A Multi-Objective Optimization Approach for the Two-Stage Helical Gearbox Design Using NSGA-II: Balancing Efficiency and Structural Compactness

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ABSTRACT

The design of two-stage helical gearboxes inherently involves trade-offs between the structural compactness and transmission efficiency. Traditional design methods often fail to capture these competing objectives simultaneously. This study presents a comprehensive multi-objective optimization approach using the Non-dominated Sorting Genetic Algorithm II (NSGA-II) to identify the optimal trade-offs between minimizing the cross-sectional area and maximizing the efficiency of a two-stage helical gearbox. Drawing from the strengths of evolutionary algorithms and integrating insights from prior literature on hybrid and decision-making-based optimization methods, the proposed model formulates the gearbox design problem with realistic constraints and evaluates the Pareto front of optimal solutions. The results demonstrate that NSGA-II provides a well-distributed set of non-dominated solutions, offering engineers greater flexibility in balancing performance and structural requirements. The comparative analysis with existing approaches highlights the effectiveness of the proposed method in simultaneously achieving compactness and energy efficiency in gearbox systems.

Keywords-NSGA-II; multi-objective optimization; gearbox design; helical gear; gear ratio; efficiency; cross-sectional area; trade-off analysis

I. INTRODUCTION

The growing demand for compact, high-performance mechanical systems has intensified the need for optimization in gearbox design. Helical gearboxes, widely used in automotive, aerospace, and industrial machinery, must be designed to balance conflicting objectives such as minimizing structural volume and maximizing efficiency. Addressing this challenge effectively requires robust multi-objective optimization techniques. Evolutionary algorithms, particularly the NSGA-II, have emerged as powerful tools for solving complex multi-objective problems due to their ability to explore a diverse set of trade-off solutions [1, 2]. NSGA-II's elitist strategy and fast sorting mechanism have made it a widely adopted method in mechanical design optimization.

Authors in [3] applied genetic algorithms for general gear train design, laying the foundation for evolutionary design strategies. Subsequent research expanded this approach into multi-objective frameworks. Authors in [4] focused on spur gearboxes, incorporating tribological considerations, and demonstrated the importance of including the surface interaction effects in design. Authors in [5] compared multiple evolutionary algorithms for optimal gear design and confirmed that NSGA-II consistently achieved superior Pareto fronts. Similarly, authors in [6] employed a genetic approach to optimize epicyclical gear trains, reinforcing the method's applicability to different transmission types.

The NSGA-II algorithm has been particularly effective for optimizing two-stage gear systems. Authors in [7] investigated two-stage spur gearboxes and highlighted significant trade-offs between torque capacity and volume. Authors in [8] extended this by integrating NSGA-II with decision-making methods, achieving balanced performance in two-stage spur gearbox optimization. Their work emphasized the need for selecting compromise solutions using decision-making techniques. Authors in [9] introduced acoustic considerations into the gearbox design optimization process using response surface methodology, adding another dimension to the performance metrics. Authors in [10] optimized gear unit design focusing on efficiency and transmission error, while authors in [11] explored multi-speed gearboxes and shift control strategies to minimize the fuel consumption and mechanical losses.

Stochastic methods have also been explored. Authors in [12] introduced a stochastic multi-objective optimization framework for synchronizer and selector mechanisms, addressing design uncertainties. Authors in [13] used the MARCOS method to optimize two-stage helical gearboxes, successfully balancing volume reduction with efficiency improvement using a novel multi-criteria approach. Recent studies have pushed the boundaries of design objectives. Authors in [14] proposed a hybrid sailfish optimization algorithm for planetary gearbox design, achieving improvements in robustness and convergence speed. Authors in [15] explored spur gear optimization, highlighting the integration of multiple design parameters, including face width, module, and pressure angle, to achieve optimal solutions.

