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Pricing and Coordination of the Extended Warranty Service Supply Chain that Considers Fairness Concerns

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Abstract: For the product dual-channel supply chain composed of one manufacturer and one retailer with fairness concerns, considering that the decision of products and extended warranty services in the supply chain is affected by the fairness concern behavior of retailers, the Stackelberg game method was used to establish the profit decision model of manufacturers and retailers, solved the product price, extended warranty service price and profit of each member of the supply chain during centralized and decentralized decision-making, and discussed the impact of retailer fairness concern on the optimal decision and profit of supply chain members. The results showed that with the increase of retailers' fairness concern coefficient, the wholesale price of products and the price of extended warranty service decrease, and the price of direct sales channel and retail channel price of products increase. Retailers' profits increase, manufacturers' profits, and the total profits of supply chain systems decrease.

Keywords: Extended warranty service, Dual channels, Pricing, Supply chain coordination, Equity concerns.

1. Introduction

At present, the global Internet economy is developing at a high speed, network payment and physical distribution services are gradually popularised and improved, more and more manufacturing industries are opening up network channels for direct sales of products, and new sales channels are integrated with traditional retail channels to form a new type of sales model. At the same time, with the continuous development of society, the economic development trend has changed, from the product economy gradually tends to the service economy, China's earnings in the service industry continues to rise [1]. Companies have begun to look for new sources of profit by introducing extended warranty services, which are a type of paid service provided by manufacturers, retailers or third-party service providers that consumers can choose to purchase when they buy a product after its warranty period has expired [2]. On the one hand, consumers can save on high maintenance costs by having the service provider provide free repair services for them when the purchased product breaks down after purchasing the extended warranty service; on the other hand, the introduction of the extended warranty service is an extension of the basic warranty service, an extension of the emerging services in the industry chain, bringing new profit growth space for more and more enterprises. With the rapid development of large retail enterprises, retailers in the downstream of the supply chain have been increasing their status, selling products and at the same time holding a large amount of information on sales and customers, their role in the provision of services is becoming more and more obvious, and in order to earn more profits, they have started to provide services attached to their products, namely extended warranty services. For example, Gome, Suning and Jingdong have launched their own extended warranty services, with Gome offering consumers the "Anjia Package", Suning launching the "Sunshine Package" and Jingdong offering its own factory warranty extension through its platform.

The research on extended warranty services by domestic and international scholars mainly includes the selection of sales channels for extended warranty services, pricing of extended warranty services.

As for the research on the sales channels of extended warranty services in supply chains, one is the selection of supply chain extended warranty service channels without competition. Scholars have mainly studied three sales channels, namely manufacturer sales, retailer sales and third-party service agent sales, and considered the influence of different factors on the selection of extended warranty service channels. For example, Wang Sujuan and Hu Qiying [3] studied how manufacturers and retailers choose the appropriate extended warranty service channel model under the influence of the extended warranty service attractiveness index. Li, et al [4] considered the impact of extended warranty service time on the cost of extended warranty service. Mai, et al [5] considered the retailer's extended warranty service repair cost and the manufacturer's product quality. The second is the choice of sales channel for extended warranty services in a competitive supply chain. For example, Heese, et al [6] studied a supply chain consisting of two competing manufacturers and retailers, in which the retailers sold products and extended warranty services. Ma, et al [7] studied a product-competitive supply chain, i.e. competition between two manufacturers and competition between two retailers, with the manufacturer providing the extended warranty service, constructed an optimal decision model for the manufacturer and the retailer, and analysed the strategy for selecting the extended warranty service model.

On the pricing of extended warranty services, scholars at home and abroad have made a lot of research results. For example, Tan Deqing, et al [8] study the optimal pricing problem considering different situations of basic warranty and extended warranty services, and give the corresponding optimal pricing strategy. For example, Zhang, et al [9] study the impact of the cost of extended warranty services on the pricing of extended warranty services in a manufacturer-led supply chain. In addition, some scholars consider the impact of different factors on the pricing of extended warranty services. Gallego, et al [10] consider the customer risk aversion factor, Kurata and Nam [11] and Dan Bin, et al [12] consider the impact of extended warranty service level on the pricing of extended warranty services, and Lu Zhen and

Zhang Jian [13] and Kou Jun, et al [14] introduce product quality into the pricing study of extended warranty services.

In addition, traditional economics studies on supply chain decision making generally assume that the firm's decision makers are in a perfectly rational state. With the rise of behavioural economics, it has been found that real-life decision makers have equity concerns and also focus on the fairness of profit distribution [15]. Unlike previous studies, decision makers when considering equity concerns do not make the optimal decision of maximising their own profits, but the optimal decision of maximising their own utility. For example, Cui, et al [16] first considered fairness concerns when studying the decision making problem of supply chain members and found that fairness concerns would reduce the profit of the supply chain system and proposed a wholesale price contract. Zhang, et al [17] studied the effect of fairness concerns on the pricing of products and extended warranty services and profits of manufacturers and retailers when extended warranty services are sold by both manufacturers and retailers. Zhou Yan, et al [18] analyse the impact of retailers' fairness concerns and product greening efficiency on the optimal decisions and profits of manufacturers and retailers in a dual-channel supply chain. Han Tongyin, et al [19] consider the effects of both government subsidies and retailer equity concerns on optimal supply chain decisions and profits when studying dual-channel green supply chains.

The above research on extended warranty services provides ideas for this paper, but there are a number of issues that need further discussion. Most previous studies have considered single-channel product situations, where the manufacturer provides the product and the retailer sells it, and have not considered the impact of dual-channel products. In addition, retailers' equity concerns can affect pricing decisions for products and extended warranty services as well as the profitability of members in the supply chain. Based on the Stackelberg game approach, we develop a profit decision model for manufacturers and traditional retailers, and analyse the product and extended warranty service prices and profits of each supply chain member under the centralised and decentralised decision models.

