Calculating Criteria Weights Using the Rank Order Centroid Method when the Ranks are Guided by the Entropy Method

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ABSTRACT

This study proposes a novel weighting approach for solving multi-objective optimization problems, called Entropy and Rank Order Centroid (ER) weighting, that integrates data-driven and preference-based weighting principles. The method consists of two sequential stages. In the first stage, the Entropy method is applied to the decision matrix to establish the priority ranking of the criteria based on their information content. In the second stage, this ranking is used to compute the final criteria weights through the Rank Order Centroid (ROC) method. To assess its effectiveness, the ER method was evaluated using a representative multi-objective optimization case: the selection of polishing machines. The results show that ER provides clear advantages over the conventional Entropy method, particularly in ensuring the stability of alternative rankings within multi-objective optimization problems.

Keywords-multi-objective optimization; weighting method; entropy method; Rank Order Centroid (ROC) method; Entropy and ROC (ER) method

I. INTRODUCTION

Multi-objective optimization is a problem-solving domain that is used in several applications, ranging from societal systems to scientific research. A prerequisite for solving such problems is a set of alternative options, each possessing input parameters and at least two output objectives [1, 2]. The two most common types of objectives for such problems are: i) either determining the optimal values of input parameters within a specified range, or ii) identifying the optimal alternative from a predefined set of options [3]. Despite their differing objectives, both types share a fundamental requirement: the weighting of criteria, where each weight expresses the relative importance of each criterion [4]. Criteria weighting methods are typically classified into two groups: objective weighting methods (Group I) and subjective weighting methods (Group II) [5]. Group I includes methods such as the Symmetry Point of Criterion (SPC), Entropy, Method based on the Removal Effects of Criteria (MEREC), and Logarithmic Percentage Change-driven Objective Weighting (LOPCOW). Among these, the Entropy method is one of the most frequently employed and recommended methods [6]. A common characteristic of these methods is that they derive criteria weights solely from numerical data in the decision matrix, without incorporating user perspectives about the relative importance of the criteria. Consequently, outcomes derived may not fully align with user expectations or decision-making preferences [7]. On the other hand, Group II includes methods like Simple Weight Calculation (SIWEC), Rank Sum (RS), and Rank Order Centroid (ROC), with the latter being the most widely used [8, 9]. These approaches determine the criteria weight based on user perceptions of criteria importance. However, the main limitation is that reliance on personal judgment can introduce bias and inconsistency, especially when users lack sufficient experience or are influenced by contextual factors [10]. Given the inherent limitations of both groups, there is a clear need to develop hybrid criteria-weighting approaches that leverage the strengths of objective (e.g., reduced bias) and subjective methods (e.g., expert input) while mitigating their weaknesses. Studies have shown that combined weighting schemes derived from two or more methods tend to yield higher reliability and efficiency than single-method approaches [11]. For instance, in [12], the subjective Analytic Hierarchy Process (AHP) and the objective Criteria Importance Through Intercriteria Correlation (CRITIC) methods were integrated to determine weights for optimizing a bank database platform. Although this hybridization improved objectivity, AHP's dependence on expert judgments introduced inconsistencies in weight estimation as the number of criteria increased [13]. In another study in [14], a hybrid approach combining the subjective Lagrange and objective Entropy methods was applied to sustainable energy option selection, yet the Lagrange method required prioritizing criteria in ascending order, contrary to most subjective methods, creating potential confusion for users [15].

In this study, a hybrid method is proposed combining Entropy and ROC (ER). The ROC method is a well-established subjective approach that minimizes individual weight assignment errors by determining the centroid of potential weights while maintaining the criteria ranking order [16]. Meanwhile, the Entropy method remains one of the most reliable and frequently adopted objective techniques, often outperforming alternatives such as MEREC and SPC [17]. Despite their complementary strengths, no previous study has reported a direct combination of ROC and Entropy for criteria weighting.

II. MATERIALS AND METHODS

A. *Entropy Method*

In the Entropy method, criteria weights are calculated from an initial dataset comprising m alternatives and n criteria. The value of the criterion j for alternative i is denoted as x_{ij}, where $i = 1, \ldots, m$ and $j = 1, \ldots, n$. The sequential steps for determining criteria weights are [18]:

- Step 1: Determine the normalized value for each criterion using (1):

$$n_{ij} = \frac{x_{ij}}{m+\sum_{i=1}^{m} x_{ij}^2} \quad (1)$$

- Step 2: Calculate the Entropy measure value for each criterion using (2):

$$e_j = \sum_{i=1}^{m}\left[n_{ij} \times ln\left(n_{ij}\right)\right] - \\ -\left(1 - \sum_{i=1}^{m} n_{ij}\right) \times ln\left(1 - \sum_{i=1}^{m} n_{ij}\right) \quad (2)$$