In addition to planetary and spur gear systems, helical gearboxes have gained attention for their quiet operation and load-bearing capacity. Authors in [16] optimized spur gear pairs for volume and efficiency, offering relevant insights transferable to helical configurations. Authors in [17] introduced robustness in gear microgeometry optimization, ensuring that solutions remain effective under uncertainty—a crucial consideration in practical engineering design. Authors in [18] combined NSGA-II with decision-making tools, such as TOPSIS and VIKOR, to guide the selection of optimal solutions. Their results demonstrated the effectiveness of combining evolutionary search with Multi-Criteria Decision-Making (MCDM). This synergy has been recently applied to two-stage helical gearboxes. Authors in [19] used the EAMR technique to optimize a two-stage design with double gear sets in the first stage, achieving reduced volume and enhanced efficiency. Authors in [20] optimized large planetary gear reducers, while authors in [21] focused on compact planetary gear trains using genetic algorithms. Authors in [20] incorporated uncertainty in planetary gear design for electric vehicles, indicating the growing trend toward robust optimization in dynamic environments.

In advanced applications, authors in [22] utilized NSGA-III to optimize angular contact ball bearings in aircraft gearboxes, demonstrating the applicability of multi-objective evolutionary algorithms beyond gears themselves. Similarly, authors in [20] proposed layered optimization for magnetic planetary gears in hybrid powertrains, showcasing novel approaches to gearbox design for emerging technologies.

Despite these advances, few studies have addressed the combined objectives of structural compactness and efficiency in two-stage helical gearboxes using a pure NSGA-II framework without relying heavily on external MCDM tools. This study aims to fill this gap by proposing a multi-objective optimization framework using NSGA-II to simultaneously minimize the overall size and maximize the efficiency of two-stage helical gearboxes. The proposed approach provides design engineers with a rich set of trade-off solutions, enhancing decision-making during gearbox development.

II. OPTIMIZATION PROBLEM FORMULATION

A. *Determining Gearbox Bottom Area*

For a two-stage helical gearbox (Figure 1), the cross-sectional area A_c is determined by:

$$A_c = (L \times H) \tag{1}$$

L and H are calculated by:

$$L = d_{w11} + d_{w21}/2 + d_{w12}/2 + d_{w22} + 2 \cdot \delta \tag{2}$$

$$H = max\,(d_{w21}, d_{w22}) + 6.5 \cdot \delta \tag{3}$$

where δ = 7÷10 (mm) [23], and d_{w1i}, d_{w2i} (i = 1÷2) are the pitch diameters of the pinion and the gear of stage *i* given by [23]:

$$d_{w1i} = 2 \cdot a_{wi}/(u_i + 1) \tag{4}$$

$$d_{w2i} = 2 \cdot a_{wi} \cdot u_i \cdot/(u_i + 1) \tag{5}$$

where a_{wi} (i = 1÷2) is the center distance of stage *i* and a_{wi} can be determined using [18]:

$$a_{wi} = k_a \cdot (u_i + 1) \cdot \sqrt[3]{T_{1i} \cdot k_{H\beta}/([AS_i]^2 \cdot u_i \cdot X_{bai})} \tag{6}$$

where X_{bai} is the wheel face width coefficient of stage i_{th}, T_{1i} (i=1÷2) is the pinion torque of stage *i,* and it can be computed using:

$$T_{1i} = \frac{T_r}{\prod_{j=i}^{3}(u_i \cdot \eta_{hg}^{3-i} \cdot \eta_{be}^{4-i})} \tag{7}$$

B. *Determining Gearbox Efficiency*

The gearbox efficiency (%) can be determined by:

$$\eta_{gb} = 100 - \frac{100 \cdot P_l}{P_{in}} \tag{8}$$

where P_l is the total gearbox power loss given by:

$$P_l = P_{lg} + P_{lb} + P_{ls} + P_{Z0} \tag{9}$$

where P_{lg}, P_{lb}, P_{ls}, and P_{Z0} are the power loss in the gears, in bearings, in seals, and in the idle motion, respectively. These components are computed as in [24].