2. Problem Description and Underlying Assumptions

2.1. Description of the problem

The product service supply chain system studied in this paper consists of manufacturers, retailers and consumers. The manufacturer has two main sales channels: one is to sell the products produced to consumers through direct sales channels; the other is to sell the products wholesale to retailers through wholesale channels. Retailers are mainly responsible for selling the wholesale products to consumers. and providing extended warranty services to consumers.

In the actual supply chain operation, the upstream manufacturer often receives more profit than the downstream retailer. Therefore, we assume that manufacturers are fair and neutral, making decisions about the wholesale price of their products and the price of their products in the direct sales channel to maximise their own profits. The retailer is not only concerned about its own profit, but also compares with the manufacturer's profit, thus creating unfairness, i.e. the retailer is assumed to have fair concern behaviour and decides its product retail channel price and extended warranty service price to maximise its own utility. Under the centralised and decentralised decision making models, the optimal decision for the extended warranty service supply chain considering fairness concerns and profit are obtained respectively. Tab. 1 represents the decision variables, product demand, extended warranty service demand and profit function, while Tab. 2 represents the specific parameters.

Table 1. Decision variables, product demand, demand for extended warranty services and profit

		Indicates meaning
Decision-making variables	w	Wholesale price per unit of product, and $w > 0$
	p_m	The price of the product in the direct sales channel, and $p_m > 0$
	p_r	Product retail channel price, and $p_r > 0$
	p_s	Unit extended warranty service prices, and $p_s > 0$
Demand functions	D_m	Direct channel product demand
	D_r	Retail channel product demand
	D_p	Total product demand
	D_s	Demand for extended warranty services
Profit function	π_m	Manufacturer's profit
	π_r	Retailer profit
	π	Total supply chain system profit

Table 2. Specific parameters and their meanings

Parameters	Meaning
a	Product market volume, and $a > 0$
k	traditional channel historical share, and $k > 0$
e	cross-price sensitivity factor, and $0 < e < 1$
c	production cost per unit of product
λ	retailer equity concern factor, and $\lambda > 0$

2.2. Basic assumptions

Hypothesis 1 For the demand functions of the two channels of the product, referring to the study on the construction of the dual-channel product demand function in the Yao and Liu model [20], the product demand functions of the traditional channel, the direct sales channel and the total product demand function are constructed directly through the linear demand function as:

$$D_r = ka - p_r + ep_m \tag{1}$$

$$D_m = (1-k)a - p_m + ep_r \tag{2}$$

$$D_p = a - (1-e)(p_m + p_r) \tag{3}$$

$a(a>0)$ is the market volume of the product, $k(0<k<1)$ is the traditional market base share of the product, and $e(0<e<1)$ is the cross-price sensitivity factor of the consumer to the channel.

Hypothesis 2 During the extended warranty service phase, the retailer offers and sells the extended warranty service to the consumer at the price of the extended warranty service p_s. The extended warranty cost is incurred when selling the extended warranty service and is assumed to be zero as the retailer's extended warranty service cost depends on its own service capability and usually remains constant over a certain period of time.

Hypothesis 3 In reality, the extended warranty service is purchased only after the product is purchased and the demand for the extended warranty service is negatively related to the price of the service. Referring to Li et al.'s study[4], a linear demand function for the extended warranty service is constructed as

$$D_s = D_p - p_s \tag{4}$$

Hypothesis 4 For ease of calculation, it is assumed that $c=0$.

From equations (1-4), the profit of the manufacturer π_m, the profit of the retailer π_r, and the overall profit of the extended warranty service supply chain π are obtained as

$$\pi_m = D_r w + D_m p_m \tag{5}$$

$$\pi_r = D_r(p_r - w) + D_s p_s \tag{6}$$

$$\pi = \pi_m + \pi_r \tag{7}$$

3. Extended Warranty Service Decision Model Analysis

3.1. Centralised decision making model

Under this model, the manufacturer and the retailer in the supply chain work together as a whole, so there is no fair concern behaviour between the supply chain members and both parties negotiate together with the common goal of maximising the total profitability of the supply chain system, and together they determine the price of the product in the retail channel, the price of the product in the direct sales channel and the price of the extended warranty service. The model is indicated by the superscript "c".

Proposition 1 When the centralized decision is made, the profit function of the supply chain system is a concave function of the optimal product direct sales price, the optimal product retail price and the optimal extended warranty service price of the supply chain, at which time the optimal decision of the supply chain system and the total profit of the supply chain system are:

$$p_m^{c*} = -\frac{a(2(e-1)k+e+1)}{4(e^2-1)} \tag{8}$$

$$p_r^{c*} = \frac{a(2(e-1)k-3e+1)}{4(e^2-1)} \tag{9}$$

$$p_s^{c*} = \frac{a}{2(e+1)} \tag{10}$$

$$\pi^{c*} = \frac{a^2\left(e(1-2k)^2 - 4(k-1)k - 3\right)}{8(e^2-1)} \tag{11}$$

Proof: For equation (1-7), find the first order derivative with π^c respect to p_m, p_r, p_s as:

$$\frac{\partial \pi^c}{\partial p_m} = -a(k-1) + 2ep_r + (e-1)p_s - 2p_m \tag{12}$$

$$\frac{\partial \pi^c}{\partial p_r} = ak + 2ep_m + (e-1)p_s - 2p_r \tag{13}$$

$$\frac{\partial \pi^c}{\partial p_s} = a - 2p_s + (p_m + p_r)(e-1) \tag{14}$$

For equations (12-14), finding the second order partial derivatives with respect to p_m, p_r, p_s, yields the Hesse matrix as;

$$H = \begin{pmatrix} -2 & 2e & e-1 \\ 2e & -2 & e-1 \\ e-1 & e-1 & -2 \end{pmatrix}$$

Since $A_{11} = -2 < 0$, $A_{22} = 4 - 4e^2 > 0$, $H = 4(e-1)(e+1)^2 < 0$, the Hessian matrix is negatively determined, there exists a unique optimal solution to the centralised decision model.