- Step 3: Compute the weight for each criterion according to (3):

$$w_j = \frac{1-e_j}{\sum_{j=1}^{n}(1-e_j)} \quad (3)$$

B. *ROC Method*

In the ROC method the weights are calculated differently compared to the Entropy method based on the following steps [19]:

- Step 1: Establish the decreasing order of importance (priority ranking) among the criteria.
- Step 2: Calculate the weights for the criteria using (4), where k denotes the priority rank of criterion j:

$$w_j = \frac{1}{n}\sum_{k}^{n}\frac{1}{k} \quad (4)$$

C. *ER Method*

In the proposed ER method, the calculation of criteria weights integrates the two constituent methods, as described in Figure 1. Specifically, in the initial phase, criteria weights are determined using the Entropy method using (1)-(3), then the criteria are ranked in decreasing order of their importance, and lastly, the final criteria weights are calculated using (4).

III. PERFORMANCE EVALUATION OF THE PROPOSED METHOD

To assess the effectiveness of the proposed ER method, a multi-objective optimization example problem for selecting the optimal alternative among 13 polishing machine types (M1-M13) was used, employing the dataset used in [21]. Each machine was assessed based on six criteria: C1 (selling price), C2 (power), C3 (polishing disk diameter), C4 (no-load speed), C5 (weight), and C6 (warranty period). The performance of the proposed ER method was directly compared with that of the standalone traditional Entropy method. The Entropy weights for each criterion C1-C6 were calculated using formulas (1)-(3), resulting in 0.13621, 0.13680, 0.14129, 0.13599, 0.25350, and 0.19622, respectively. Consequently, the criteria were ranked in descending order of importance as C5 > C6 > C3 > C2 > C1 > C4. Then, using this ranking in the ROC formulation (4), the criteria weights derived via the ER method were computed as 0.06111, 0.10278, 0.15833, 0.02778, 0.40833, and 0.24167 for C1 to C6, respectively. To quantitatively compare the Entropy and ER methods, five Multi-Criteria Decision-Making (MCDM) techniques were used: Simple Additive Weighting (SAW), Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), Proximity Indexed Value (PIV), Range of Value (ROV), and Root Assessment Method (RAM) [22, 23]. Figures 2 and 3 illustrate the ranking of polishing machines determined by the Entropy and ER methods across these five optimization techniques.

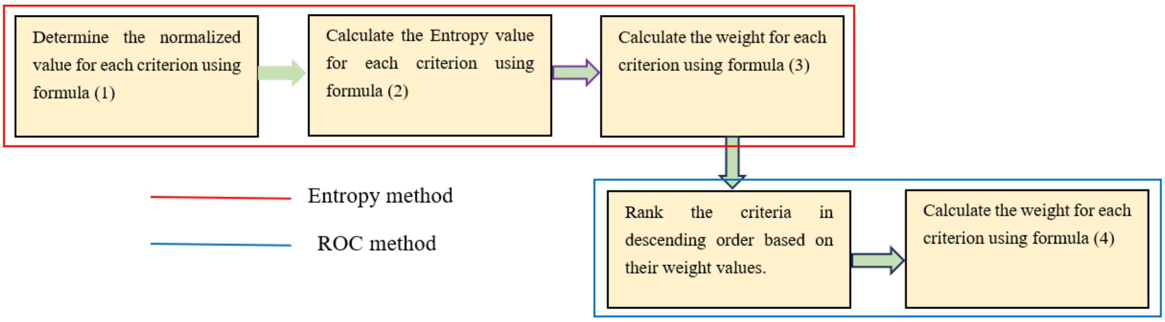

Fig. 1. Block diagram of the ER method.

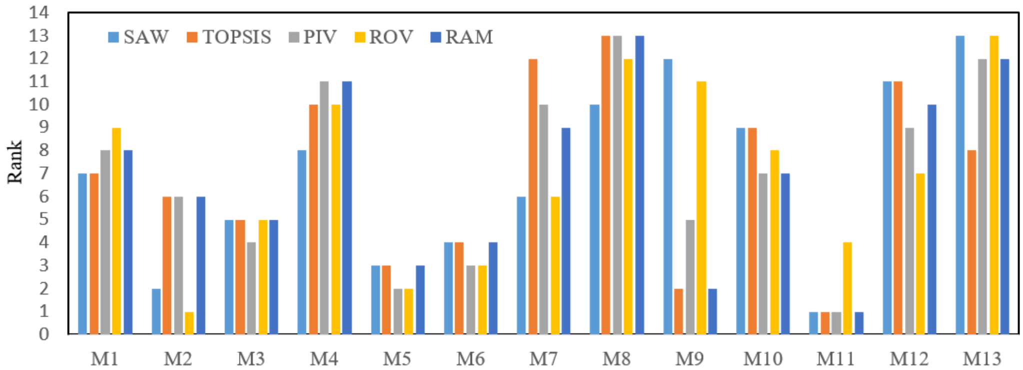

Fig. 2. Ranking of polishing machines by optimization methods using Entropy for criteria weighting.

