Reliability Evaluation of the Impact of Pile Positioning Deviation on Pile Cap Punching Shear Resistance: A Case Study from Vietnam

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ABSTRACT

The deviations of the pile center coordinates (*Δx*, *Δy*) from the designed position during construction are a common issue affecting the structural safety. Although Vietnamese Standards (VNS) permit such deviations, their impact on the reliability of the pile caps is not quantitatively evaluated in the current design calculations. This study presents a quantitative reliability assessment of the punching shear resistance of a pile cap designed in accordance with VNS, based on a case study utilizing statistical data (mean μ, standard deviation σ) from 510 field measurements in a project in Vietnam. The reliability theory is employed to establish the limit state functions for punching shear cases (due to column and edge piles) and one-way shear (on inclined sections), with coordinate deviations (*Δx*, *Δy*) modeled as random variables. The reliability index (β) and probability of failure (P_f) for the load cases are determined using the First-Order Reliability Method (FORM) and verified by Monte Carlo Simulation (MCS). The analysis evaluates whether the pile cap design, when accounting for construction deviations, achieves the target reliability level referenced in Eurocode. The results clarify the actual safety level, providing a basis for recommendations to improve the design procedures and quality control for the pile foundation construction in Vietnam.

Keywords-pile position deviation; punching shear resistance; pile cap; reliability; Vietnam standards (VNS); reliability theory; FORM; Monte Carlo Simulation (MCS)

I. INTRODUCTION

The pile foundations are widely used as an effective solution, particularly in projects involving heavy structural loads or soft ground conditions. However, during the pile construction (including bored piles, pressed piles, and driven piles), the deviations in the actual pile center position (*Δx*, *Δy*) from the design coordinates are common and often unavoidable. Such deviations can alter the distribution of the internal forces in the pile cap, increase the local stresses, and potentially reduce its punching resistance, which can affect the progress, costs, and overall quality of the project.

The calculation methods outlined in [1], particularly the punching resistance check, mainly depend on the deterministic method. This approach assumes that the pile position is accurate according to the design and has not yet quantitatively evaluated the safety level, considering the inherent uncertainty and randomness in the pile position deviations during construction. Although the allowable deviation limits in pile construction are specified in [2], neglecting this uncertainty can pose potential risks to the punching resistance. This may occur even when the design complies with the nominal deviation limits, especially given the significant deviation data recorded during construction.

The pile cap behavior has been examined, with a focus on the effects of the positional deviations and the use of the reliability theory. Authors in [3] investigated how the eccentric loads—resulting from pile position deviations—impact the pile cap resistance through experimental studies and comparisons with design standards. Authors in [4] developed the Softened Strut-and-Tie Model to assess the shear strength of reinforced concrete pile caps, considering the force balance, deformation compatibility, and the stress-strain behavior of cracked concrete. Authors in [5] introduced a methodology for analyzing pile caps designed to correct pile placement. Authors in [6] investigated the impact of the eccentricity and pile damage from the construction deviations on the pile cap, proposing to restore the system's center of gravity by increasing the number of piles and repositioning replacement piles. Authors in [7, 8] determined the optimal number and placement of replacement piles to achieve pile cap safety given the damage within the pile group. It was stressed that the overall eccentricity of the pile group—not the individual pile eccentricity—is the key factor affected by the pile-driving process, and that a fixed limit of 75 mm can lead to unsafe conditions.

Numerous Vietnamese authors have also researched pile foundations to support the rational design of piles and rafts. For example, authors in [9] utilized 3D finite element analysis to investigate the influence of raft thickness and pile configurations on the behavior of piled raft foundations, while authors in [10] focused on the rational design of the raft thickness, specifically for heavily loaded skyscrapers. Further research in Vietnam has focused on the rational pile design. Authors in [11] investigated the pile design considering the downdrag effect and proposed a modification method [12]. The use of computer-based programs to aid in this rational design process was also explored in [13]. The application of reliability methods has also been a key research focus. For instance, authors in [14] conducted a reliability analysis of pile foundations situated on soft soil grounds. Authors in [15] applied MCS in conjunction with the Modified Unified Method for the pile design. Authors in [16] investigated random factors to improve the design method for low-pile cap foundations based on reliability criteria. Authors in [17] proposed a detailed process for calculating the pile cap puncture based on [2]. Authors in [18] analyzed the behavior of piles within the cap, considering the deviations in pile axis position during construction.