Letting equations (12-14) equal 0, the optimal product direct channel price, the optimal product retail channel price and the optimal extended warranty service price for the supply chain are obtained as:

$$p_m^{c*} = -\frac{a(2(e-1)k+e+1)}{4(e^2-1)} \tag{15}$$

$$p_r^{c*} = \frac{a(2(e-1)k-3e+1)}{4(e^2-1)} \tag{16}$$

$$p_s^{c*} = \frac{a}{2(e+1)} \tag{17}$$

Substituting (15-17) into (1-7) gives an overall maximum

profit for the supply chain:

$$\pi^{c*} = \frac{a^2\left(e(1-2k)^2 - 4(k-1)k - 3\right)}{8\left(e^2-1\right)} \tag{18}$$

Proposition 1 is proved.

3.2. Decentralised decision making model

In this model, based on the Stackelberg game, the manufacturer, as the dominant player, first decides on the wholesale price of the product and the price of the direct sales channel in order to maximise its own profit. The retailer, as the follower, considers its own equity concerns and decides on the retail channel price of the product and the price of the extended warranty service in order to maximise its utility. Referring to [19], the retailer's fair concern utility is obtained as $u_r = (1+\lambda)\pi_r - \lambda\pi_m$:

$$u_r = (1+\lambda)(D_r(p_r - w) + D_s p_s) - \lambda(D_r w + D_m p_m) \tag{19}$$

λ denotes the retailer fair concern coefficient and u_r denotes the retailer utility.

Proposition 2 There is a unique optimal solution for the extended warranty service supply chain under decentralised decision making when the retailer is equitably concerned, at which point the optimal decision and optimal profit for the manufacturer and retailer, respectively, are

$$w^{d\lambda*} = -\frac{a\begin{pmatrix}\lambda\left(e^3-7e^2+(e-1)^2((e-4)e+9)k+19e+3\right)+\\ e(e((e^2-2e+2)k+e-3)-10k+7)+9k+3\end{pmatrix}}{\left(e^2-1\right)(e((e-3)e(\lambda+1)-13\lambda-5)+47\lambda+23)} \tag{20}$$

$$p_m^{d\lambda*} = \frac{a\begin{pmatrix}e^4(-k)(\lambda+1)+e^3(4k-1)(\lambda+1)+e^2((4k+3)\lambda+3)+\\ e(-16k(2\lambda+1)+5\lambda+1)+k(25\lambda+13)-23\lambda-11\end{pmatrix}}{\left(e^2-1\right)(e((e-3)e(\lambda+1)-13\lambda-5)+47\lambda+23)} \tag{21}$$

$$p_r^{d\lambda*} = \frac{a\begin{pmatrix}e\left(e\left(-e^2+(e-2)^2k+3e+3\right)+16k-15\right)+\\ \lambda(e(e(e(e(k-1)-6k+3)+6k+9)+34k-31)-35k)-17k+4\lambda+2\end{pmatrix}}{\left(e^2-1\right)(e((e-3)e(\lambda+1)-13\lambda-5)+47\lambda+23)} \tag{22}$$

$$p_s^{d\lambda*} = \frac{a(\lambda(e(-e((e-5)k+2)+k+4)-5k)+e(e(2k-1)+2)-2k+14\lambda+7)}{(e+1)(e((e-3)e(\lambda+1)-13\lambda-5)+47\lambda+23)} \tag{23}$$

$$\pi_m^{d\lambda*} = \frac{2a^2\begin{pmatrix}e^3k^2\lambda+e^2k(-k(7\lambda+2)+3\lambda+1)+e(k(13k\lambda+6k-14\lambda-6)+2\lambda+1)\\ +k(-k(7\lambda+4)+11\lambda+5)-6\lambda-3\end{pmatrix}}{\left(e^2-1\right)(e((e-3)e(\lambda+1)-13\lambda-5)+47\lambda+23)} \tag{24}$$

$$\pi_r^{d\lambda*} = \frac{a^2\begin{pmatrix}-4e^4k^2+2e^4k+24e^3k^2-20e^3k+3e^3+2\lambda A_1-32e^2k^2+40e^2k-15e^2\\ +\lambda^2A_2-8ek^2+20ek+5e+52k^2-42k+162\lambda+39\end{pmatrix}}{(e+1)(e((e-3)e(\lambda+1)-13\lambda-5)+47\lambda+23)^2} \tag{25}$$

Proof: To find the game equilibrium, the analysis is carried out using the inverse order method and for equations (1-4), (6), (19) , find the first order partial derivative with π_r respect to p_r , p_s :

$$\frac{\partial\pi_r^d}{\partial p_r} = ak(1+\lambda) - (2p_r+p_s)(1+\lambda) + e(p_m+p_s+p_s\lambda) + w + 2\lambda w \tag{26}$$

$$\frac{\partial\pi_r^d}{\partial p_s} = (\lambda+1)(a+(e-1)(p_m+p_r)-2p_s) \tag{27}$$

For equations (26), (27), find the second order partial derivatives with π_r respect to p_r , p_s ,yields the Hesse matrix as:

$$H = \begin{pmatrix} -2\lambda-2 & (e-1)(\lambda+1) \\ (e-1)(\lambda+1) & -2\lambda-2 \end{pmatrix}$$

Since $A_{11} = -2\lambda-2 < 0$, $H = (3-e)(e+1)(\lambda+1)^2 > 0$, there exists a unique optimal solution to the objective function.