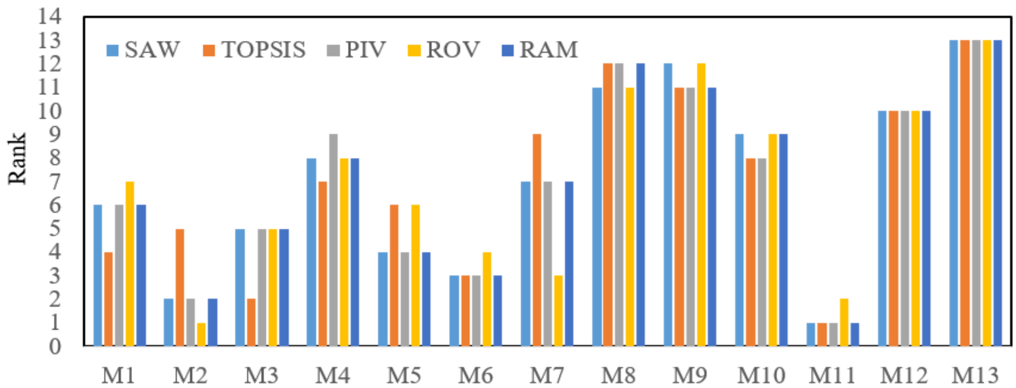

Fig. 3. Ranking of polishing machines by optimization methods using ER for criteria weighting.

As shown in Figure 2, the Entropy method exhibits considerable variation in the ranking of polishing machines, with no single machine maintaining a consistent position across all five optimization methods. For example, M11 ranked first under SAW, TOPSIS, PIV, and RAM, but dropped to sixth under ROV. In contrast, Figure 3 shows much higher consistency when ER weights were applied: M11 remained the top-ranked option in four methods and second in ROV, while M12 and M13 consistently occupied the 10th and 13th positions, respectively. For a quantitative comparison between the Entropy and ER methods, the Spearman's rank correlation coefficient between MCDM methods was calculated using (5):

$$S = 1 - \frac{6D_i^2}{m(m^2-1)} \tag{5}$$

where D_i represents the difference in rank for the alternative i (M1-M13) across the five methods [23]. The calculated Spearman correlation matrices for both weighting methods are presented in Table I and Table II.

TABLE I. SPEARMAN'S CORRELATION COEFFICIENT BETWEEN MCDM METHODS USING THE ENTROPY METHOD

Method	TOPSIS	PIV	ROV	RAM
SAW	0.4780	0.6923	0.8846	0.5879
TOPSIS	-	0.8846	0.4505	0.9121
PIV	-	-	0.7363	0.9615
ROV	-	-	-	0.6154

TABLE II. SPEARMAN'S CORRELATION COEFFICIENT BETWEEN MCDM METHODS USING THE ER METHOD

Method	TOPSIS	PIV	ROV	RAM
SAW	0.9066	0.9890	0.9341	0.9945
TOPSIS	-	0.9066	0.7912	0.9121
PIV	-	-	0.9231	0.9945
ROV	-	-	-	0.9286

For the Entropy method, the average Spearman coefficient among all optimization methods was 0.7203, while for the ER method it was 0.9280. Moreover, the lowest coefficient when using ER was 0.7912, whereas for Entropy it was 0.4505. Additionally, 9 out of 10 coefficients exceeded 0.9 under ER, compared to only 2 out of 10 under Entropy.

To further assess the robustness of the proposed model, two sensitivity analysis scenarios were conducted. In scenario S1, the alternative M7 was removed, a similar tactic used in [24]. Then, formulas (1)-(3) yielded new Entropy weights for criteria C1-C6 of 0.1367, 0.1373, 0.1418, 0.1365, 0.2490, and 0.1987, respectively. Corresponding ER weights were 0.0611, 0.1028, 0.1583, 0.0278, 0.4083, and 0.2417. The final ranking results are summarized in Table III, while the average Spearman correlation coefficient among the five MCDM methods was 0.7239 when using the Entropy method and 0.9629 when using the ER method.

