Optimizing Investment Portfolio Allocation: Analyzing Trends and Dynamics of Alternative Investments in Estate Planning

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ABSTRACT

Private Equity (PE) plays a unique role in estate planning due to its potential for high returns and complexity. Therefore, understanding its pricing dynamics is essential for effectively incorporating it into an estate plan. The current research examines the complexities of optimizing investment portfolio allocation by analyzing the pricing dynamics of PE funds in secondary markets. The study acknowledges the challenges posed by the inherent risks, illiquidity, and long-term nature of PE investments. It focuses on the impact of PE investments on Limited Partners' (LP) optimal allocations. Typically, LPs commit capital to PE funds, which is gradually called and eventually distributed back to them. The research reveals that the PE investments significantly influence LPs' optimal allocations, with differing strategies being observed among LPs with varying risk aversion levels. The study's findings reveal that PE allocations do not consistently decrease in risk aversion, suggesting the presence of nuanced decision-making processes. Depending on an LP's risk aversion, they adopt one of two distinct investment strategies. A conservative LP, characterized by higher risk aversion, holds relatively more liquid reserves of stocks and bonds compared to illiquid PE investments. This LP tends to be unconstrained and to remain close to an interior optimum, leaving it largely unaffected by the illiquid and long-term nature of PE investments. Moreover, the model extends to incorporate a secondary market for PE partnership interests, allowing an exploration into the implications of trading in this market and the pricing dynamics of the Net Asset Value (NAV) and unfunded liabilities. The proposed analysis provides valuable insights into the intricate interplay between the risk aversion, PE investments, and secondary market dynamics, offering guidance to LPs navigating the complex landscape of alternative investments.

Keywords-Private Equity (PE); estate planning; limited partners; Net Asset Value (NAV); unfunded liabilities; investments

I. INTRODUCTION

The efficient allocation of investment portfolios is of critical importance to the financial management of high-net-worth individuals and institutional investors. Fundamentally, portfolio allocation is the process of dividing assets across different investment categories in a planned manner to minimize risks and reach the desired financial goals. This procedural aspect is indispensable for the long-term preservation of wealth and the optimization of returns [1]. Investing involves the diversification of an investor's portfolio, thereby reducing market volatility. This diversification occurs through the ownership of a variety of asset types, including bonds, real estate, stocks, and different kinds of investments, like money managers and PE. Moreover, portfolio allocation enables investors to align their investment plans with their time horizon, risk tolerance, and financial objectives, whether they are focused on wealth creation, income generation, or capital preservation. Investors who allocate their investment portfolio optimally can capitalize on opportunities arising from shifting market circumstances and economic cycles [2]. In light of the dynamic nature of financial markets, it is imperative for investors to employ a flexible approach to asset allocation, one that can adapt to the changing market conditions, interest rates, inflation forecasts, and geopolitical issues. Investors can modify their asset allocation and rebalance their portfolios to preserve targeted risk-return profiles and capitalize on new investment themes when they employ a dynamic technique to portfolio management. Efficient portfolio management has also been shown to improve the effectiveness of capital distribution by limiting the exposure to failing assets or industries and allocating resources to the most attractive investment possibilities. Investors from institutional sources, including governments, pension funds, and foundations, recognize the significance of portfolio allocation in achieving their long-term financial objectives and fulfilling the stakeholder obligations [4]. These entities often have multifaceted investment objectives and obligations, which necessitates the formulation of sophisticated asset allocation strategies tailored to their distinct requirements [5]. To optimize portfolio allocation, it is essential to adhere to financial management principles and regulatory obligations, while achieving a balance between the capital development, income generation, and risk management. Inadequate resource allocation can result in suboptimal investment outcomes, jeopardizing the long-term viability and financial stability of these organizations [6]. The benefit of diversification is particularly pronounced in times of market or economic uncertainty, as asset prices may exhibit heightened degrees of correlation during such periods [7]. A substantial segment of the investment alternatives/alternative investment market comprises of PE funds, which play a critical role in the portfolios of institutional and high-net-worth investors [8].

