

# Research on the Optimal Strategy of Supply Chain System Considering Carbon Emission Reduction Level

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**Abstract:** In response to the decision-making problem of a secondary supply chain system composed of manufacturers and retailers, the optimal strategy of a master-slave game was studied, considering carbon emissions reduction. We compared and analyzed the differences in optimal strategies with and without coordination mechanisms, and explored the impact of sensitivity on the optimal strategy. Research has shown that the optimal profit of manufacturers and retailers under coordination mechanisms is greater than that without coordination mechanisms; Wholesale prices, carbon reduction levels, retail prices, and manufacturer profits are all negatively correlated with price sensitivity and positively correlated with carbon reduction sensitivity, while retailer profits are the opposite; There is an optimal cost sharing ratio that maximizes retailer profits.

**Keywords:** Supply chain system, Carbon reduction, Optimal strategy, Game.

## 1. Introduction

With the proposal of carbon peaking and carbon neutrality strategies, the issue of supply chain carbon reduction has attracted the attention of scholars. A large number of scholars considered the impact of consumers' environmental awareness and low-carbon preference on the degree of carbon emission reduction and decision-making in the supply chain. Liu et al. [1] discussed the impact of carbon emission reduction cost sharing on the profit of the supply chain based on the carbon emission trading mechanism and consumers' low-carbon preference. Research shows that the increase of consumers' preference for low-carbon products can improve the income of supply chain enterprises, retailers have seen an increase in order quantity and profits. Han et al. [2] respectively established a Stackelberg game model under centralized decision-making, decentralized decision-making and revenue sharing contracts, and studied the impact of consumer environmental awareness and carbon emission reduction costs on the emission reduction efforts of upstream manufacturers in the supply chain under the carbon tax policies. The study found that enhancing consumer environmental awareness is conducive to improving the total revenue of the supply chain, while increasing carbon emission reduction costs will damage the profits of supply chain members. Zhang and Qin [3] studied the cooperative strategy of carbon reduction among supply chain members in a three-level supply chain consisting of a manufacturer, a transportation enterprise, and a retailer, considering carbon limit trading policies and consumer environmental awareness. The proposed cost sharing contract can achieve Pareto improvement, while the two-part pricing contract can achieve Pareto optimization. On this basis, they also studied the pricing, carbon emission reduction and social welfare decisions of the supply chain system, considering transport carbon emissions and social welfare factors under the carbon tax policy, and proposed a cross ownership contract for coordination and optimization[4].

In addition, scholars have also conducted research on the coordination and optimization of low-carbon supply chains. Yin and Liu [5] explored the impact of carbon emission

trading policies on carbon emission reduction decisions and coordination strategies in the closed-loop supply chain of new energy vehicles in the three-level supply chain participated by recyclers, and designed a revenue cost sharing contract based on the decentralized decision-making model. The results show that the contract can achieve complete coordination of the supply chain by reasonably setting the revenue and cost allocation ratio. Zhu et al. [6] explored the impact of retailers' horizontal and vertical fairness awareness on carbon reduction and pricing decisions in dual channel supply chains, taking into account consumer channel preferences, based on carbon tax policies. They utilized two pricing contracts to achieve a win-win situation for supply chain members.

In summary, scholars have achieved rich research results on consumer low-carbon preferences, carbon emissions reduction, supply chain decision-making and coordination, which lays a solid foundation for this study. On the basis of existing research, this paper compares and analyzes the most effective strategies with and without coordination mechanisms, and provides an analytical formula for the most effective strategies; On the other hand, the impact of price sensitivity and carbon emission reduction level sensitivity on the optimal strategy was explored through numerical simulation.

## 2. The Model

This paper studies a secondary supply chain system consisting of a manufacturer and a retailer. Among them, the manufacturer is responsible for the product, the production cost per unit product is  $c$ , the carbon emission reduction level is  $e$ , the product is sold wholesale to retailers at price  $w$ , the retail price is  $p$ , and the market demand for the product is  $q$ .

Based on the above description, the following assumptions are made for the model:

(1) The higher the level of carbon reduction, the higher the degree of low-carbon of the product; In markets where consumers have a low-carbon preference, the product demand is positively correlated with the level of carbon reduction,  $e > 0$ .