However, there is still a lack of a quantitative assessment of the punching and shear failure reliability of the pile caps designed strictly according to [1], particularly when subjected to the statistical uncertainty of actual pile coordinate deviations observed in Vietnamese construction practice. The current design procedures rely on deterministic checks and prescribed deviation limits [2], but they do not provide insights into the actual probability of failure or the reliability index. This oversight means that a design deemed "safe" by the standard may not meet the target reliability levels expected for critical structural components. Therefore, the primary goal and novelty of this study are to fill this gap by developing a procedure and conducting a quantitative assessment of the reliability of a pile cap designed according to TCVN 5574:2018 [1]. This is achieved by using a large dataset of 510 actual pile deviation measurements from a construction site in Vietnam as the primary random variables in a reliability analysis framework based on FORM and MCS [19].

The research results are expected to establish a scientific basis for evaluating the impacts of the construction deviations in design and improving the reliability of the pile foundation structures.

II. MATERIALS AND METHODS

A. Behavior of Pile Caps Against Punching

The punching shear behavior of an example pile cap is illustrated in Figure 1. The punching shear resistance of a pile cap is calculated according to TCVN 5574:2018 [1]. For a pile cap with four or more piles, the condition is expressed as:

$$F_{per} \le \frac{R_{bt}h_0}{\alpha} + \sum_{i=1}^{m} u_i \frac{h_0}{c_i} \qquad (1)$$

where F_{per} is the calculated punching force, equal to the sum of the reactions of the spiles outside the bottom range of the punching shear zone, R_{bt} is the design tensile strength of concrete, h_0 is the effective depth of the pile cap, u_i is the average value of the upper and lower bottom edges of the i^{th} side of the punching shear zone (the punching shear zone can have multiple sides, not always four), c_i is the distance from the edge of the column to the side surface of the pile outside the punching shear zone, and α is the coefficient monolithic pile cap.

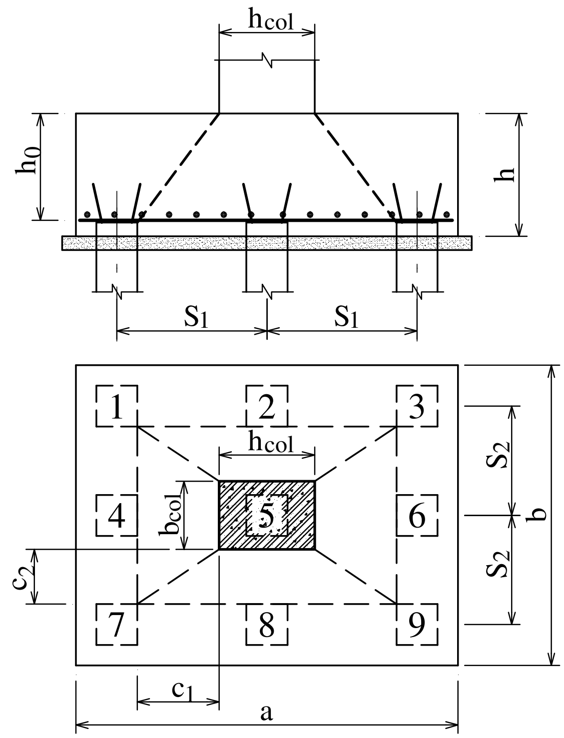

Fig. 1. Punching behavior of an example pile cap, showing the development of the punching failure zone beneath a reinforced concrete column.

The punching shear capacity of a pile subjected to a central load with a rectangular column can be expressed as:

$$F_{per} \le \frac{2R_{bt}h_0}{\alpha}\left[\frac{h_0}{c_1}(b_{col}+c_2) + \frac{h_0}{c_2}(h_{col}+c_1)\right] \qquad (2)$$

where b_{col} and h_{col} Are the width and height of the column section, respectively, c_1 and c_2 are the distances from the edge of the column (in two column cross-section directions) to the inner edge of the nearest column row outside the bottom of the punching shear zone, respectively. Also, $F_{per} = 2\sum F_i$, where $\sum F_i$ is the sum of the reactions of all piles located on one side of the column axis in the higher stressed part minus the reactions of the piles located within the punching shear zone on the same side of the column axis, and, $1 \le h_0/c_i \le 2.5$.