In the context of India's expanding economy, authors in [9] examined the dynamics between the holding times, return multiples, fund categories, and exit strategies for a range of Venture Capital (VC) and PE investments. The dataset, encompassing the period from January 2004 to March 2021, was meticulously analyzed, leading to the observation that there is a negative correlation between the holding duration and returns [10]. The study also reveals that there is no statistically significant difference in the average holding duration of foreign and India-dedicated funds. Furthermore, the study revealed that India-focused funds outperform their foreign counterparts in terms of generating returns. The results of the study call into question the widely held notion that certain exit routes invariably yield higher returns, suggesting that the outcomes may be comparable across all routes. The Russian conflict against Ukraine is regarded in [11] as an unanticipated exogenous crisis event, and it is examined how it has affected the field of entrepreneurial financing. Beyond a mere emotional and impact analysis, the study delves into the underlying causes and mechanisms, as well as the range of reactions and coping strategies employed by investors in entrepreneurial financing. The study's findings, based on polls of European PE and VC investors, reveal a substantial negative impact of the conflict on the participants and the funds associated with them. Both VC and PE investors have identified increased risk aversion and LPs' exiting the market as significant obstacles that reduce LPs' willingness to invest. Overall, VC portfolio businesses appear to face greater challenges than PE investors. In order to provide investors and PE specialists with transparent and intelligible decision criteria, authors in [12] propose the development of automatically generated recommitment techniques based on the growth of symbolic expressions. This approach marks a pioneering contribution in the field of evolutionary learning, offering a novel methodology for teaching recommitment methods. Empirical evidence indicates that the proposed methodology can yield dependable and effective strategies, ensuring a high level of investment while mitigating the risk of overinvestment. However, the approach may also present certain disadvantages, such as the potential complexity and difficulty in accurately modeling the dynamic nature of PE investments. An examination of the changes in service-line provision at hospitals purchased by PE was conducted in [13]. In contrast to hospitals that were not acquired, PE acquisitions were associated with an increased likelihood of introducing specific profitable hospital-based services (such as hemodialysis, labor and delivery, and robotic cardiac catheterization), as well as profitable technologies (like digital mammography and robotic surgery), and freestanding or satellite emergency rooms. Furthermore, the analysis revealed a heightened probability of PE acquisition in cases where the provision of care was previously deemed unprofitable but has recently demonstrated financial viability, such as in the context of mental health services. Authors in [14] examined the strategies employed by commercial funders of construction projects to ensure the security of their investment returns. To this end, the researchers conducted semi-structured interviews with construction bankers and their consultants to explore their qualitative perspectives on infrastructure investment. The study's findings, derived from a range of diversified property and risk-mitigated portfolio approaches for building investments, underscore nine control mechanisms employed by financiers. These mechanisms, as highlighted in the study, are found to be contingent on governance related to the project's environment, relationships, knowledge, and expertise. A potential limitation of the study is its restricted generalizability, attributable to the qualitative nature of the research method and its exclusive focus on construction bankers and their consultants.

II. INVESTMENT PORTFOLIO ALLOCATION AND PRIVATE EQUITY FUNDS IN SECONDARY MARKETS

The methodology delineated in the provided text/current paper comprises several crucial components. Firstly, it details the institutional framework underlying the PE fund transactions, emphasizing the role of LPs and General Partners (GPs), as well as the mechanics of the secondary market for transferring fund stakes. The employed dataset, which is of a London-based financial intermediary provenance, is described with a focus being placed on the buyout funds in Western Europe and North America. The subsequent analysis delves into the behavior of LPs, introducing a model of LP preferences and liquidity shocks. It also explores the functioning of the secondary market for PE claims, considering different investor types and liquidity states. The analysis further presents a method for classifying funds based on characteristics, such as age, size, and region of focus. The empirical model of bid arrivals is outlined, employing Poisson distributions and parametric functions to estimate the bid rates. The text then goes on to discuss portfolio management strategies, including the construction of diversified portfolios with private market assets and the simulation of investment behaviors. The text then moves on to address the measurement of portfolio performance, taking into account the challenges posed by appraisal-based NAVs and the smoothing effect on returns. It proposes a method to estimate true economic returns. The methodology integrates theoretical frameworks, empirical analysis, and simulation techniques to explore various aspects of PE fund transactions and portfolio management.