(2) This paper only considers the one-time carbon reduction costs invested by manufacturers (such as production process improvement, etc.), which are not related to the quantity of products. The cost of achieving carbon emission reduction  $e$  is  $\gamma e^2/2$ , and  $\gamma$  is the cost coefficient,  $\gamma > 0$ .

(3) To ensure that manufacturer and retailer are profitable, it is necessary to meet  $p > w > c > 0$ .

(4) This paper studies the Stackelberg game model, in which the manufacturer as the leader, first decides the wholesale price  $w$  and carbon emission reduction level  $e$ ; Retailer as follower, decides the retail price  $p$  last.

Based on the above description, the market demand function of the product is:

$$q = a - \alpha p + \beta e \quad (1)$$

where  $a$  is the potential maximum demand for the product,  $\alpha$  is the sensitivity coefficient of demand to price, and  $\beta$  is the sensitivity coefficient of demand to carbon emission reduction level.

### 3. Model Analysis

Suppose that both manufacturers and retailers are perfect rationality decision makers, and they can obtain all the information needed for decision-making. Manufacturers need to bear high carbon reduction costs in order to improve the carbon reduction level of their products. This paper considers two scenarios: there is no coordination mechanism, where the manufacturer bears the cost of carbon reduction alone, represented by subscript 1. There is a coordination mechanism, where retailers share the cost of carbon reduction, represented by subscript 2. In addition, the superscript \* represents the optimal strategy, and  $\pi_m$  and  $\pi_r$  represent the profits of manufacturers and retailers, respectively.

#### 3.1. Manufacturers bear the cost of carbon reduction alone

Under this uncoordinated mechanism, the profit functions of manufacturers and retailers are:

$$\pi_{m1}(w_1, e_1) = (w_1 - c)q - \frac{1}{2}\gamma e_1^2 \quad (2)$$

$$\pi_{r1}(p_1) = (p_1 - w_1)q \quad (3)$$

**Proposition 1:** When  $\beta^2 - 4\alpha\gamma < 0$ , there are optimal strategies  $w_1^*$ ,  $e_1^*$ , and  $p_1^*$  that maximize both the manufacturer's and retailer's profits. where

$$w_1^* = \frac{c\beta^2 - 2\alpha\gamma - 2c\alpha\gamma}{\beta^2 - 4\alpha\gamma}, \quad e_1^* = \frac{c\alpha\beta - a\beta}{\beta^2 - 4\alpha\gamma},$$

$$p_1^* = \frac{c\beta^2 - 3\alpha\gamma - c\alpha\gamma}{\beta^2 - 4\alpha\gamma}$$

Based on the reverse solution method, first, according to the profit function Eq. (3) of the retailer, the optimal retail

price  $p_1(w_1, e_1)$  is obtained when ensuring  $\pi_{r1}$  maximum. Secondly, bring it into the manufacturer's profit function Eq. (2), and obtain  $w_1^*$  and  $e_1^*$  when ensuring  $\pi_{m1}$  maximum and condition (I):  $\beta^2 - 4\alpha\gamma < 0$  is met. Finally, bring both into  $p_1(w_1, e_1)$  to obtain  $p_1^*$ . Based on  $w_1^*$ ,  $e_1^*$  and  $p_1^*$ , the optimal profit is obtained as follows:

$$\pi_{m1}^* = -\frac{(a - c\alpha)^2 \gamma}{2(\beta^2 - 4\alpha\gamma)}, \quad \pi_{r1}^* = \frac{\alpha(a - c\alpha)^2 \gamma^2}{(\beta^2 - 4\alpha\gamma)^2}$$

where manufacturers and retailers must meet condition (II)  $a \neq c\alpha$  to ensure  $\pi_{m1}^* > 0$  and  $\pi_{r1}^* > 0$  in order to profit.