Letting equations (26), (27) equal 0, the response function for retailing is obtained as

$$p_r = -\frac{a(\lambda+1)(e+2k-1)+p_m\left(e^2+(e-1)^2\lambda+1\right)+4\lambda w+2w}{(e-3)(e+1)(\lambda+1)} \tag{28}$$

$$p_s = -\frac{a(\lambda+1)((e-1)k+2)+(e-1)(p_m(e+2\lambda+2)+2\lambda w+w)}{(e-3)(e+1)(\lambda+1)} \tag{29}$$

Substituting equations (28), (29) into equation (5), find the first order derivative with π_m respect to w , p_m :

$$\frac{\partial\pi_m^d}{\partial w} = \frac{\begin{matrix}a(\lambda+1)(((e-2)e-1)k+e-1)+\\ p_m(e((e-1)e(\lambda+1)-9\lambda-5)+\lambda+1)+8\lambda w+4w\end{matrix}}{(e-3)(e+1)(\lambda+1)} \tag{30}$$

$$\frac{\partial\pi_m^d}{\partial p_m} = \frac{\begin{matrix}-a(\lambda+1)\left(\left(e^2-3\right)k+e+3\right)-2ep_m\left(e^2+((e-1)e-1)\lambda+e-1\right)+\\ w(e((e-1)e(\lambda+1)-9\lambda-5)+\lambda+1)+6p_m(\lambda+1)\end{matrix}}{(e-3)(e+1)(\lambda+1)} \tag{31}$$

For equations (30), (31), find the second order partial derivatives with respect to w , p_m , and obtain the Hesse matrix as

$$H = \begin{pmatrix} \frac{8\lambda+4}{(e-3)(e+1)(\lambda+1)} & \frac{e((e-1)(\lambda+1)e-9\lambda-5)+\lambda+1}{(e-3)(e+1)(\lambda+1)} \\ \frac{e((e-1)(\lambda+1)e-9t-5)+\lambda+1}{(e-3)(e+1)(\lambda+1)} & -\frac{2e\left(e^2+(e-1)^2\lambda+1\right)}{(e-3)(e+1)(\lambda+1)}-2 \end{pmatrix}$$

$$A_{11} = \frac{8\lambda+4}{(e-3)(e+1)(\lambda+1)} < 0 \quad ,$$

$$H = \frac{(1-e)\left(e\left(\left(e-3\right)e\left(\lambda+1\right)-13\lambda-5\right)+47\lambda+23\right)}{(e-3)^2\left(\lambda+1\right)} > 0 \ ,$$

there is a unique optimal solution to the objective function.

Letting equations (30), (31) equal 0, we obtain $w^{d\lambda*}$, $p_m^{d\lambda*}$:

$$w^{d\lambda*} = -\frac{a\begin{pmatrix}\lambda\left(e^3-7e^2+(e-1)^2((e-4)e+9)k+19e+3\right)+\\ e(e((e^2-2e+2)k+e-3)-10k+7)+9k+3\end{pmatrix}}{\left(e^2-1\right)(e((e-3)e(\lambda+1)-13\lambda-5)+47\lambda+23)} \tag{32}$$

$$p_m^{d\lambda*} = \frac{a\begin{pmatrix}e^4(-k)(\lambda+1)+e^3(4k-1)(\lambda+1)+e^2((4k+3)\lambda+3)+\\ e(-16k(2\lambda+1)+5\lambda+1)+k(25\lambda+13)-23\lambda-11\end{pmatrix}}{\left(e^2-1\right)(e((e-3)e(\lambda+1)-13\lambda-5)+47\lambda+23)} \tag{33}$$

Substituting this into equations (28), (29), we obtain $p_r^{d\lambda*}$ and $p_s^{d\lambda*}$

$$p_r^{d\lambda*}=\frac{a\begin{pmatrix}e\left(e\left(-e^2+(e-2)^2k+3e+3\right)+16k-15\right)+\\ \lambda(e(e(e(e(k-1)-6k+3)+6k+9)+34k-31)-35k)-17k+4\lambda+2\end{pmatrix}}{\left(e^2-1\right)(e((e-3)e(\lambda+1)-13\lambda-5)+47\lambda+23)} \quad (34)$$

$$p_s^{d\lambda*}=\frac{a(\lambda(e(-e((e-5)k+2)+k+4)-5k)+e(e(2k-1)+2)-2k+14\lambda+7)}{(e+1)(e((e-3)e(\lambda+1)-13\lambda-5)+47\lambda+23)} \quad (35)$$

Substituting into equations (1-7), (19), we obtain π_m^{d*}, π_r^{d*}:

$$\pi_m^{d\lambda*}=\frac{2a^2\begin{pmatrix}e^3k^2\lambda+e^2k(-k(7\lambda+2)+3\lambda+1)+e(k(13k\lambda+6k-14\lambda-6)+2\lambda+1)\\ +k(-k(7\lambda+4)+11\lambda+5)-6\lambda-3\end{pmatrix}}{\left(e^2-1\right)(e((e-3)e(\lambda+1)-13\lambda-5)+47\lambda+23)} \quad (36)$$

$$\pi_r^{d\lambda*}=\frac{a^2\begin{pmatrix}-4e^4k^2+2e^4k+24e^3k^2-20e^3k+3e^3+2\lambda A_1-32e^2k^2+40e^2k-15e^2\\ +\lambda^2A_2-8ek^2+20ek+5e+52k^2-42k+162\lambda+39\end{pmatrix}}{(e+1)(e((e-3)e(\lambda+1)-13\lambda-5)+47\lambda+23)^2} \quad (37)$$

Of which:

$$A_{1=}-8\left(e^2((e-6)e+10)-17\right)k^2+4(e-6)(e-3)(e-1)(e+1)k+e(5(e-5)e+3)$$

$$A_2=(e+1)(e(e(e((e-20)e+118)-308)+337)k^2+8(e-5)(e-3)(e-1)(e+1)k+8(e-3)((e-2)e-7)$$

Proposition 2 is proved.