A. Data

The present study employed a secondary dataset that was established using two conventional asset classes: U.S. equity (S&P 500 Composite – Total Return Index) and U.S. government bonds (JPM United States Govt. Bond – Total Return Index). The study incorporated five non-traditional forms of asset management, including asset-backed securities (IBOXX Coll. ABS – Total Return Index), buyout funds (CepreX US Buyout), and commodities (S&P G). The time series data for the commodities, ranging from January 1998 to July 2006, were corrected for potential sources of bias, such as survivorship bias. To this end, monthly mean hedge fund returns were lowered by 2. The mean returns of the S&P 500 were increased by 5% per year to reflect historical evidence, and the autocorrelations in hedge fund and transaction-based indices were tested and handled by Ljung-Box portmanteau. These corrections once again provided the mean return of 2.81% and the standard deviation of 8.79% for buyout funds, while for VC funds it was 2.01%. This renders the data reliable for the purpose of studying the performance and characteristics of the assets. Comparable approaches to the conditioning of dataset biases have been employed in previous studies to enhance the reliability of the analysis of the performance of alternative assets [14, 15].

B. Limited Partners

The three periods of the model are denoted by the notation $t = 0, 1, 2$. The economic environment is populated by two distinct categories of individuals: executives, also known as GPs, who possess the capacity to engage in long-term initiatives but are constrained by their financial resources, and investors, also referred to as LPs, who possess substantial financial assets. GPs seek funding from LPs, leveraging their investment acumen. However, fluctuations in liquidity can influence the propensity of LPs to contribute cash to long-term projects. The total number of LPs, denoted by N, consumes during periods 1 and 2 and is risk-neutral. At time 0, each LP possesses one unit of cash, accessible at $s = 0$, with no risk. In time 1, LPs may encounter a sudden increase in liquidity, which would raise their discounted rate. The mathematical values of an LP with probability λ of experiencing a liquidity shock are:

$$v(d_1, d_2) = d_1 + \bar{\delta}_{d_2}, where\ \bar{\delta} = \begin{cases} 1\ (1-\lambda) \\ 0 \quad \lambda \end{cases} \qquad (1)$$

where $d_t$represents utilization during time t.

The rate at which a liquidity-preferring LP will reduce its date two pay-outs' date is contingent upon the probability of a sudden increase in liquidity (λ):

$$\underline{s}(\lambda) = \frac{1}{1-\lambda} - 1 \qquad (2)$$

C. Fund Classification

It has been observed that certain funds within the specified dataset receive a considerable proportion of bids, while other funds with comparable attributes and dimensions acquire a limited number or none at specific periods. Researchers have devised a methodology to ensure that the demand for each category of funds adheres to a Poisson distribution. This is achieved by calculating the monthly bid volume across all funds with analogous attributes and subsequently summing them. When executed in an appropriate manner, this categorization of funds into classes encompasses all funds that are homogeneous from the perspective of the bidder, in addition to the tractability of a Poisson distribution in the estimation of a large demand system. To identify the categorization process, researchers employ a statistical method. The probability of a fund with a particular set of characteristics receiving at least one bid in a given quarter is estimated through a piece-wise Logit modeling approach. The results of this study indicate that: fund age, fund size, and area of investment emphasis are the three features that stand out and fund growth and fund duration have non-linear impacts.

D. An Empirical Model of Bid Arrivals

1) Demand Model

For each month t ranging from 1 to 88, let y_{jt} represent the total number of bids placed for funds of category j ranging from 1 to 24. Assuming that y_{jt} follows a Poisson distribution, the study finds that the average monthly number of bids, denoted by λ_{jt}, is contingent on an array of time-dependent, type-specific controlling factors, Z_{jt}, such as the typical historical achievement of funds of kind j, and a group of time-dependent visible variables, M_t, which are prevalent to every one of the funds. Given the restriction that the number of bids must be nonnegative and that the monthly bid distribution of each investment type is substantially skewed, the Poisson assumption is logical. Furthermore, the total number of bids for each account in a collection, or the total number of individual arrivals, will also exhibit a Poisson distribution if the total number of bids across all funds in the group is considered, given the mean arrival percentage. This is true even if the Poisson expectation regarding the emergence of bids appears more probable for a single fund compared to a collection of funds. The conditioned density of y_{jt} is:

$$f(y_{jt}|t, M_t, Z_{jt}, 1_j) \equiv Qs(Y_{jt} = y_{jt}|M_t, Z_{jt}, 1_j) = \frac{\lambda_{jt}^{y_{jt}} \exp(-\lambda_{jt})}{y_{jt}!} \qquad (3)$$