#### 3.2. Retailers share the cost of carbon reduction

Under this coordination mechanism, retailers share the cost of carbon reduction, the sharing ratio is  $\mu$ ,  $\mu \in (0, 1]$ . At this point, the retailer, as a follower, decides on the retail price  $c$  and the sharing ratio  $\mu$ . The profit functions of manufacturers and retailers are:

$$\pi_{m2}(w_2, e_2) = (w_2 - c)q - (1 - \mu)\frac{1}{2}\gamma e_2^2 \quad (4)$$

$$\pi_{r2}(p_2, \mu) = (p_2 - w_2)q - \mu\frac{1}{2}\gamma e_2^2 \quad (5)$$

**Proposition 2:** When  $3\beta^2 - 8\alpha\gamma < 0$ , there are optimal strategies  $w_2^*$ ,  $e_2^*$ ,  $p_2^*$  and  $\mu^*$  that maximize the profits of manufacturers and retailers. where

$$w_2^* = \frac{(a + 5c\alpha)\beta^2 - 8\alpha(a + c\alpha)\gamma}{2\alpha(3\beta^2 - 8\alpha\gamma)},$$

$$e_2^* = -\frac{2(a - c\alpha)\beta}{3\beta^2 - 8\alpha\gamma}, \quad \mu^* = \frac{\beta^2}{8\alpha\gamma},$$

$$p_2^* = \frac{3(a + 3c\alpha)\beta^2 - 8\alpha(3a + c\alpha)\gamma}{4\alpha(3\beta^2 - 8\alpha\gamma)}$$

**Proof:** According to the reverse solution method, due to  $\frac{\partial^2 \pi_{r2}(p_2, \mu)}{\partial p_2^2} = -2\alpha < 0$ , there is an optimal retail price

that maximizes the retailer's profit. Let  $\frac{\partial \pi_{r2}(p_2, \mu)}{\partial p_2} = 0$

obtain:

$$p_2(w_2, e_2) = \frac{a + w_2\alpha + e_2\beta}{2\alpha} \quad (6)$$

By introducing Eq.(6) into  $\pi_{m2}$ , when condition (III):  $\beta^2 - 4\alpha\gamma(1 - \mu) < 0$  is satisfied, the Hesse matrix is negative definite, and there exists an optimal wholesale price and carbon reduction level that maximizes the manufacturer's

profits. Combining  $\frac{\partial \pi_{m2}(w_2, e_2)}{\partial w_2} = 0$  and

$\frac{\partial \pi_{m2}(w_2, e_2)}{\partial e_2} = 0$  yields:

$$w_2(\mu) = \frac{c(\beta^2 + 2\alpha\gamma(\mu-1)) + 2a\gamma(\mu-1)}{\beta^2 - 4\alpha\gamma(1-\mu)} \quad (7)$$

$$e_2(\mu) = -\frac{\beta(a-c\alpha)}{\beta^2 - 4\alpha\gamma(1-\mu)} \quad (8)$$

Bringing Eq.(7) and Eq.(8) into Eq.(6) yields:

$$p_2(\mu) = \frac{c(\beta^2 + \alpha\gamma(\mu-1)) + 3a\gamma(\mu-1)}{\beta^2 - 4\alpha\gamma(1-\mu)} \quad (9)$$

Bring Eqs.(7) - (9) into Eq.(5), and when condition (IV):  $8\alpha\gamma(1+2\mu) - 5\beta^2 > 0$  is met, there exists an optimal carbon reduction cost sharing ratio that maximizes retailer profits. The optimal sharing ratio is:

$$\mu^* = \frac{\beta^2}{8\alpha\gamma} \quad (10)$$

By bringing Eq. (10) into Eqs. (7) - (9), we can obtain:

$$p_2^* = \frac{3(a+3c\alpha)\beta^2 - 8\alpha(3a+c\alpha)\gamma}{4\alpha(3\beta^2 - 8\alpha\gamma)} \quad (11)$$

$$e_2^* = -\frac{2(a-c\alpha)\beta}{3\beta^2 - 8\alpha\gamma} \quad (12)$$

$$w_2^* = \frac{(a+5c\alpha)\beta^2 - 8\alpha(a+c\alpha)\gamma}{2\alpha(3\beta^2 - 8\alpha\gamma)} \quad (13)$$

Bring Eq. (10) into the above conditions (III) and (IV) to obtain condition (V):  $3\beta^2 - 8\alpha\gamma < 0$ .