Similarly, the punching shear calculation for the foundation caused by the boundary pile is carried out using:

$$F_{ai} \le R_{bt}h_{01}\left[\beta_1\left(b_{02}+\frac{c_{02}}{2}\right)+\beta_2\left(b_{01}+\frac{c_{01}}{2}\right)\right] \qquad (3)$$

where F_{ai} is the reaction force of the edge pile, b_{01} and b_{02} are the distances from the inner edges of the boundary piles to the outer edges of the pile cap, c_{01} and c_{02} are the distances from the inner edges of the boundary piles to the nearest column edge in two directions, and β_1 and β_1 are values taken from Table I [17].

The strength calculation for the one-way shear across an inclined section is performed according to:

$$Q \leq 1.5bh_0R_{bt}\frac{h_0}{c} \quad (4)$$

where $Q = \sum F_i$ is the total shear force from the piles outside the considered section, b is the width of the pile cap base, and c is the projection length of the inclined section, as illustrated in Figure 1.

B. Reliability Analysis

In reliability analysis, the primary uncertainty considered is the deviation of the actual pile positions, using field measurement data. Following the reliability theory, the structural safety is described by a limit state function, which is established based on VNS [1] design conditions as:

$$g(X) = R(X) - S(X) \quad (5)$$

where R is the resistance, S is the load effect, and X is a vector of basic random variables.

The failure probability (P_f) of the structure is defined as the probability of the failure state:

$$P_f = P[g(X) \leq 0] \quad (6)$$

The reliability index (β) is another measure of safety, usually related to P_f through the normal distribution function Φ:

$$\beta = -\Phi^{-1}(P_f)$$

For this study, the primary random variables in the vector X are the pile position deviations (Δx, Δy) from their design coordinates. Their statistical characteristics (mean μ, standard deviation σ) are determined from 510 field measurements. The deviations are assumed to follow a normal distribution, a reasonable assumption given the symmetrical, bell-shaped nature of the data histograms shown in Figure 3. Other parameters, such as the material strengths and applied loads, are treated as deterministic design values. This simplification, necessary due to the limited availability of local statistical data, means that the calculated reliability index should be interpreted as an upper-bound estimate.

C. Limit State Function

Based on the VNS [1], three limit state functions were defined to model the primary failure modes: the punching shear caused by the column load (G_1), the punching shear caused by an edge pile reaction (G_2), and the one-way shear across the pile cap (G_3):

$$G_1(X) = R_1(X) - S_1(X) = \frac{2R_{bt}h_0}{\alpha}\begin{bmatrix}\frac{h_0}{c_1(X)}\left(b_{col} + c_2(X)\right) \\ + \frac{h_0}{c_2(X)}\left(h_{col} + c_1(X)\right)\end{bmatrix} - F_{per}(X) \quad (7)$$

$$G_2(X) = R_2(X) - S_2(X) = R_{bt}h_{01}\begin{bmatrix}\beta_1(X)\left(b_{02}(X) + \frac{c_{02}(X)}{2}\right) \\ +\beta_2(X)\left(b_{01}(X) + \frac{c_{01}(X)}{2}\right)\end{bmatrix} - F_{ai}(X) \quad (8)$$

$$G_3(X) = R_3(X) - S_3(X) = 1.5bh_0R_{bt}\frac{h_0}{c(X)} - Q(X) \quad (9)$$

In these limit state functions, both the resistance $R_j(X)$ and the load effect $S_j(X)$ are functions of the pile coordinate deviation vector X. This is because all their constituent parameters—including the geometric distances (c_1, c_2, b_{01}, b_{02}, c_{01}, and c_{02}) and calculated forces (F_{per}, Q, and F_{ai})—are altered by the pile misplacement. Therefore, each step in the reliability analysis requires a re-calculation of the pile reactions (P_i) and the corresponding geometry to accurately define the resistance and load effect for that specific deviation scenario.