In the context of the fund's category, $j = 1,\ldots,9$, an array of binary parameters, such as Small (yes/no), Old (yes/no), US (yes/no), etc., is represented by 1_j. The bid rates for arrival:

$$\lambda_{jt} = \exp\left(\tau t + M_t'\beta_j^M + Z_{jt}'\beta_j^Z + \sum_{d=1}^{9} 1\{d = j\}\gamma_d\right) \forall j, t \qquad (4)$$

The exponential function guarantees that the number of arrivals is positive. It is important to note that β_j^Z is type-specific, thereby allowing for a demand for different types of funds to differently respond to macroeconomic shocks. The parameter τ is a common time trend, and the parameters γ are fixed effects. Assuming that the bids across the fund types are independently distributed conditional on M_t and Z_{jt}, then the log-likelihood function for the number of bids y_{jt} for each fund type in each month is as follows: consequently, the demands are estimated jointly as a system using the full panel rather than as independent time-series. Substantial arrivals are made certain by the exponential algorithm. Consequently, rather than being determined as separate time-series, the requirements are collectively calculated as a system employing the entire panel. The most probable likelihood estimation of β will be determined to be unique under the condition that the Poisson arrival percentage of bids, λ, is an exponential expression of its determinant M_t and its variables β.

E. Management of Portfolios

The study's objective is to develop portfolios for simulation studies, with the aim of allocating to three distinct categories: United States fixed-income funds, United States privatized markets funding, and United States public equity funds. A standard portfolio is established, exclusively comprising public equities, with an intended allocation of sixty percent towards PE and forty percent towards publicly fixed-income assets. To maintain this sixty-percent to forty-percent split, an investor restores equilibrium to the portfolio at the conclusion of each quarter. In instances where an investor is disproportionately equipped or underequipped in fixed-income assets, they are required to liquidate these assets and allocate the proceeds into public equity fund stocks. The primary study operates under the assumption that an investor allocates twenty percent of their portfolio, equivalent to one-third of their equity allocation, to private fund assets. Accordingly, the objective allocations in the aforementioned portfolio are as follows: twenty percent is allocated to private investment funds, forty percent is allocated to publicly fixed-income assets, and forty percent is allocated to public stocks. Given the presumed insolvency associated with private funds, claims in public funds may be transferred in resetting to conduct the studies using them. Consequently, at the conclusion of each quarter, the portfolio undergoes a rebalancing process, with forty percent allocated to publicly fixed-income investments and sixty percent to private funds and public equities. The remaining percentage is determined by the valuation of private funds. The study results in annual contributions at the start of each subsequent year. To articulate this in a different manner, the objective of the investors is to attain a constant state "40-40-20" targeted distribution, signifying that twenty percent of the entire portfolio is allocated to private funds. The remaining twenty percent is allocated arbitrarily, yet it aligns with the typical allocations made by several sizable financial institutions, including the public pension systems in the United States. Given the inherently unpredictable nature of capital decisions, payouts, and net asset values in the private fund sector, it is impractical to adhere to a fixed twenty percent allocation to private investments. The necessity of investing in a timely manner constitutes the primary factor contributing to the initial deviation from the intended allocations, as it necessitates a certain time period to accumulate a private fund portfolio. The study examines two distinct stages of the portfolios, which are manually produced by the arranged obligations and unique cash flow characteristics of the investments. The initial stage, designated as the "ramp-up" phase, is characterized by the gradual accumulation of assets, while the subsequent stage, referred to as the "steady-state" phase, is marked by the maintenance of a consistent asset allocation, typically equivalent to the capital requirements for new fund obligations. During this period, the investor's primary objective is to augment the allocated capital until it attains the desired level. However, as subsequently discussed, the investor may also aspire to continue diversifying over the course of the subsequent years. A substantial portion of the research was predicated on the assumption that the ramp-up period transpired from 1987 to 1996. The subsequent analysis will examine the effectiveness of the advanced stage, which transpired from 1997 to 2018. While a private fund's typical investment horizon is five years, it is important to note that not all of the allocated capital is necessarily spent in the initial year. As a result, it may require a longer period for the portfolio to approach a target allocation. It is reasonable to conclude that a steady-state portfolio with diversification throughout the historical years will annually allocate approximately 25% of its target allocation. The study made the assumption that the investor allocates an equivalent amount on an annual basis. Furthermore, researchers presuppose that at the beginning of each historical decade, the investor makes an arbitrary selection of private market funds and recognizes the existence of and has accessibility to the complete array of these funds. It is reasonable to presume that investors frequently know which GPs will be collecting fresh funds in the upcoming year, given the normal solicitation timeframe for ther private funds. However, it should be noted that not all investors possess equivalent levels of access to capital. The robustness study examined the impact of certain restrictions, such as the fact that high-performing VC funds are practically inaccessible to investors who have not made investments in prior funds. The degree of total private fund diversity is determined by the number of funds an investor decides to make a commitment to in every historical year. The majority of the study's findings are centered on commitments to ten funds during each period. However, the researchers also examine the impact of contributing to varying amounts each year in the robustness assessment. The design of the study is further evidenced in practice by the existence of commitment levels, whether minimal or excessive, for individual funds. These thresholds often correspond to the total assets of the fund in question. However, it is uncertain whether an investor would employ an equal-weight commitment approach, given the inherent vintage variation in private fund allocations. Consequently, researchers have postulated that there are no constraints on the actual fund contribution amounts and that the shareholder value-weights are allocated across funds.