Based on Eqs. (4)-(5) and Eqs.(10)-(13), the optimal profits for manufacturers and retailers are:

$$\pi_{m2}^* = \frac{(a-c\alpha)^2(\beta^2 - 8\alpha\gamma)}{8\alpha(3\beta^2 - 8\alpha\gamma)} \quad (14)$$

$$\pi_{r2}^* = -\frac{(a-c\alpha)^2(\beta^2 + 8\alpha\gamma)}{16\alpha(3\beta^2 - 8\alpha\gamma)} \quad (15)$$

### 3.3. Comparative analysis

According to propositions 1-2, the optimal strategy with or without coordination mechanism is shown in Table 1.

**Table 1.** Optimal strategies in two cases

|   | $w^*$  | $e^*$   | $p^*$  | $\mu^*$                         | $\pi_m^*$  | $\pi_r^*$  |
|---|--|---|--|---------------------------------|--|--|
| N | $\frac{c\beta^2 - 2a\gamma - 2c\alpha\gamma}{\beta^2 - 4\alpha\gamma}$                     | $\frac{c\alpha\beta - a\beta}{\beta^2 - 4\alpha\gamma}$ | $\frac{c\beta^2 - 3a\gamma - c\alpha\gamma}{\beta^2 - 4\alpha\gamma}$                        | —                               | $-\frac{(a-c\alpha)^2\gamma}{2(\beta^2 - 4\alpha\gamma)}$                          | $\frac{\alpha(a\gamma - c\alpha\gamma)^2}{(\beta^2 - 4\alpha\gamma)^2}$              |
| Y | $\frac{(a+5c\alpha)\beta^2 - 8\alpha(a+c\alpha)\gamma}{2\alpha(3\beta^2 - 8\alpha\gamma)}$ | $-\frac{2(a-c\alpha)\beta}{3\beta^2 - 8\alpha\gamma}$   | $\frac{3(a+3c\alpha)\beta^2 - 8\alpha(3a+c\alpha)\gamma}{4\alpha(3\beta^2 - 8\alpha\gamma)}$ | $\frac{\beta^2}{8\alpha\gamma}$ | $\frac{(a-c\alpha)^2(\beta^2 - 8\alpha\gamma)}{8\alpha(3\beta^2 - 8\alpha\gamma)}$ | $-\frac{(a-c\alpha)^2(\beta^2 + 8\alpha\gamma)}{16\alpha(3\beta^2 - 8\alpha\gamma)}$ |

Notes: N represents no coordination mechanism, Y represents coordination mechanism.

**Theorem 1:** When  $a > c\alpha$ ,  $p_1^* < p_2^*$ ,  $w_1^* < w_2^*$ ,  $e_1^* < e_2^*$ ; When  $a < c\alpha$ ,  $p_1^* > p_2^*$ ,  $w_1^* > w_2^*$ ,  $e_1^* > e_2^*$ .

The optimal strategy difference between the two scenarios can lead to theorem 1. Theorem 1 indicates that when the market demand for products is high, the optimal wholesale price, optimal retail price, and optimal carbon emission reduction level under the coordination mechanism are greater than those without the coordination mechanism. When the market demand for a product is low, the result is the opposite.

**Theorem 2:** When  $a \neq c\alpha$ ,  $\pi_{m1}^* < \pi_{m2}^*$ ,  $\pi_{r1}^* < \pi_{r2}^*$ .

From Proposition 1, it can be inferred that  $a \neq c\alpha$  and theorem 2 indicate that the optimal profits of both manufacturers and retailers under a coordination mechanism are greater than those without a coordination mechanism. From the perspective of manufacturers and retailers, they

prefer to achieve higher profits by establishing a cost sharing and coordination mechanism.

## 4. Numerical Simulation

In order to more intuitively demonstrate the impact of retail price sensitivity, carbon emission reduction level sensitivity, and coordination ratio on the optimal strategy, the parameter values are:  $a = 1$ ,  $\alpha = 0.8$ ,  $\beta = 0.6$ ,  $\gamma = 0.4$ ,  $c = 0.1$ , based on meeting the conditional assumptions, conditions (I), conditions (II), and conditions (V). At this point, there is  $a = 1 > c\alpha = 0.08$ . According to theorem 1 and theorem 2, the optimal strategies under the coordination mechanism is greater than those without the coordination mechanism. After calculation, the optimal strategies for the two cases are shown in Table 2.