D. Reliability Analysis Method

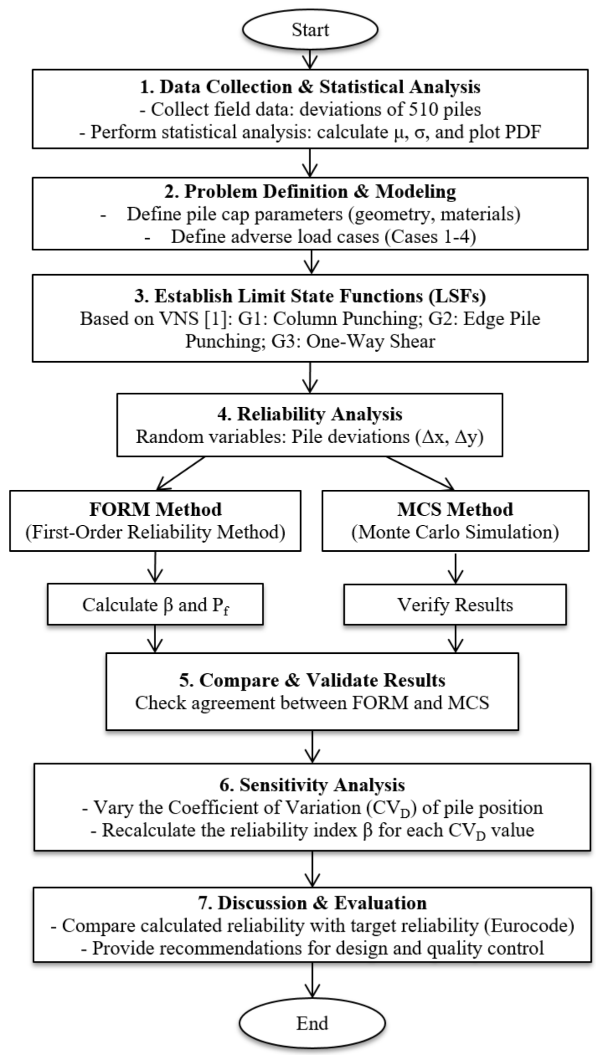

Fig. 2. Flowchart of the proposed research methodology.

The reliability index (β) and failure probability were primarily determined using FORM, an efficient analytical approach that identifies the most probable failure point. To verify the accuracy of the FORM results for this application, a rigorous validation was performed using MCS. Figure 2 displays the methodology of the study.

E. Validation Method

To validate the accuracy of the results obtained from the FORM, an independent MCS analysis was conducted. MCS is considered a benchmark for verification in reliability analysis due to its ability to handle potentially nonlinear limit state functions without requiring analytical simplifications. A large sample size of 10^6 simulations was used to ensure the convergence and stability of the results. A detailed comparison between the outcomes of the FORM and MCS methods, including the reliability index (β) and probability of failure (P_f), is presented in Table IV and Figure 4. The excellent agreement between the two methods confirms the suitability of FORM for this study.

III. EXAMPLE PROBLEM DESCRIPTION

A. Example

Let us consider a rectangular pile cap (labeled PC1) supporting a rectangular column subjected to eccentric loading. The pile cap consists of 9 piles arranged in a 3 × 3 grid, as depicted in Figure 1. Its dimensions are a = 2.8 m, b = 2.4 m, and h = 0.9 m. The column cross-section measures b_{col} = 0.5 m and h_{col} = 0.8 m. The piles are reinforced concrete with square sections of 0.3 m × 0.3 m, constructed from B25 concrete, each with a design bearing capacity of P_{tk} = 970 kN.

To model the uncertainty of the pile location in reliability analysis, the study used measured data of the pile center coordinates from 510 actual piles in a construction project. The statistical analysis of the deviation data compared to the design coordinates is presented using the histograms in Figure 3 and the descriptive statistics in Table I. Table I summarizes the key statistical characteristics of the 510-sample dataset, including the mean, standard deviation, and range of observed deviations. Based on the histogram of the measured data, the deviations of the variables are assumed to follow a normal distribution in the reliability analysis model. The four different load cases acting on the top of the pile cap, as listed in Table II, represent adverse load combinations that could occur during the project's operation.