1) Assessing the Effectiveness of a Portfolio

The present study aims to measure the risks and profitability features of portfolios that contain private markets assets. Additionally, it seeks to explore both the cross-sectional and time-series data variance in the allocations of diverse portfolios. Despite the preference among investors for higher returns adjusted for risk, the absence of market pricing for private funds complicates the estimation of returns and risk using periodic portfolio performance. Consequently, the majority of investors rely on the NAV of private market funds, which are provided by GP on a quarterly basis. Significant reporting distortions by general practitioners might affect the appraisal-based value. A widely recognized observation is that private fund NAVs are significantly reduced in comparison to general market prices of comparable assets. This is also crucial to the research. Consequently, estimates of the PE fund variability according to revenues and NAV movements are substantially lower compared to similar publicly available assets. Consequently, the outcomes of portfolio investments are filtered out and presented. Sharpe ratios, the metric of choice for quantifying risk-adjusted returns, exhibit a propensity to increase in portfolios that include private fund allocations. The time-series data of portfolio returns exhibit autocorrelation due to the distortion of underlying returns caused by appraisal-based NAVs. The present study employs a methodological approach that deploys a technique to obtain estimations of authentic, non-auto-correlated financial benefits. According to the approach, a weighted average of the actual returns for the last p+1 periods, which are the observable returns at time t, is:

$$S_t^{oar} = \sum_{j=0}^{p} \theta_j S_{t-j}^{true} \quad (5)$$

In this case, the process of loading θ_j indicates the amount of data incorporated into the measured return S_t^{oar} from the initial return S_{t-j}^{true}. It is implied that every variable affecting the portfolio growth throughout period t is completely represented in the observable return, however it can extend a maximum of p+1 intervals, by incorporating the extra limitation that the weightings equal to one, i.e. $\sum_{j=0}^{p} \theta_j S_{t-j}^{true} = 1$. Therefore, the situation without any smoothing is represented by θ_j =1. The study uses the Advanced Information Criteria for determining the number of MA-lags, yet it does not assume any particular eliminating characteristic. The study was initiated within the last year, allowing for a maximum duration latency of three sections, which is adequate to provide results with minimal autocorrelation.

III. RESULTS AND DISCUSSION

The study employs a multifaceted approach to comprehending PE fund transactions. It commences with an examination of the institutional framework encompassing LPs and GPs, accompanied by an analysis of the mechanics of secondary market operations for fund stake transfers. The dataset, procured from a London-based financial intermediary, focuses on buyout funds in Western Europe and North America. The analysis delves into the behavior of LPs, incorporating a model of preferences and liquidity shocks, while also examining secondary market dynamics across various investor types and liquidity conditions. The methodology introduced in this study involves a fund classification method based on age, size, and regional focus. This is followed by an empirical model that uses Poisson distributions and parametric functions to estimate bid arrivals. The text goes on to discuss portfolio management strategies, including diversified portfolio construction and investment behavior simulation, as well as the challenges in portfolio performance measurement, notably the impact of appraisal-based NAVs and return smoothing. Figure 1 is composed of four panels. Each panel presents the mean values for different metrics across various bidder types. The data from panel A and panel B are compared with data from the Preqin cash flow. The initial panel presents the mean bid placed by bidder type, thereby demonstrating the disparities in bidding behavior across both panels. The second panel shows the fraction of US funds by bidder type, highlighting any disparities in the geographical distribution of funds across bidder types. The bottom row of Figure 1 contains two more panels. The third of these displays the mean fund size by bidder type, providing insights into the average size of funds associated with each bidder type in panels A and B. Finally, the fourth panel presents the mean fund age by bidder type, indicating the average age of funds associated with different bidder types in both panels. By juxtaposing data from panels A and B, a comparative analysis of the bidder characteristics and their relationship with key metrics across the two panels is facilitated, allowing for a deeper understanding of the bidder behavior and fund attributes in the context of PE transactions.