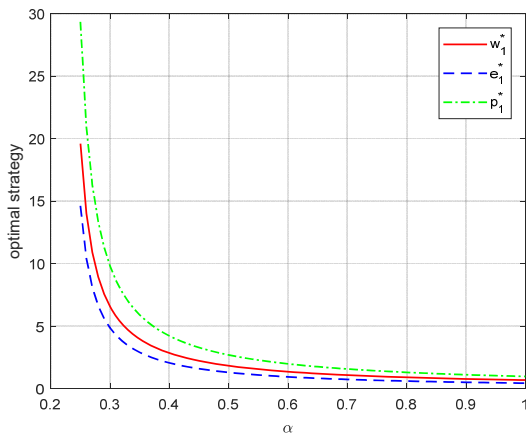
**Table 2.** Optimal strategy values for two cases

|   | $w^*$  | $e^*$  | $p^*$  | $\mu^*$ | $\pi_m^*$ | $\pi_r^*$ |
|---|--------|--------|--------|---------|-----------|-----------|
| N | 0.9000 | 0.6000 | 1.3000 | —       | 0.1840    | 0.1280    |
| Y | 0.9547 | 0.7459 | 1.3821 | 0.1406  | 0.1966    | 0.1305    |

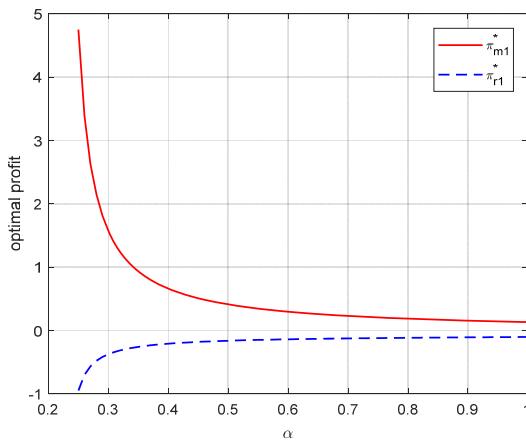
Notes: N represents no coordination mechanism, Y represents coordination mechanism.

#### 4.1. The impact of price sensitivity coefficient $\alpha$ on the optimal strategy without coordination mechanism

Calculate  $\alpha > 0.225$  from condition (I), where  $\alpha$  starts at 0.25. The impact of  $\alpha$  on the optimal strategy is shown in Figure 1. and Figure 2.



**Figure 1.** The impact of  $\alpha$  on the optimal prices and carbon reduction levels



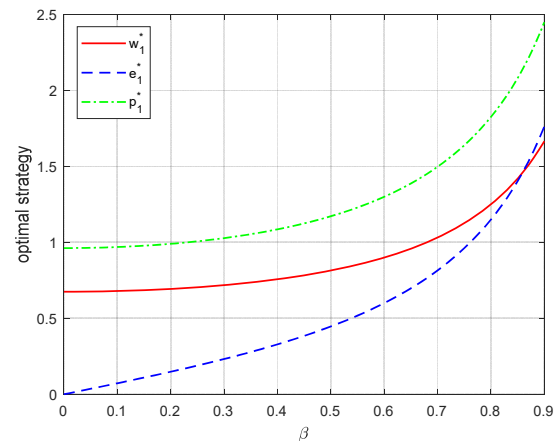
**Figure 2.** The impact of  $\alpha$  on the optimal profits for manufacturers and retailers

Figure 1 and Figure 2 indicate that as retail price sensitivity increases, wholesale prices, carbon reduction levels, and retail prices decrease, manufacturers' profits gradually decrease, retailers' profits gradually increase, and finally tend to be relatively stable. Therefore, retailers and consumers are more inclined towards high price sensitivity, while manufacturers are the opposite.

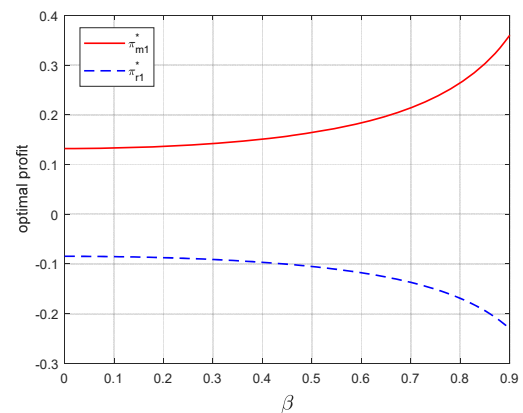
#### 4.2. The influence of sensitivity coefficient of carbon emission reduction level on the optimal strategy without coordination mechanism

According to condition (V), it can be seen that  $\beta < \sqrt{8\alpha\gamma/3} = 0.9238$ ,  $\beta \in [0, 0.9]$  is set here.