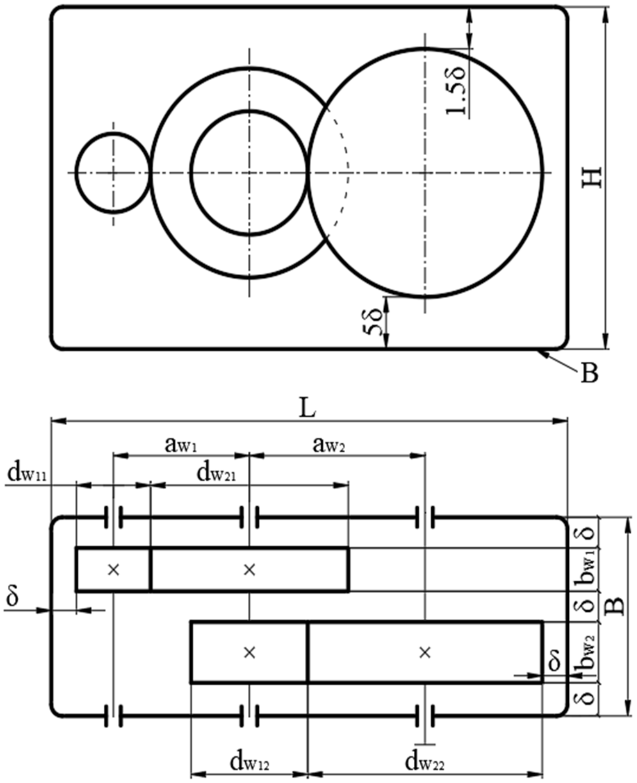

Fig. 1. Schematic for the calculation of the gearbox cross-sectional area.

C. *Objective Functions*

This work formulates the optimization problem as a bi-objective minimization issue, representing two critical performance metrics of a two-stage helical gearbox:

- Minimizing the gearbox cross-sectional area:

$$min f_1(X) = A_c \quad (10)$$

- Maximizing the gearbox efficiency:

$$min f_2(X) = \eta_{gb} \quad (11)$$

The design variable vector *X* includes the key geometrical and performance-related parameters of the gearbox. Five parameters are typically employed to define the geometry: u_1, X_{ba1}, X_{ba2}, AS_1, and AS_2 [24]. Based on findings in [15], the optimal values of AS_1 and AS_2 approach their maximum allowable values. As a result, the three most sensitive and adjustable parameters are identified as decision variables: u_1, X_{ba1}. The following is established:

$$X = \{u_1, Xba_1, Xba_2\} \quad (12)$$

D. *Constrains*

For a two-stage helical gearbox, $u_i = 1 \div 9$, $X_{bai} = 0.25 \div 0.4$ ($i = 1 \div 2$) [23]. The MOOP includes the subsequent limitations:

$$1 \le u_i \le 9 \quad (13)$$

$$0.25 \le Xba_i \le 0.4 \quad (14)$$

III. OPTIMIZATION METHODOLOGY

A. *Optimization Approach Using NSGA-II*

To optimize gearbox design, the NSGA-II is employed due to its proven performance in handling conflicting objectives and generating well-distributed Pareto fronts [1]. The main features of NSGA-II include:

- Fast non-dominated sorting for population ranking.
- Crowding distance calculation for diversity preservation.
- Elitism to retain the best solutions over generations.

For each fixed u_h, the algorithm searches the design space for optimal combinations of u_1, X_{ba1}, and X_{ba2}. The implementation has been developed using MATLAB, incorporating the parameters outlined in Table I.

TABLE I. NSGA-II ALGORITHM PARAMETERS IN MATLAB IMPLEMENTATION

Parameter	Value
Population size	100
Generations	200
Crossover rate	0.9
Mutation rate	0.1
Selection method	Binary tournament
Encoding	Real-valued

B. *Multi-Scenario Optimization by Sweeping u_h*

To investigate the influence of the total transmission ratio u_h on gearbox performance, a multi-scenario optimization strategy is employed. Specifically, the NSGA-II algorithm is executed independently for a set of pre-defined transmission ratios:

$$u_h \in \{5, 10, 15, 20, 25, 30, 35\} \quad (15)$$

For each scenario, the optimization is carried out with the same algorithmic settings, objective functions, and constraints, while treating uh as a fixed parameter.

This strategy allows for a systematic exploration of how increasing transmission demands affect the shape and range of the Pareto fronts between efficiency and structural compactness. Unlike single-scenario studies, this approach provides a more complete perspective on the design trade-offs across application contexts. The resulting Pareto fronts for all u_h values are later visualized and compared in Figure 1, and their implications are analyzed in detail below. This separation ensures that the methodology remains general and scalable, while the discussion of trends and engineering interpretation is deferred to the results section.