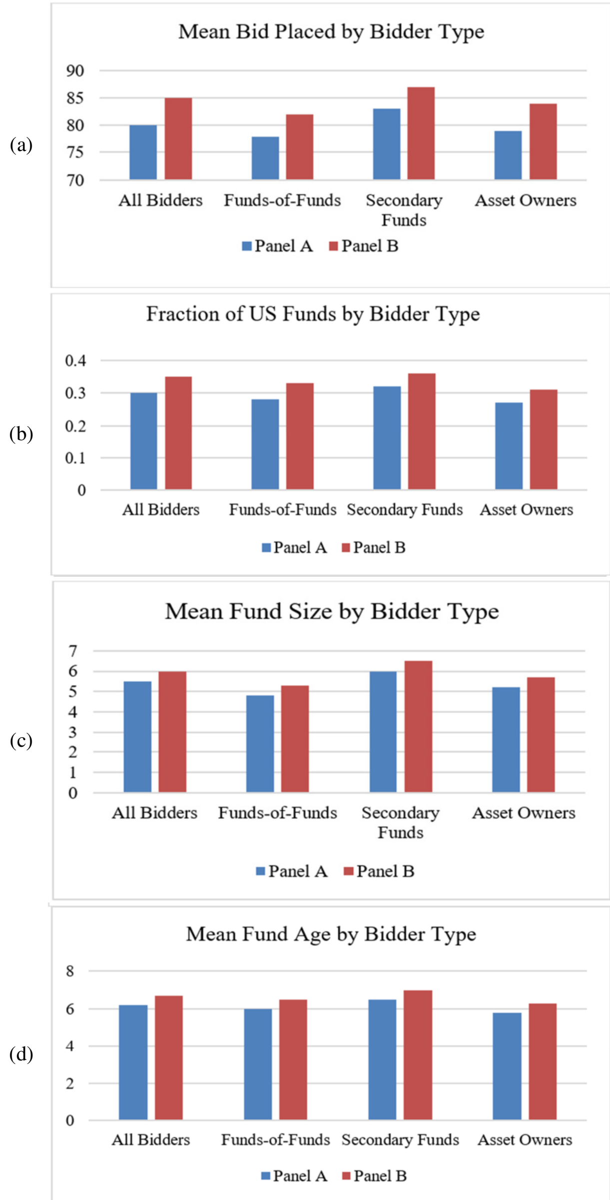

Fig. 1. Mean values for different metrics across various bidder types: (a) mean bid placed by bidder type, (b) fraction of US funds by bidder type, (c) mean fund size by bidder type, (d) mean fund age by bidder type.

As displayed in Figure 2, the mean allocation percentage for distinct asset categories offers insights into the distribution of investment across various categories. Each bar represents an asset class, including equity, fixed income, real estate, commodities, and other. The height of each bar corresponds to the average allocation percentage for the respective asset class. The data are presented visually in the form of a bar chart, which allows for clear interpretation of the allocation percentages. The bar charts are further enhanced by the presentation of the actual percentage values at the summit of each bar, hence facilitating a clear interpretation of the allocation figures. The *y*-axis of the bar chart represents the average allocation percentage, while the *x*-axis denotes the different asset classes.

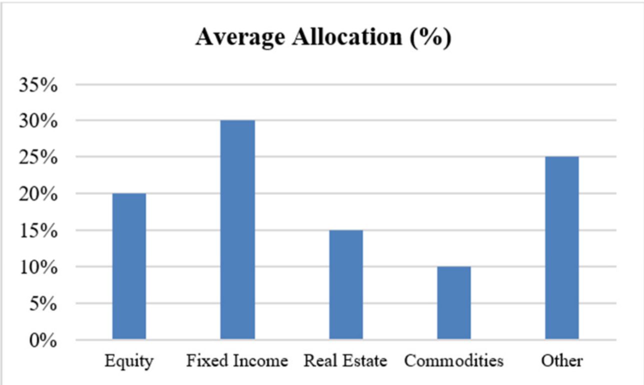

Fig. 2. Average allocation by asset class.