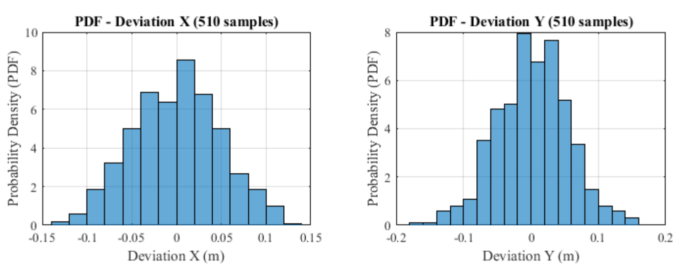

Fig. 3. Pile coordinate deviation distribution chart in X and Y directions.

The deterministic method is used to check the punching shear resistance and shear resistance of the pile cap according to VNS [1], assuming that the pile is in the correct design position, and it is performed for 04 load cases. The results, presented in Table III, indicate that the pile cap design generally meets the resistance conditions specified in the standard.

TABLE I. DESCRIPTIVE STATISTICS OF PILE POSITION DEVIATION (510 SAMPLES)

Statistical parameter	Deviation in X-direction Δx, (m)	Deviation in Y-direction Δy, (m)
Mean μ_{obs}	0.0012	-0.0036
Standard deviation σ_{obs}	0.0510	0.0542
Minimum value	-0.1398	-0.2031
Maximum value	0.1784	0.1539

TABLE II. ANALYZED LOAD CASES

Load cases	N (kN)	M_x (kNm)	M_y (kNm)	$P1_{max}$ (kN)	$\frac{P_1}{P_{tk}}$
Case 1	5,660.0	270	-970	825.9	0.95
Case 2	5,350.0	260	-950	786.5	0.9
Case 3	5,100.0	250	-850	741.8	0.85
Case 4	4,845.0	228	-795	701.1	0.8

$P1_{max}$ is the maximum pressure on the pile head. $\frac{P_1}{P_{tk}}$Is the ratio showing the reserve level of pile bearing capacity?

B. Reliability Analysis and Results

The reliability analysis results, obtained using FORM and verified with MCS ($N = 10^6$), are summarized for all load cases and limit states in Table IV and Figure 4.

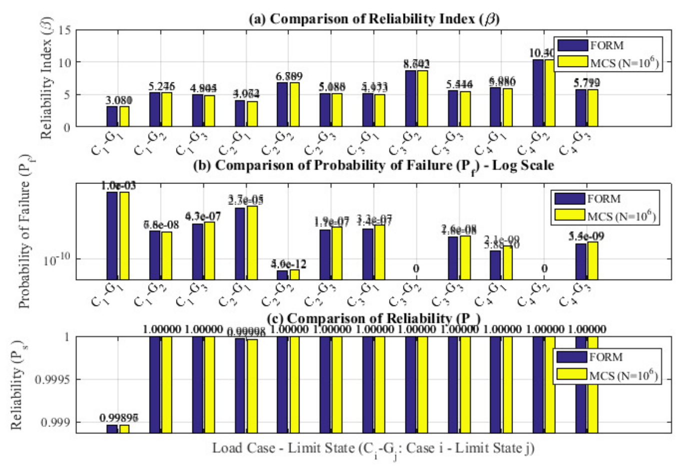

Fig. 4. Comparison of β, P_f, and P_s of the pile cap according to FORM and MCS.

The excellent agreement between the FORM and MCS results validates the accuracy of the FORM method for this problem.

C. Evaluation of the Influence of Pile Coordinate Deviation on Pile Cap Punching Shear Resistance

To further investigate the relationship between the construction quality and structural safety, a sensitivity analysis was performed by varying the coefficient of variation (CV_D) of the pile position. This coefficient is defined as the ratio of the standard deviation of the position deviation (σ) to the pile dimension (D), i.e., $CV_D = \frac{\sigma}{D}$. The analysis was carried out for CV_D values ranging between 0.05 and 0.4. This range was chosen to represent a wide spectrum of construction quality scenarios: from very high precision ($CV_D \approx 0.05$) and typical compliance with VNS limits [1] ($CV_D \approx 0.10–0.25$), to cases with lower quality control or challenging conditions, where deviations may exceed the permissible limits (CV_D>0.25). For each value of CV_D in the surveyed range, the FORM method was used to recalculate the reliability index (β) for the three limit states (G_1, G_2, and G_3) and four load cases (Case 1-4). The results, as detailed in Table V and visualized in Figures 5 and 6, demonstrate a clear decline in the reliability as the construction deviation increases.