Proposition 3 The retailer is fairly neutral an $\lambda=0$, instantaneously, the optimal decision, and the optimal profit, for the manufacturer and the retailer, respectively, are:

$$w^{d0*}=-\frac{a(e(e(((e-2)e+2)k+e-3)-10k+7)+9k+3)}{(e-1)(e+1)(e((e-3)e-5)+23)} \quad (38)$$

$$p_m^{d0*}=\frac{a(-e(e((e-4)ek+e-3)+16k-1)+13k-11)}{(e-1)(e+1)(e((e-3)e-5)+23)} \quad (39)$$

$$p_r^{d0*}=\frac{a\left(e\left(e\left(-e^2+(e-2)^2k+3e+3\right)+16k-15\right)-17k+2\right)}{(e-1)(e+1)(e((e-3)e-5)+23)} \quad (40)$$

$$p_s^{d0*}=\frac{a(e(e(2k-1)+2)-2k+7)}{(e+1)(e((e-3)e-5)+23)} \quad (41)$$

$$\pi_m^{d0*}=\frac{2a^2\left(-2(e-2)(e-1)k^2+(e-5)(e-1)k+e-3\right)}{(e-1)(e+1)(e((e-3)e-5)+23)} \quad (42)$$

$$\pi_r^{d0*}=\frac{a^2\begin{pmatrix}3e^3-15e^2-4(e+1)(e((e-7)e+15)-13)k^2\\ +2(e-7)(e-3)(e-1)(e+1)k+5e+39\end{pmatrix}}{(e+1)(e((e-3)e-5)+23)^2} \quad (43)$$

Corollary 1 (1) $\frac{\partial w^{d0*}}{\partial k}>0,\frac{\partial p_m^{d0*}}{\partial k}<0,\frac{\partial p_r^{d0*}}{\partial k}>0,\frac{\partial p_s^{d0*}}{\partial k}<0$

(2) $\frac{\partial w^{d\lambda*}}{\partial k}>0,\frac{\partial p_m^{d\lambda*}}{\partial k}<0,\frac{\partial p_r^{d\lambda*}}{\partial k}>0,\frac{\partial p_s^{d\lambda*}}{\partial k}<0$

Proof: The proof is obtained by a derivative analysis.

Corollary 1 shows that when retailers are fair neutral and fair concerned, the traditional channel market share has an impact on the optimal wholesale price of the product, the extended warranty price, the direct channel price of the product and the retail channel price of the product. In particular, the gradual expansion of the traditional channel's market share will lead to a gradual reduction in the wholesale price and extended warranty price of the product. At the same time, the expansion of the traditional channel market will also lead to an increase in the direct and retail prices.

Corollary 2 $\frac{\partial w^{d\lambda*}}{\partial \lambda}<0,\frac{\partial p_m^{d\lambda*}}{\partial \lambda}>0,\frac{\partial p_r^{d\lambda*}}{\partial \lambda}>0,\frac{\partial p_s^{d\lambda*}}{\partial \lambda}<0$

Proof: The proof is obtained by a derivative analysis.

Corollary 2 shows that retailers will continue to increase their own fairness concerns in order to balance their own fairness mentality, which will inevitably cause the optimal wholesale price of the product and the price of the extended warranty service to decrease, and likewise cause the price of the product in the direct sales channel and the price of the product in the retail channel to increase.

Corollary 3 $\frac{\partial \pi_m^{d\lambda*}}{\partial \lambda}<0,\frac{\partial \pi_r^{d\lambda*}}{\partial \lambda}>0,\frac{\partial \pi^{*}}{\partial \lambda}<0$

Proof: The proof is obtained by a derivative analysis.

Corollary 3 shows that an increase in the retailer's fairness concern causes a decrease in the manufacturer's profit and an increase in the retailer's profit, and the combined effect of the two ultimately causes a loss in the total profit of the supply chain system.

From the above conclusions, it can be concluded that the optimal decisions of manufacturers and retailers are influenced by the retailer's fairness concerns and that the retailer's fairness concerns reduce the total profit of the supply chain system, making the double marginal effect of the supply chain system more prominent. It is important for the supply chain members, especially the dominant manufacturer, to consider the common interests of both parties in the operation of the supply chain, so that both parties cooperate and design a reasonable contract to coordinate the profit of the supply chain system to achieve a win-win situation.

4. Numerical Examples

4.1. The impact of traditional market channel share on optimal pricing and profitability

Based on the conclusions drawn above, the impact of the parameters in question on the manufacturer and retailer and on the supply chain system as a whole is verified by means of numerical simulation. From the range of parameters used in the solution of the above model, it is assumed that the initial values of the parameters are set to: $a=100$, $k=0.4$, $e=0.5$.

(1) Influence of traditional channel market share on optimal pricing

Let $k\in[0,1]$, respectively, the effects of traditional market channel share on product wholesale price, product direct sales channel price, product retail channel price and extended warranty service price be obtained, as shown in Fig. 1-4.