In Figure 3, the distribution of the allocation to a PE fund over the span of several years is represented, thereby providing insights into the evolution of investment in this fund over time. The data are presented in the form of a line graph, with each data point denoting the allocation percentage for a specific year. The *x*-axis corresponds to the years from 2010 to 2016, while the *y*-axis represents the allocation percentage, ranging from 0% to 30%. To enhance the clarity and visibility of the data, each data point is demarcated by a blue circle, and a continuous blue line connects these points to illustrate the trend in allocation over the years.

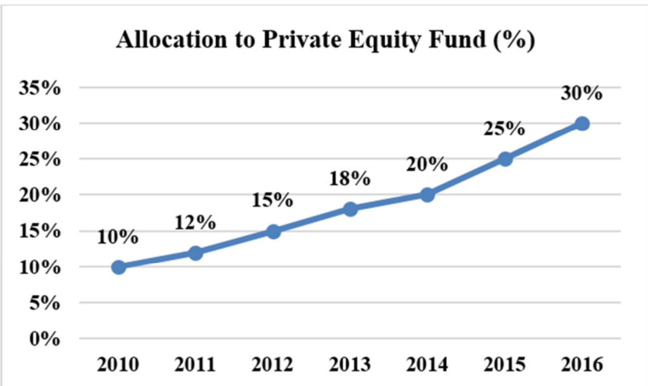

Fig. 3. Distribution of allocation to PE fund over years.

In Figure 4, the return distribution of a diversified portfolio is depicted, with the frequency of the mean returns across a range of values being shown. The *x*-axis corresponds to the mean returns in percentage format, ranging from 6.0% to 9.0%, with tick marks at 0.5% intervals. Each bin on the histogram represents a specific range of mean returns, thus allowing for the visualization of how frequently different return levels occur within the portfolio. The *y*-axis denotes the frequency of occurrence for each bin, indicating the number of funds in the portfolio exhibiting mean returns falling within the corresponding range. To enhance visual clarity, the histogram bars are displayed with a transparency of 0.7, while the edge color of the bars is set to black for enhanced delineation.

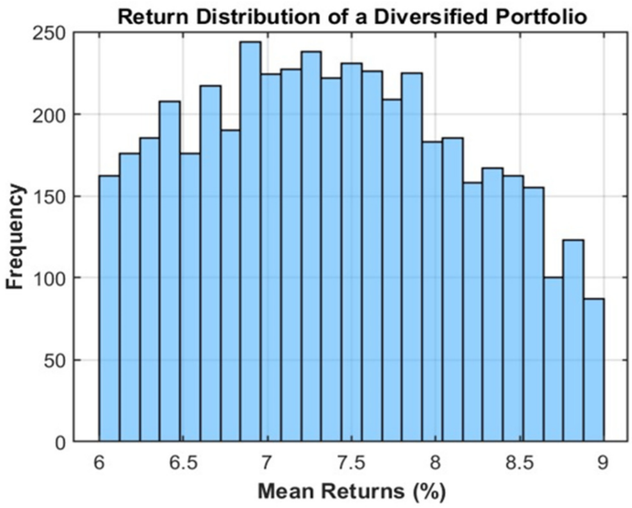

Fig. 4. Return distribution of a diversified portfolio.