TABLE III. THE DETERMINISTIC ANALYSIS RESULTS OF THE PILE CAP RESISTANCE

Load cases	Column punching (G_1)		Edge pile punching (G_2)		One-way shear (G_3)		Evaluation
	R_1 (kN)	F_{per} (kN)	R_2 (kN)	F_{ai} (kN)	R_3 (kN)	Q (kN)	
Case 1	14,585.3	3,083.3	913.5	825.3	3,262.5	1,541.7	Satisfactory
Case 2	14,585.3	2,568.0	913.5	782.7	3,262.5	1,284.0	Satisfactory
Case 3	14,585.3	2,426.0	913.5	740.0	3,262.5	1,213.0	Satisfactory
Case 4	14,585.3	2,290.0	913.5	697.1	3,262.5	1,145.0	Satisfactory

TABLE IV. RELIABILITY ANALYSIS RESULTS FROM FORM AND MCS METHODS

Load cases	Limit states	FORM method			MCS method ($N = 10^6$)		
		β_{FORM}	$P_{f\,FORM}$	$P_{S\,FORM}$	β_{MSC}	$P_{f\,MSC}$	$P_{S\,MSC}$
Case 1	G_1	3.080	0.00103	0.998965	3.081	0.001032	0.998968
	G_2	5.246	7.77E-08	0.999999	5.275	6.64E-08	1.000000
	G_3	4.904	4.69E-07	0.999999	4.845	6.33E-07	0.999999
Case 2	G_1	4.072	2.33E-05	0.999976	3.964	3.68E-05	0.999963
	G_2	6.809	4.91E-12	1.000000	6.789	5.65E-12	1.000000
	G_3	5.188	1.06E-07	0.9999999	5.080	1.88E-07	1.000000
Case 3	G_1	5.133	1.42E-07	0.999999	4.973	3.29E-07	1.000000
	G_2	8.642	0.00000	1.000000	8.703	0.000000	1.000000
	G_3	5.514	1.75E-08	1.000000	5.446	2.57E-08	1.000000
Case 4	G_1	6.086	5.78E-10	1.000000	5.880	2.05E-09	1.000000
	G_2	10.309	0.00000	1.000000	10.402	0.000000	1.000000
	G_3	5.792	3.48E-09	1.000000	5.719	5.36E-09	1.000000

Where β is the reliability index, P_f is the probability of failure, and P_s is the Probability of Reliability, calculated as $P_s = 1 - P_f$.

TABLE V. RELIABILITY CHARACTERISTICS OF LOAD CASES WITH COEFFICIENT OF VARIATION OF PILE COORDINATES

Load case	Limit state		Coefficient of variation, CV_D					
			0.05	0.1	0.15	0.2	0.3	0.4
Case 1	G_1	β	10.47	5.23	3.49	2.618	1.745	1.309
		P_s	1.000	1.000	0.99975	0.99558	0.95951	0.90473
	G_2	β	17.83	8.91	5.945	4.459	2.973	2.229
		P_s	1.000	1.000	1.000	0.99999	0.99852	0.98709
	G_3	β	16.67	8.33	5.55	4.169	2.779	2.084
		P_s	1.000	1.000	1.000	0.99998	0.99727	0.9814
Case 2	G_1	β	13.84	6.92	4.615	3.461	2.308	1.731
		P_s	1.000	1.000	0.999	0.99973	0.9895	0.95827
	G_2	β	23.15	11.57	7.717	5.788	3.859	2.894
		P_s	1.000	1.000	1.000	1.000	0.99994	0.99809
	G_3	β	17.64	8.82	5.88	4.410	2.94	2.205
		P_s	1.000	1.000	1.000	0.99999	0.99836	0.98627
Case 3	G_1	β	17.45	8.72	5.817	4.363	2.909	2.182
		P_s	1.000	1.000	1.000	0.99999	0.99818	0.98544
	G_2	β	29.38	14.69	9.794	7.346	4.897	3.673
		P_s	1.000	1.000	1.000	1.000	1.000	0.9998
	G_3	β	18.74	9.37	6.249	4.687	3.125	2.343
		P_s	1.000	1.000	1.000	0.99999	0.99911	0.99043
Case 4	G_1	β	20.69	10.34	6.898	5.174	3.449	2.587
		P_s	1.000	1.000	1.000	1.000	0.99972	0.99516
	G_2	β	35.05	17.52	11.684	8.763	5.842	4.382
		P_s	1.000	1.000	1.000	1.000	1.000	0.99999
	G_3	β	19.69	9.84	6.564	4.923	3.282	2.462
		P_s	1.000	1.000	1.000	1.000	0.99948	0.99309