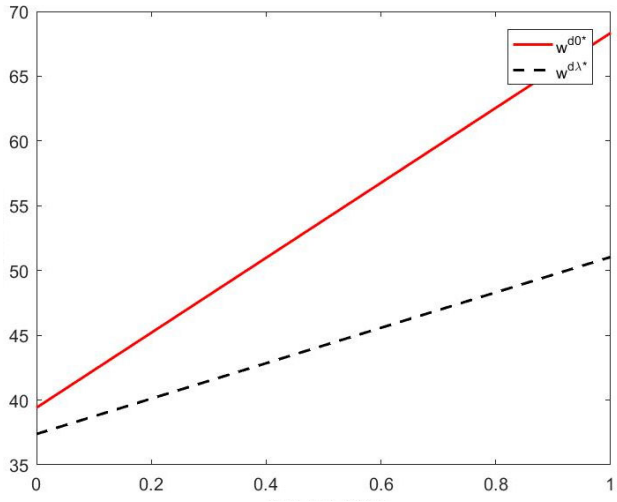

Figure 1. Effect of k on wholesale prices

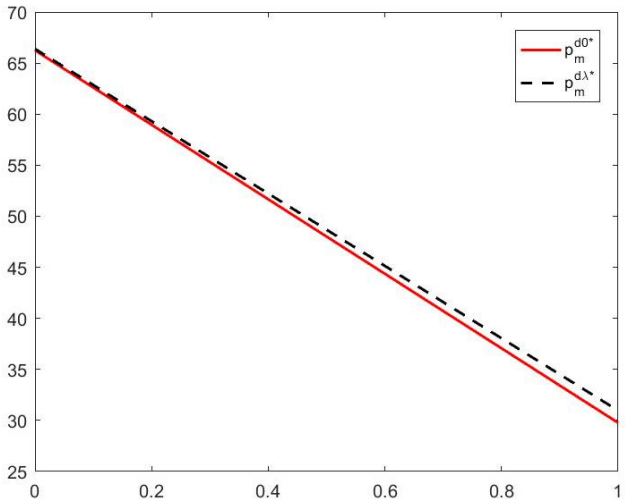

Figure 2. Effect of k on prices in direct product sales channels

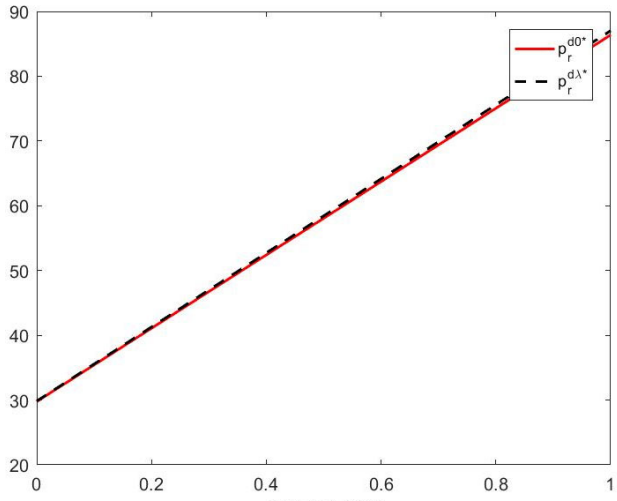

Figure 3. Effect of k on the price of the product in the retail channel

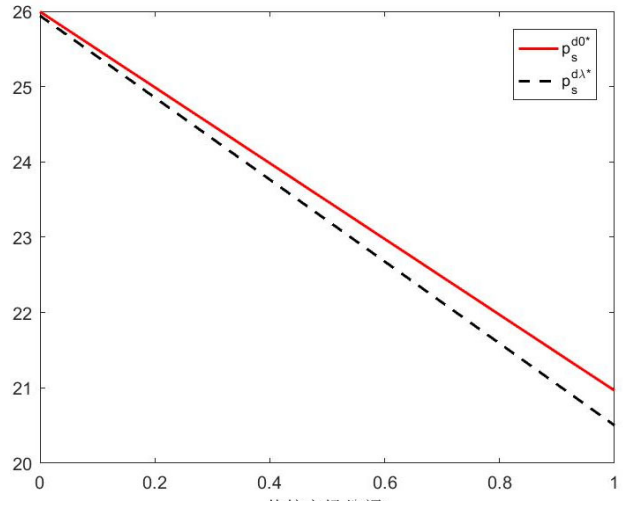

Figure 4. Effect of k on the price of the extended warranty service

Fig. 1-4 shows that in the decentralisation decision, regardless of the retailer's fairness concerns, as the market share of the traditional channel increases, the wholesale price of the product and the retail channel price of the product tend to increase, while the direct channel price of the product and the price of the extended warranty service tend to decrease. As the market share of the traditional channel increases, the potential market for the product occupied by the retailer increases, and the retailer increases the retail channel price of the product, which increases its own profit. For manufacturers, an increase in market share in the traditional channel will inevitably lead to a decrease in market share in the manufacturer's direct channel, so manufacturers will lower their direct channel prices to increase demand for their direct channel products. It is important to note that when the retail channel price is higher, the demand for the product in the retail channel will decrease and in order to balance the ultimate profit, the retailer will consider lowering the price of the extended warranty service to promote the sale of the product and the extended warranty service to gain higher profit.

(2) Impact of traditional channel market share on profits

Fig. 5 and Fig. 6 depict the impact of traditional channel market share on manufacturer and retailer profits respectively.

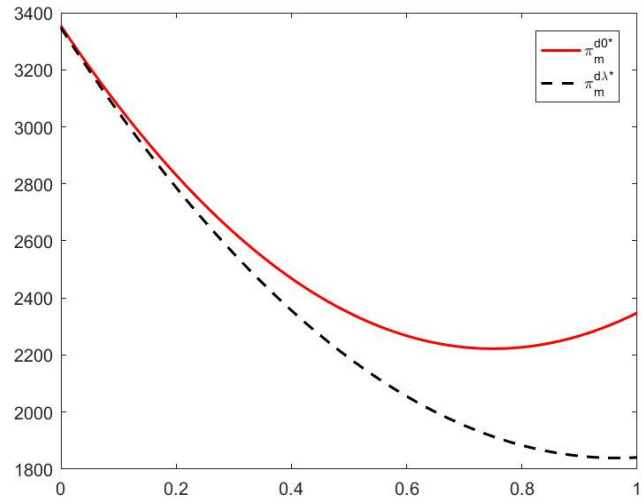

Figure 5. Effect of k on manufacturer's profit

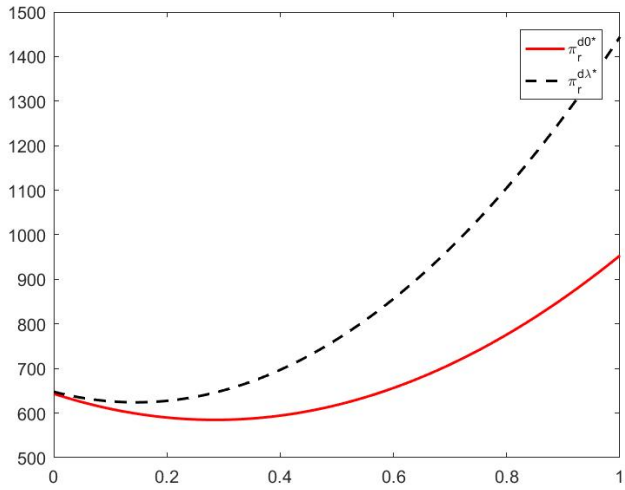

Figure 6. Effect of k on retailer's profit

Fig. 5 shows that when decentralisation is decided, regardless of whether the retailer is fairly concerned or not, an increase in the traditional channel's market share causes a significant reduction in the manufacturer's profit when it is small, and then causes an increase in the manufacturer's profit when the traditional channel's market share is large. This is because an increase in market share in the traditional channel means that demand for the manufacturer's products in the direct channel decreases, which, combined with an increase in the price of the product in the direct channel, ultimately results in a decrease in profits.