By aggregating optimal solutions from multiple scenarios, this framework also supports the development of empirical design trends, such as the linear relationship between the stage-1 gear ratio u_1 and total ratio u_h, which can accelerate future design workflows and reduce the need for repeated optimization.

IV. RESULTS AND DISCUSSION

A. Trade-Off Behavior under Varying Transmission Ratios

Figure 2 illustrates the Pareto fronts obtained across different total transmission ratios (u_h), revealing a consistent pattern: as efficiency improves, the gearbox cross-sectional area increases. Notably, the shape and height of the Pareto fronts degrade with increasing u_h, indicating that higher transmission ratios limit the ability to achieve both compact and efficient designs.

This trend contrasts with the findings in [3] on spur gearboxes, where the deterioration in efficiency was less significant due to lower gear interaction losses. Besides, while authors in [9] explored the trade-offs between vibration and performance using RSM, they did not provide a comprehensive view of the structural impact. In contrast, this work explicitly quantifies how the efficiency loss is associated with the structural growth, providing a clearer map for design decision-making.

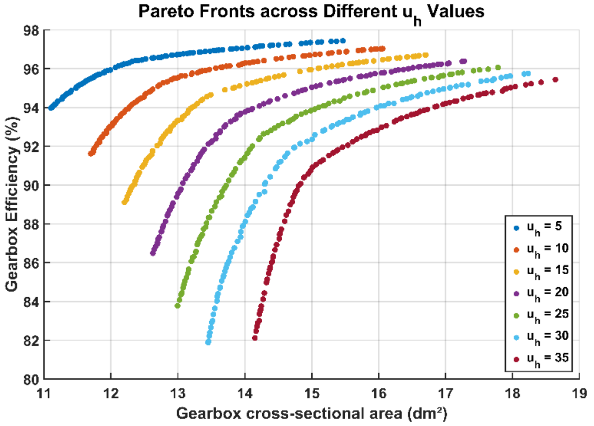

Fig. 2. Pareto fronts across varying u_h values.

B. Optimal Configurations and Efficiency–Compactness Relationship

Optimal design configurations were extracted using two strategies: (i) minimum cross-sectional area, and (ii) maximum efficiency-to-area ratio. These configurations consistently demonstrate that increasing u_h leads to larger gearbox dimensions and a decline in efficiency—mirroring the trend shown in Figure 3.

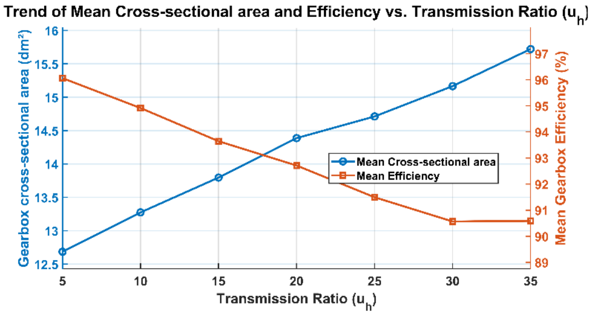

Fig. 3. Trend of mean bottom area and efficiency versus gearbox ratio (uh).

The efficiency drop—from 97.4% to 90.3% as u_h increases from 5 to 35—highlights an important engineering constraint. This decline is more gradual and better managed in the proposed NSGA-II approach compared to other studies on MCDM-based approach, which showed steeper efficiency degradation without an explicit trade-off structure.

These findings are significant for applications such as robotics or electric drivetrains, where space is limited and efficiency is critical. By providing a continuous solution space, this method supports more nuanced engineering compromises.

C. Efficiency Loss versus Structural Growth: A Strategic Balance

One of the most practically valuable outcomes is the ability to visualize and quantify the balance between efficiency and size. The mean trend observed in Figure 2 shows a clear inverse relationship between the two objectives.

The resulting Pareto fronts demonstrate that moderate transmission ratios (e.g., u_h = 10 to 20) offer a desirable compromise: compact designs (13–14.5 dm²) while maintaining high efficiencies (~92–94%). This provides a design guideline for early-stage gearbox sizing, especially in multidisciplinary design environments.