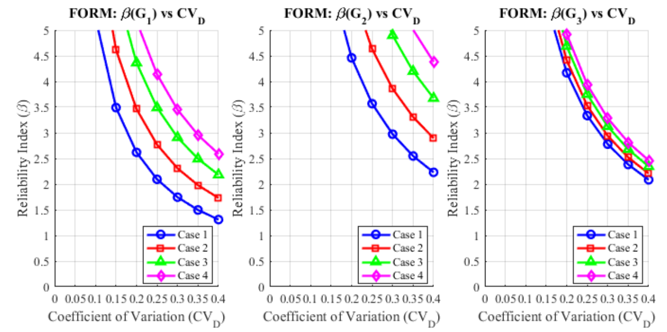

Fig. 5. Relationship between the reliability index (β) and the coefficient of variation of pile position (CV_D) with each subplot corresponding to a specific limit state (G_1, G_2, and G_3) , with the curves representing the four different load cases.

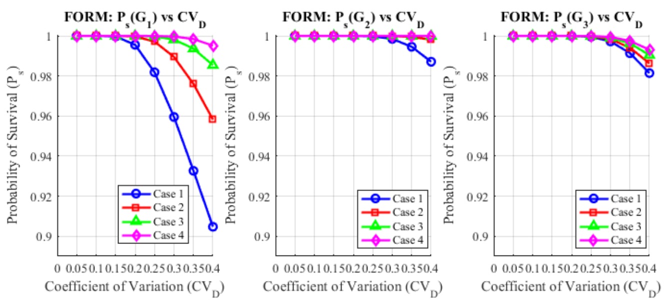

Fig. 6. Relationship between Reliability (P_s) and the coefficient of variation of pile position (CV_D).

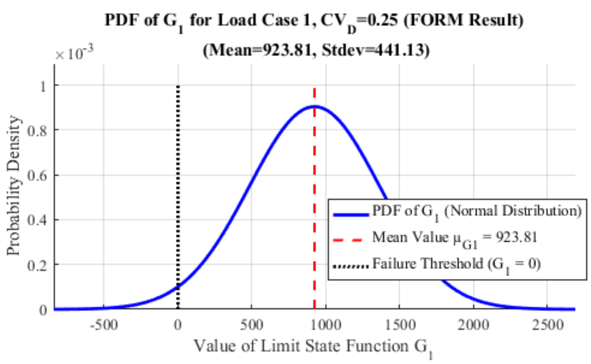

Fig. 7. Probability Density Function (PDF) of limit state G_1 (Case 1), showing an increased failure probability with a rising CV_D.

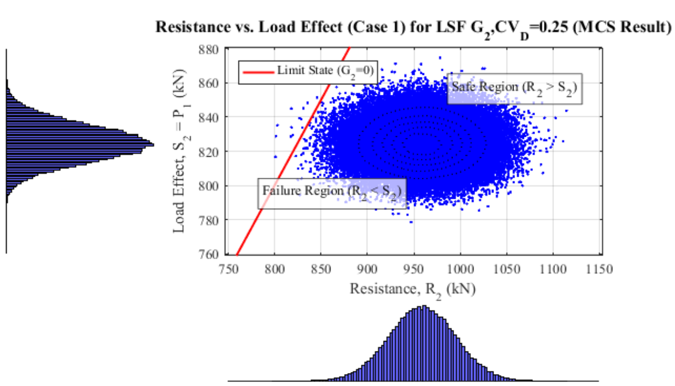

Fig. 8. Resistance versus load effect for limit state G_2 (Case 1), illustrating the safe and failure regions.