As portrayed in Figure 5, the return distribution of a portfolio diversified across all fund's ranges from a mean return of 6.0% to 9.0%. Each bar on the plot represents a specific mean return value, spanning from 6.0% to 9.0% in increments of 0.5%. The height of each bar corresponds to the frequency of occurrence for the respective mean return, as indicated on the *y*-axis. For instance, the bar representing a mean return of 8.0% is the tallest, with a frequency of 110. This indicates that 110 funds in the portfolio exhibit a mean return of 8.0%. The bars positioned to the left and right of the central bar represent the frequencies for mean returns of 6.0% and 9.0%, respectively, with the bars becoming smaller as the mean returns deviate from the central value. The *x*-axis, which denotes the mean returns in a percentage format, facilitates the interpretation of the return distribution. In Figure 6, the median quarter of the maximum NAV per vintage is shown for different types of funds, specifically buyout, VC, and real estate funds, across four vintage quarters denoted as Q1, Q2, Q3, and Q4. Each line on the plot corresponds to a specific fund type, with buyout funds being represented by the color blue, VC funds by green, and real estate funds by red. The *x*-axis denotes the vintage quarters, therefore facilitating the identification of the respective time periods. The *y*-axis, on the other hand, represents the median NAV values. The markers, which are circles for buyout, squares for VC, and triangles for real estate, highlight the data points for each fund type within each vintage quarter.

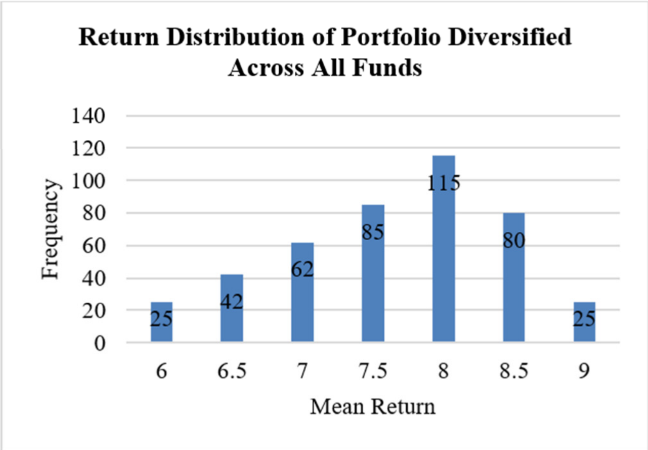

Fig. 5. Return distribution of portfolio diversified across all funds.

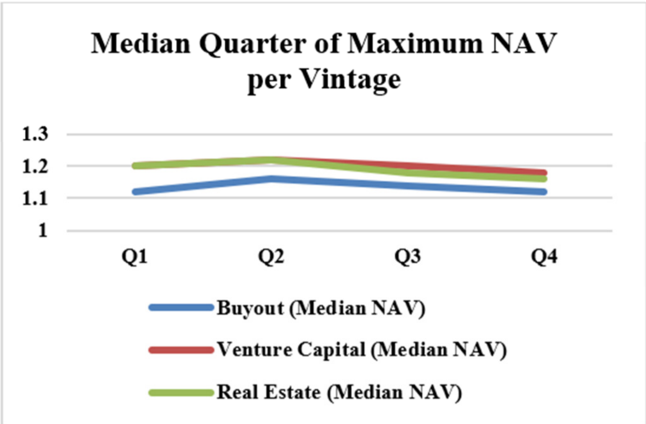

Fig. 6. Median quarter of maximum NAV per vintage.

IV. CONCLUSIONS AND FUTURE WORKS

The bid arrival modeling and other secondary market characteristics of Product Environmental Footprint (PEF) transactions were extended, and efficient notions of portfolio management were incorporated to enhance the current knowledge about transactions. Using a dataset obtained from Carbury Limited, which encompasses 101 buyout funds from Western Europe and North America, the study proposes a novel Limited Partner (LP) classification system based on time horizon, fund size, and geographical area. The paper provides a comprehensive analysis of the LP behavior under constrained and unconstrained liquidity environments. The study's primary findings address the intricacies of bid arrivals, the ramifications of appraisal-based Net Asset Value (NAV) smoothing, and the incorporation of Private Equity (PE) into diversified portfolios. The subsequent studies should overcome the limitations of geographic and sectoral coverage and richness by employing additional cross-sectional data and by extending the scope of the study to other emerging markets and non-traditional forms of funds so as to facilitate the generalization of these insights to other PE markets. This could expand the knowledge on bid dynamics and investors' behavior as well as the performance benchmarks to aid strategic decisions in this highly speculative area of investment. Future research endeavors could address this limitation by incorporating datasets covering a more extensive range of geographic regions and industry sectors, allowing for a more comprehensive understanding of the global PE market dynamics and investor behavior. By expanding the scope of data analysis, future studies can offer more robust insights into the bid dynamics, portfolio management strategies, and performance evaluation metrics, enhancing individuals' understanding of the PE landscape and promoting more informed investment decisions.

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