Fig. 6 shows that, regardless of the retailer's focus on fairness, when the traditional channel market share is small and gradually increases, the retailer's profit tends to decrease, and then the increase in traditional market share leads to a significant increase in the retailer's profit. When the manufacturer opens a direct sales channel for product sales, some of the retailer's product demand is shifted to the manufacturer, reducing the retailer's revenue from product sales, but as the retailer's product market increases, and as the extended warranty service has a higher marginal profit, higher product pricing and lower extended warranty service pricing can boost the retailer's product and extended warranty service sales, resulting in an increase in the retailer's profitability.

4.2. The impact of retailer fairness concerns on optimal pricing and profitability

(1) Influence of retailer fairness concern on optimal pricing

When analysing the impact of retailer fairness concern coefficients on supply chain pricing, other parameters are fixed such that, respectively, the changes in product wholesale price, product direct channel price, product retail channel price and extended warranty service price are obtained, as shown in Fig. 7-10.

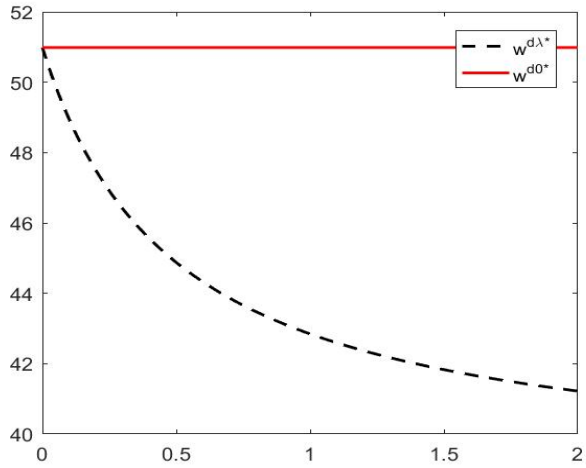

Figure 7. Effect of λ on wholesale prices

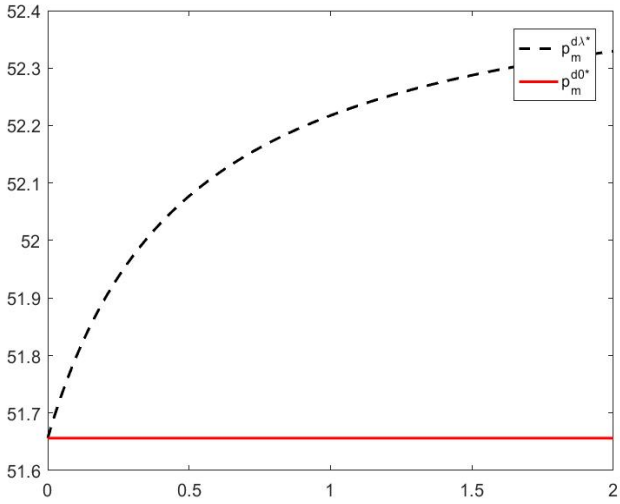

Figure 8. Effect of λ on prices in direct product sales channels

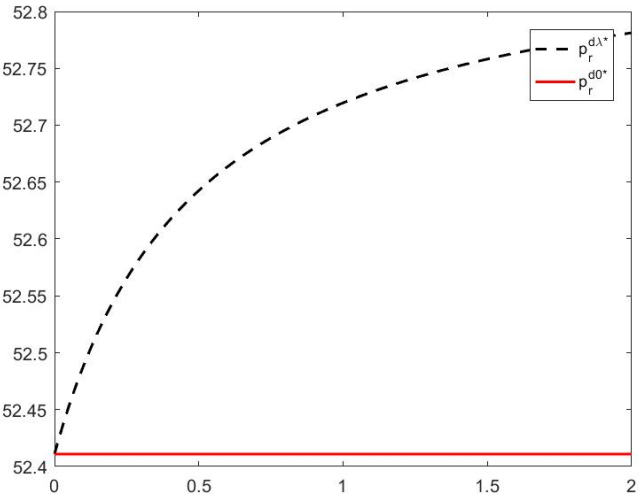

Figure 9. Effect of λ on the price of the product in the retail channel

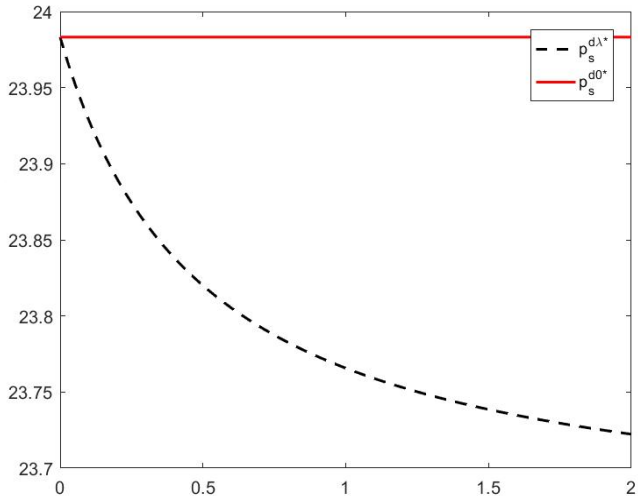

Figure 10. Effect of λ on the price of the extended warranty service

Fig. 7 shows that as the fairness concern factor increases, the retailer will bargain with the upstream manufacturer, at which point the manufacturer will reduce the wholesale price of the product to appease the unfairness of the retailer due to its own smaller profits, thus compensating for the retailer's utility, in line with the conclusion obtained from Corollary 2.

Fig. 8 shows that as the retailer's equity concern factor increases, the direct channel price increases, indicating that the manufacturer will reduce the wholesale price of the product in order to reduce the retailer's equity concern, which will inevitably result in a reduction in profit, and in order to ensure its own profit, the manufacturer will appropriately increase the direct channel price.