IV. DISCUSSIONS

A. Comparison with Design Codes and Implications for Quality Control

A primary finding of this study is the quantitative assessment of the pile cap's actual safety level when accounting for the construction deviations. For the most unfavorable load case (Case 1), the analysis reveals a reliability index of $\beta \approx 3.08$, which is significantly below the target of $\beta_{target} = 3.8$, as proposed by Eurocode 0 for the structures in this consequence class [20]. This discrepancy indicates that designs compliant with the VNS deterministic methods may not ensure the intended safety level when accounting for real-world construction deviations.

This leads to a critical implication for the construction quality control. The sensitivity analysis of this study identifies that to achieve the target reliability of $\beta \approx 3.8$, the coefficient of variation of the pile position (CV_D) must be maintained below 0.15. For the D = 300 mm piles, this translates to a required standard deviation of a placement error of 45 mm or less. This tolerance is considerably stricter than the 75 mm limit permitted by VNS [2], suggesting that merely complying with the current nominal deviation limits is insufficient to guarantee the required structural reliability.

While previous studies, such as [3], have highlighted the effects of the eccentric loads, the current research provides a complementary, quantitative perspective. By integrating a large dataset of field measurements from Vietnam, this study bridges the gap between the deterministic codes and the statistical reality of construction, offering a basis to re-evaluate the safety margins and quality control thresholds in the region.

B. Implications for Design Practice

A key implication of this analysis for design engineers is a counterintuitive finding regarding the governing failure mode. While attention is often given to the edge pile punching limit state (G_2), which is highly sensitive to individual pile misplacement, this study reveals a different critical mechanism. For the most unfavorable load case (Case 1), the column punching state (G_1) becomes the governing failure mode as the construction deviation increases. This trend is visually supported by the sensitivity analysis results exhibited in Figure 5.

The underlying cause of this finding is the combined effect of the load eccentricity and construction deviation. The analyzed load case involves a high axial force along with a large eccentric moment, which already results in a highly uneven distribution of the pile reactions. When the entire pile group shifts from its intended position, the center of gravity of the pile reactions moves as well, further increasing the eccentricity of the total load. This causes a significant redistribution of forces and concentrates a much larger shear force on the critical punching perimeter around the column. As a result, the safety condition at the column face is compromised before any individual edge pile reaches its punching limit. This highlights the importance for designers to thoroughly check the column punching resistance, especially in structures subjected to high load eccentricity.

C. Limitations and Future Research

The findings of this study should be considered alongside its limitations. First, the statistical data were collected from a single construction site and may not represent the entire Vietnamese industry, which has diverse conditions and technologies. Second, the model only considered the uncertainty from the pile positions, treating the material strength and loads as fixed values. Ignoring the variability in these parameters, which have been proved important [3, 19], probably led to an overestimation of the reliability index. Therefore, future research should focus on two main areas: (1) creating a national database of the pile construction deviations to develop more accurate probabilistic models, and (2) incorporating probabilistic models for the material strength and applied loads to perform a more thorough reliability analysis.

V. CONCLUSIONS

This study used the First-Order Reliability Method (FORM) to assess the reliability, and the results were validated through Monte Carlo Simulation (MCS) to quantitatively evaluate how the deviations in pile center coordinates affect the punching and shear resistance of reinforced concrete pile caps designed in accordance with Vietnamese Standards (VNS) [1]. Based on the analysis, the following key conclusions can be drawn:

- For the most unfavorable load case analyzed in this case study, the calculated reliability index ($\beta \approx 3.08$) was below the target level of 3.8 proposed by Eurocode. This indicates that a pile cap design, which is compliant with the deterministic methods in VNS, may not provide the required level of safety under realistic construction deviations.
- The reliability of the pile cap is highly sensitive to the construction quality. The investigation revealed a sharp decrease in the reliability index as the coefficient of variation (CV_D) of the pile positions increased. This quantitatively confirms that the accuracy of pile placement is a key factor directly impacting the cap's reliability.
- The analysis identified that the limit state for the edge pile punching (G_2) is highly sensitive to deviations and the column punching state (G_21) is the governing failure mechanism for the most unfavorable load case. The reliability index for column punching (β_1) drops below the target of 3.8 when the coefficient of variation (CV_D) of the pile positions exceeds approximately 0.15.
- The results indicate a strong agreement between the FORM method and MCS in estimating the reliability for this problem, confirming the feasibility of using reliability methods to assess theb pile cap safety.

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