9	345	0.0113	1.66	2
10	380	0.0113	1.33	3
11	275	0.0113	0.99	4
12	310	0.0113	0.66	5
13	380	0.0131	0.99	2
14	345	0.0131	0.66	3
15	310	0.0131	1.66	4
16	275	0.0131	1.33	5

This study focuses on EN 8 steel and alumina ceramics because of their significant characteristics. EN 8 steel is widely used in many engineering applications, including in the automotive industry. The nozzle was routinely evaluated during the research and replaced if it showed signs of wear. Table I lists the water pressure, mass of the abrasive, traverse speed, and standoff distance for the chosen experiment combinations. EN 8 steel is typically used when superior qualities are needed over those of mild steel. Alumina ceramics are becoming increasingly popular in the electrical, electronics, and biomedical fields due to their natural strength, hardness, and wear resistance, as well as their well-established electrical, optical, and magnetic properties. The extensive use of alumina ceramics, as well as the need for rapid prototyping and precise dimensional control, often necessitates post-sintering machining. In the present study, a CNC AWJ machine (make: FLOW; model: MACH 200) was employed to cut alumina ceramics and EN 8 steel under various process parameters. Figure 1 illustrates the AWJ machining process.



Fig. 1. AWJ machining process.

Consequently, in the current work, a comprehensive experimental analysis was conducted to assess the depth of penetration and Material Removal Rate (MRR) in AWJ machining. The experiments were performed over a range of process parameters, including water pressure, standoff distance, and traverse speed, with the objective of cutting the target materials.

IV. RESULTS AND DISCUSSION

The cutting-edge software Design-Expert 13 was used to perform an Analysis of Variance (ANOVA). The results indicated that a linear model provides the best fit, as illustrated in Figure 2. The pressure and mass of the abrasive were identified as significant parameters, with p-values below 0.005.

ANOVA for Linear model

Response 1: Depth of Penetration

Source	Sum of Squares	df	Mean Square	F-value	p-value	
<b>Model</b>	5.64	4	1.41	5.90	0.0087	significant
A-Mass of Abrasive	0.1161	1	0.1161	0.4864	0.5000	
B-Standoff Distance	0.0249	1	0.0249	0.1041	0.7530	
C-Pressure	1.30	1	1.30	5.44	0.0398	
D-Traversal Speed	4.20	1	4.20	17.58	0.0015	
<b>Residual</b>	2.63	11	0.2388			
<b>Cor Total</b>	8.26	15				

Fig. 2. ANOVA results highlighting key parameters.

Figure 3 compares the predicted and experimental penetration depths, where the nearly symmetrical residual distribution provides strong validation for the model. The results show that the penetration depth increases with abrasive mass flow rate, decreases at shorter standoff distances, and increases at longer ones.

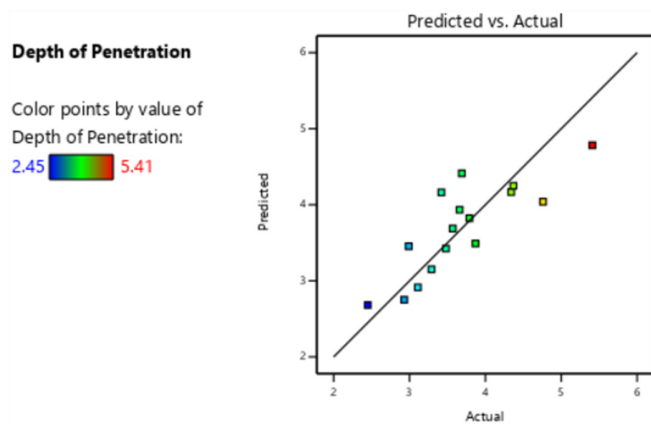


Fig. 3. Comparison of predicted and experimental penetration depths.

Taguchi analysis was performed using Minitab for penetration depth, where a greater depth is preferred. Figure 4 presents the main effects plot for the Signal-to-Noise (SN) ratio, indicating that the highest penetration depth (5.41 mm) was achieved with a water pressure of 380 MPa, abrasive mass flow rate of 0.091 kg/s, traverse speed of 0.66 mm/s, and standoff distance of 4 mm. After the Taguchi analysis, an Artificial Neural Network (ANN) model was developed using MATLAB R2016a to further predict penetration depth. The network architecture consisted of 10 hidden neurons, two input parameters, and one output parameter. Twelve samples (about 70%) were randomly distributed by the software for training, two samples (15%) were used for validation, and two samples

(15%) were used for testing. The training algorithm used was Bayesian regularization.