In scenario S2, instead of removing an alternative, criterion C5 was removed. The basis for its removal was that in S1, it had the highest weight of 0.2490, and removing it would show how each methodology adjusts to this change. With C5 excluded, C6 showed the highest data dispersion among the alternatives and received the highest Entropy weight (0.2646). Using the ER method, the weights for the remaining criteria (C1, C2, C3, C4, C6) were 0.0900, 0.1567, 0.2567, 0.0400, and 0.4567, respectively, resulting in the priority order C6 > C3 > C2 > C1 > C4. This distinct differentiation in weights highlights ER's capability to focus on the "center of potential weights", which reflects the most reasonable assessment of each criterion's importance without being disproportionately influenced by extreme values. This contrasts with conventional Entropy weighting, which relies solely on dispersion and may produce less balanced weight distributions. Afterwards, the ranking of alternatives in S2 was conducted accordingly, and the resulting ranking is shown in Table IV. Based on these rankings, the corresponding average Spearman correlation coefficient using the ER and the Entropy method was 0.9324 and 0.6978, respectively.

Overall, across all analyses, the proposed ER method displayed both more reliable and balanced results than the standalone Entropy method, achieving significantly higher Spearman coefficients regardless of the MCDM method employed.

TABLE III. RANKING OF POLISHING MACHINE TYPES IN SCENARIO S1

Alt.	Entropy weight					ER weight				
	SAW	TOPSIS	PIV	ROV	RAM	SAW	TOPSIS	PIV	ROV	RAM
M1	6	7	9	8	8	6	5	6	6	6
M2	2	6	6	1	6	2	4	2	1	2
M3	5	5	4	5	5	5	2	5	4	5
M4	7	11	10	9	11	7	7	8	7	8
M5	3	3	2	2	3	4	6	4	5	4
M6	4	4	3	3	4	3	3	3	3	3
M8	9	12	12	11	12	10	11	11	10	11
M9	11	2	5	10	2	11	10	10	11	10
M10	8	9	7	7	7	8	8	7	8	7
M11	1	1	1	4	1	1	1	1	2	1
M12	10	10	8	6	9	9	9	9	9	9
M13	12	8	11	12	10	12	12	12	12	12

TABLE IV. RANKING OF POLISHING MACHINE TYPES IN SCENARIO S2

Alt.	Entropy weight					ER weight				
	SAW	TOPSIS	PIV	ROV	RAM	SAW	TOPSIS	PIV	ROV	RAM
M1	10	12	12	11	12	8	9	11	8	11
M2	1	9	6	1	6	1	1	2	1	2
M3	7	8	7	5	8	6	6	5	5	6
M4	11	11	11	10	11	7	8	9	7	9
M5	2	2	2	2	2	2	2	1	2	1
M6	5	4	4	4	4	4	4	4	4	4
M8	3	6	9	9	9	9	10	8	10	10
M9	6	1	1	8	1	11	11	10	11	8
M10	9	10	10	6	10	5	7	7	6	7
M11	8	7	5	7	7	10	5	6	9	5
M12	4	5	3	3	3	3	3	3	3	3
M13	12	3	8	12	5	12	12	12	12	12

IV. CONCLUSION

This study introduces a novel criterion weighting method, the Entropy and Rank Order Centroid (ER) method, in which criterion weights are determined using the Rank Order Centroid (ROC) method, with the Entropy method serving as a supporting tool to establish the priority order of criteria. The effectiveness of the ER method was evaluated using a classic multi-objective optimization example, the selection of polishing machines, using five different Multi-Criteria Decision-Making (MCDM) methods, including Simple Additive Weighting (SAW), Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), Proximity Indexed Value (PIV), Range of Value (ROV), and Root Assessment Method (RAM).

Results show that ER outperforms Entropy in maintaining stable rankings across the five MCDM techniques, while also achieving considerably higher Spearman's rank correlation coefficient. Specifically, the average Spearman coefficient among SAW, TOPSIS, PIV, ROC, and RAM methods was 0.9280 using ER, compared to 0.7203 using Entropy. Additionally, a sensitivity analysis was conducted with two scenarios: in S1, one alternative was removed from the ranking, and in S2, the criterion with the highest weight was removed. In both scenarios, ER consistently outperformed Entropy, demonstrating greater robustness and reliability in ranking stability.

Future research could further evaluate the performance of the ER method by comparing it with alternative weighting approaches, such as the Method based on the Removal Effects of Criteria (MEREC) method or the Equal Weighting method. Additionally, the current study only compared the ER and Entropy methods using specific numerical criteria. To enhance the generalizability of these findings, future studies should extend the comparison to scenarios involving fuzzy or qualitative criteria, as these types of data are common in real-world decision-making contexts.

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