Figure 3 and Figure 4 indicate that product demand is more sensitive to carbon reduction levels, leading to significant increases in wholesale prices, carbon reduction levels, and retail prices. Manufacturers' profits gradually increase, while retailers' profits continue to decline. This indicates that the low-carbon market (where consumers prefer low-carbon products) is more advantageous for manufacturers, but consumers need to pay relatively high purchasing costs.



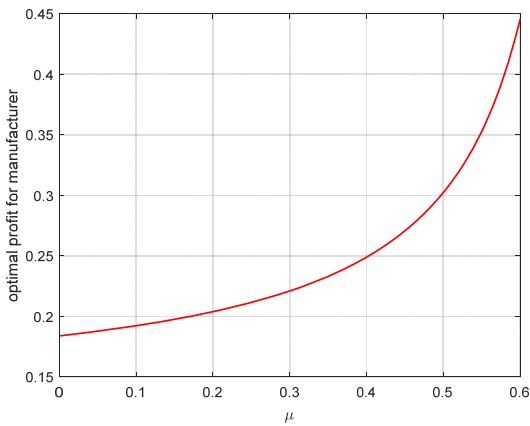
**Figure 3.** The impact of  $\beta$  on optimal prices and carbon reduction levels



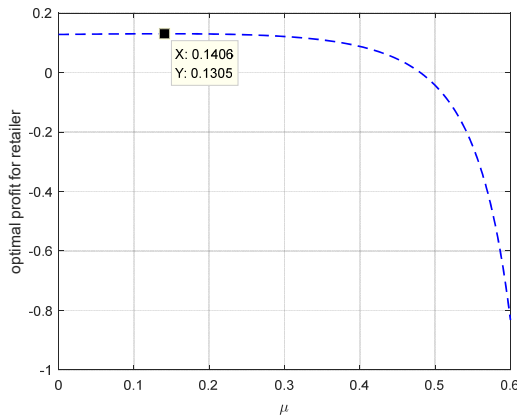
**Figure 4.** The impact of  $\beta$  on optimal profits for manufacturers and retailers

### 4.3. The influence of cost sharing coefficient on the optimal profits with coordination mechanism

For clearer graphical display, set  $\beta \in [0, 0.6]$  here.



**Figure 5.** The impact of  $\mu$  on the optimal profits for manufacturer



**Figure 6.** The impact of  $\mu$  on the optimal profits for retailer

Figure 5 shows that the higher the cost sharing ratio, the more profitable the manufacturer will be, but retailers cannot always increase the sharing ratio. According to theorem 2, cost sharing is beneficial for both manufacturers and retailers. Specifically, for retailers, there is an optimal sharing ratio that maximizes its profit, as shown in Figure 6. When  $\mu = 0.1406$ , the maximum profit for the retailer is 0.1305, while the profit for the manufacturer is 0.1966, which is consistent with Table 1.

## 5. Conclusions

On the basis of considering the level of carbon emission reduction, the optimal strategy of the secondary supply chain system is studied in depth. By constructing the Stackelberg game model, the optimal strategy and the analytical formula of the optimal carbon emission reduction cost sharing ratio

are given, ensuring the profit maximization of manufacturers and retailers. Research has shown that establishing a sharing and coordination mechanism for carbon emission reduction costs can help improve the profits of manufacturers and retailers. However, the optimal wholesale price, carbon reduction level, and retail price are influenced by the demand for products and are not related to the coordination mechanism. The sensitivity of demand to price is negatively correlated with manufacturers' profits, but positively correlated with retailers' profits. The sensitivity of demand to carbon reduction levels is positively correlated with manufacturers' profits, while negatively correlated with retailers' profits. There is an optimal cost sharing ratio for carbon reduction that enables manufacturers and retailers to achieve optimal profits.

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## Conflicts of Interest

The authors declare no conflict of interest.

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