Figure 5 shows the regression plots from the neural network analysis. The coefficient of determination (R = 0.8235) shows the strong agreement between predicted and experimental values, confirming the reliability of the ANN model. The validated analytical model for EN 8 steel was then utilized to calculate the penetration depth using (4) as the reference equation:

$$h_d = \beta_3 \frac{\epsilon_\omega m_t P}{u \sigma_f d_n \rho_\omega} \left(\frac{d}{h_s}\right)^{c_2} \left(\frac{p}{\sigma_f}\right)^{d_2} \quad (5)$$

Using (5), the calculated penetration depth was 2.92 mm, and similar computations were performed for other test conditions. A comparison of the calculated and experimental values is presented in Table II, showing close agreement between the two.

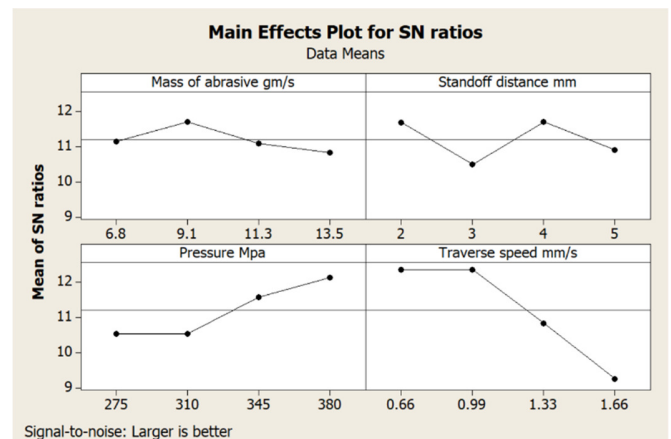


Fig. 4. Main effects plots for SN ratio.

TABLE II. COMPARISON OF CALCULATED AND EXPERIMENTAL PENETRATION DEPTHS

Observation	Penetration depths	
	Calculated (mm)	Experimental (mm)
1	3.84	4.34
2	2.70	3.66
3	2.12	3.57
4	1.78	2.99
5	2.75	3.48
6	2.00	2.45
7	6.07	5.41
8	3.77	4.76
9	2.92	3.29
10	3.79	3.79
11	4.11	3.87
12	6.57	3.42
13	6.21	4.37
14	8.60	3.69
15	3.16	2.93
16	3.62	3.11

Figure 6 illustrates the comparison of the experimental and analytical penetration depths for different process parameters,

such as pressure, abrasive mass flow rate, traverse speed, and standoff distance. The figure shows that the penetration depth increases with higher pressure and abrasive mass flow rate but decreases with larger standoff distances due to the increase in

the nozzle's cone angle. Consequently, the target substance cannot be removed by an AWJ. Overall, the graph shows that the analytical and experimental penetration depths are in line.

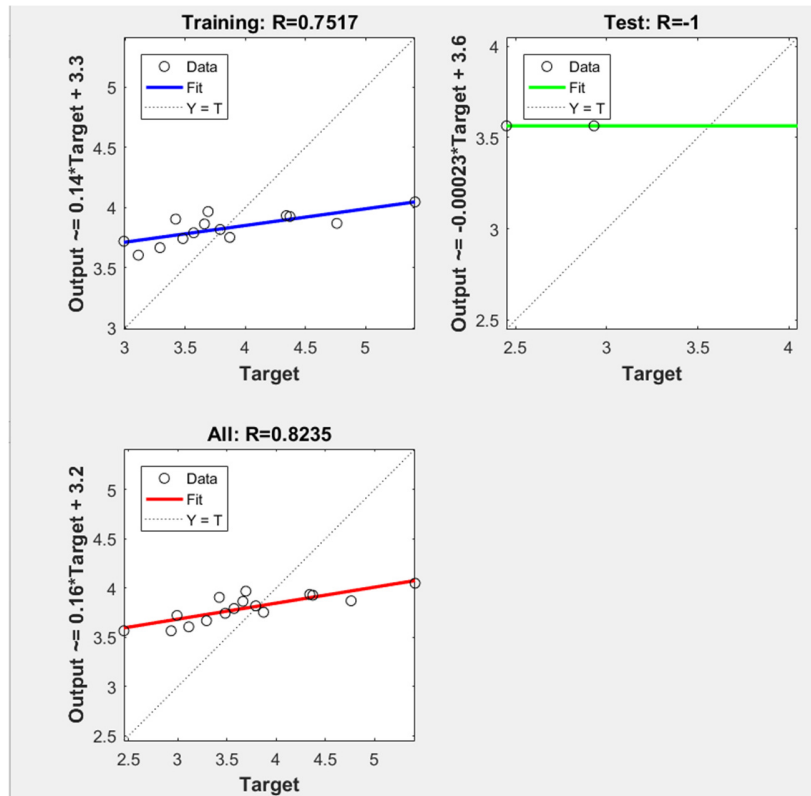


Fig. 5. Regression analysis of penetration depth prediction by ANN.