D. Design Variable Behavior and Ratio Decomposition Strategy

Figure 4 presents a strong linear relationship between stage-1 ratio u_1 and total transmission ratio u_h with $R^2 = 0.9773$, captured by the following regression:

$$u_1 = 0.1798 \cdot u_h + 1.6953 \qquad (16)$$

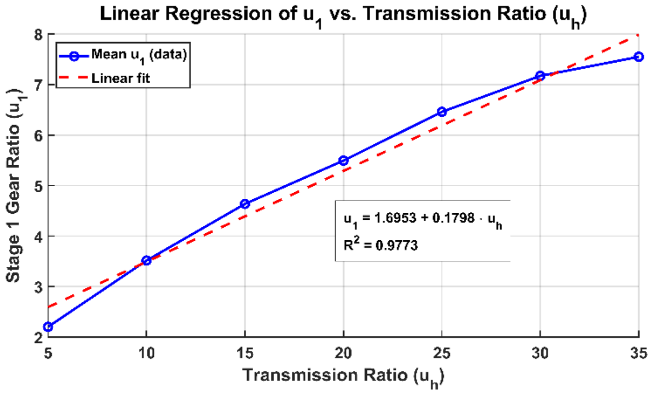

Fig. 4. Linear regression of u_1 versus gearbox ratio (u_h)

TABLE II. OPTIMAL GEARBOX DESIGNS AT DIFFERENT TRANSMISSION RATIOS *uh*

u_h	u_1	u_2	X_{iba1}	X_{iba2}	A_c (dm²)	η (%)
5	1.27	3.95	0.25	0.40	15.46	97.43
10	2.04	4.91	0.25	0.40	16.06	97.03
15	2.66	5.63	0.25	0.40	16.70	96.70
20	3.22	6.21	0.25	0.40	17.28	96.38
25	3.73	6.70	0.25	0.40	17.78	96.06
30	4.21	7.13	0.25	0.40	18.23	95.75
35	4.66	7.51	0.25	0.40	18.64	95.44

This finding allows rapid estimation of u_1 without re-optimization for each u_h, which is a useful feature. The design variable X_{iba1}, representing the torque distribution in the first stage, tends to shift toward lower values (0.25–0.30) for high-efficiency designs, as presented in Table II. This suggests an intelligent torque-splitting strategy that reduces losses in early transmission stages. Compared to the MARCOS-based results of [13], which did not explore the internal power flow in depth, this study brings to light a critical design insight for achieving efficiency without structural overload.

E. Significance of the Study

By integrating NSGA-II with a detailed mechanical model and focusing on dual-objective optimization (efficiency versus size), this study contributes a complete, data-driven framework for helical gearbox design. It extends beyond previous works by:

- Providing a continuous Pareto front instead of single-point solutions.
- Offering predictive models for gear ratio decomposition.
- Uncovering the role of torque allocation variables such as X_{iba1} in managing power loss.

These insights are not only theoretically valuable but also offer practical utility for mechanical engineers dealing with constrained spaces, energy efficiency demands, and high transmission requirements.

V. CONCLUSIONS

This study presented a comprehensive multi-objective optimization framework for the design of two-stage helical gearboxes using the Non-dominated Sorting Genetic Algorithm II (NSGA-II). By formulating a bi-objective problem—minimizing the gearbox cross-sectional area and maximizing mechanical efficiency—the proposed approach systematically explored the trade-offs involved in gearbox design under various total transmission ratios u_h. The optimization was conducted across a range of $u_h \in \{5, 10, 15, 20, 25, 30, 35\}$, allowing for detailed analysis of the impact of transmission ratio on design performance. The key findings of the study include:

- The shape and extent of Pareto fronts deteriorate with increasing u_h, indicating stronger trade-offs and performance penalties for compact designs at higher transmission demands.
- A clear inverse relationship between cross-sectional area and efficiency was observed, with the most balanced designs occurring in the moderate range of $u_h = 10$–20.
- A strong linear relationship between u_1 and u_h was established, enabling predictive estimation of gear ratio decomposition for future designs.
- The torque-splitting variable X_{ba1} consistently remained near its lower bound in high-efficiency solutions, underscoring its significance in internal power loss control.

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