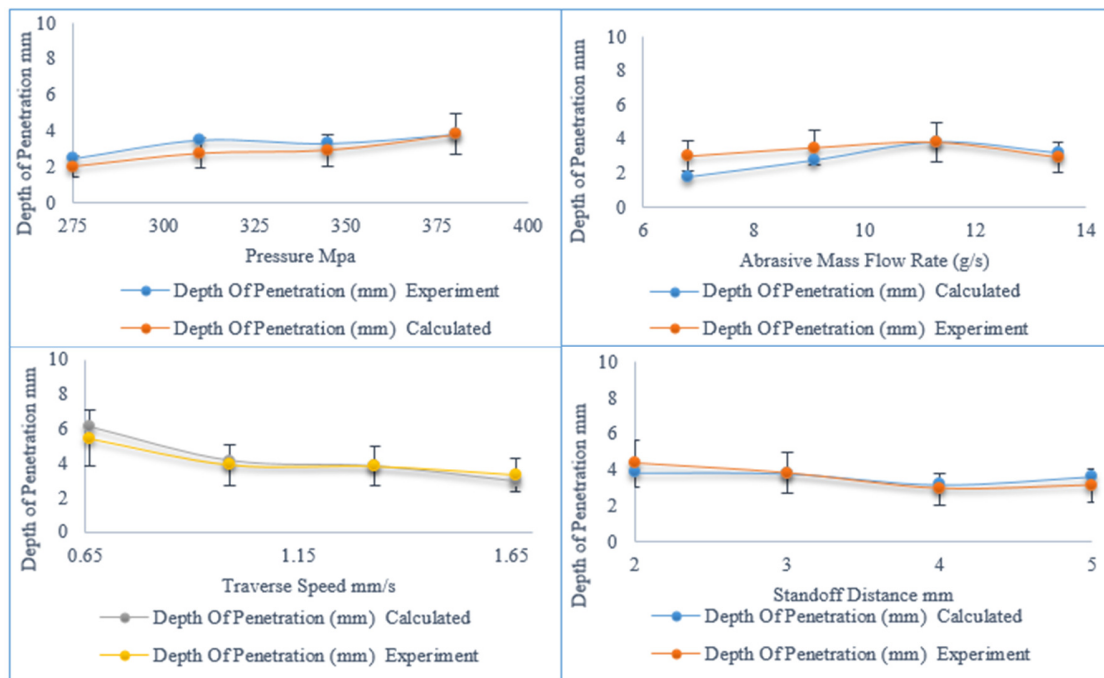


Fig. 6. Comparison of predicted and experimental penetration depths.

## V. CONCLUSION

The present study examined the complex interactions between process parameters in the Abrasive Water Jet (AWJ) machining process. These parameters include pressure, traverse speed, standoff distance, and abrasive mass flow rate. A comprehensive understanding of the impact of each parameter on the outcome of the machining process was achieved by employing a predictive dimensional model for the depth of penetration. The study revealed that the depth of penetration in AWJ machining is significantly influenced by variations in pressure, abrasive mass flow rate, traverse speed, and standoff distance:

- It has been demonstrated that alterations in pressure significantly influence machining efficiency, with higher pressures generally resulting in greater depth of penetration.
- In a similar manner, the depth of penetration is primarily determined by the abrasive mass flow rate, thereby underscoring the significance of precise machining.
- Traverse speed was identified as a significant parameter influencing the total machining process efficiency. The study demonstrated the careful balancing required to achieve optimal results, as excessive traverse rates can compromise cutting quality, whereas inadequate speeds can negatively impact the output.
- It was shown that standoff distance also had a significant influence on the machining outcomes. The study emphasizes the critical importance of selecting the standoff distance with precision to maintain a balance between Material Removal Rate (MRR) and jet stability.
- The outputs of the prediction model demonstrate a high degree of alignment with experimental results, indicating the model's accuracy and reliability. This agreement between expected and observed results shows how well the model captures the complex interactions between process parameters in AWJ machining. The predictive model's potential as a useful tool for streamlining and improving the machining process is highlighted by the effective matching of results. It offers engineers and practitioners a predictive framework to assist them in achieving their objectives.

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