Fig. 9 shows that the retailer's product retail channel price increases with its own fairness concern factor, and is greatest when the fairness concern factor is 2. This is because when a retailer is faced with unfairness due to a large profit differential with the manufacturer, it will increase its own retail channel price to enhance its own utility given a certain market demand, and therefore, the retail channel price of the product will increase due to the enhanced fairness concern preference of the retailer.

Fig. 10 shows that the retailer's extended warranty price decreases as its own equity concern factor increases, as the retailer raises the retail channel price in order to balance its equity mentality with that of the upstream manufacturer's profit differential. In order to increase the demand for the product and the extended warranty service, the retailer will reduce the price of the extended warranty accordingly. In addition, from the consumer's point of view, when consumers buy a product at a higher price, they are less willing to pay for the extended warranty service, otherwise the opposite is true.

(2) The impact of retailer fairness concern on optimal profit

When analysing the impact of retailer fair concern coefficient on supply chain profit, fix other parameters,set $\lambda \in [0,2]$, respectively, the change of manufacturer's profit and retailer's profit is obtained, as shown in Fig. 11-12.

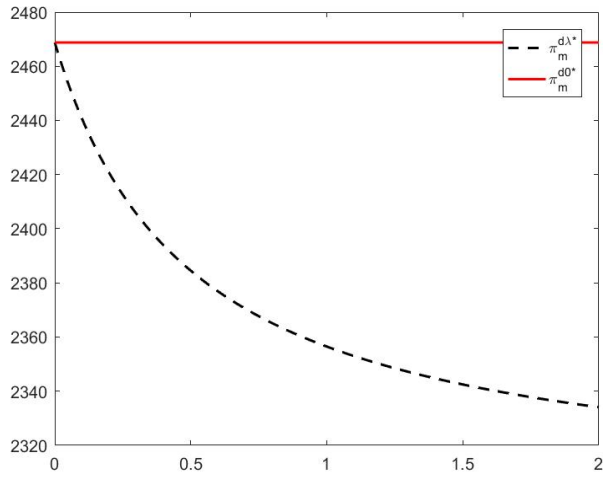

Figure 11. Effect of λ on manufacturer's profit

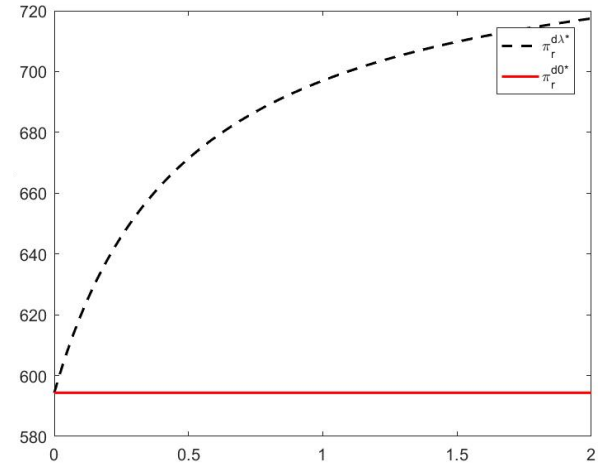

Figure 12. Effect of λ on retailer's profit

As can be seen from Fig. 11 and Fig. 12, in a dual-channel supply chain of products where retailers provide extended warranty services, the manufacturer's profit is negatively correlated with the retailer's equity concern coefficient, while the retailer's profit is positively correlated. This indicates that when the retailer's equity concern increases, the manufacturer reduces the wholesale price of the product and increases the price of the product in the direct sales channel, but as the increase in the price of the product in the direct sales channel increases the profit to a lesser extent than the decrease in the wholesale price decreases the profit, which ultimately leads to a decrease in profit. For retailers, the reduction in wholesale prices and the increase in prices in the retail channel lead to an increase in product revenues in the retail channel, which, combined with a reduction in the price of extended warranty services, boosts demand for extended warranty services and inevitably leads to an increase in retailer profits. However, the increase in retailer equity concerns results in a reduction in total supply chain system profits, which means that the retailer's equity concerns result in a loss of supply chain performance.

5. Conclusion

In this paper, a dual-channel product supply chain consisting of a manufacturer and a retailer is constructed, and the main consideration is that the product and extended warranty service decisions in the extended warranty service supply chain are influenced by the fair concern behaviour of the retailer, and the profit decisions of the manufacturer and

the retailer when the fair concern of the retailer is not considered are firstly established, and based on the Stackelberg game method, the pricing of the product and extended warranty service, the profit of the manufacturer and the retailer when the decision is centralised and decentralised are solved, The impact of traditional channel market share on the optimal decisions and profits of supply chain members under decentralised decision-making is also analysed. Then the optimal decisions and profits of manufacturers and retailers in the case of retailer equity concerns were established and solved, and the effects of traditional channel market share and retailer equity concerns on product pricing, extended warranty service pricing and profits of supply chain members in this case were analysed. The main findings of this paper are as follows:

(1) Under decentralised decision making, regardless of whether retailers pursue fairness perceptions, an increase in traditional channel market share causes product wholesale prices and product retail channel prices to increase; product direct channel prices and extended warranty service prices to decrease. The manufacturer's profit decreases first, and when the traditional channel share is larger, the manufacturer's profit appears to increase. For the retailer's profit, it tends to decrease when the market share of the traditional channel is small; then it increases and increases more and more.

(2) As the retailer's equity concern factor increases, the retailer will raise the retail channel price of the product and lower the price of its own extended warranty service. The manufacturer will reduce the wholesale price of the product and increase the price of the product in the direct sales channel. At this point, the manufacturer's profit decreases and the retailer's profit increases, resulting in a decrease in the profitability of the supply chain system as a result of the combined effect of the two. This means that the retailer's fairness concerns lead to a loss in supply chain performance, so the retailer should set its own fairness concerns factor while pursuing profit.

In this paper, we only consider the impact of retailers' fairness concerns on the dual-channel supply chain of products for which retailers provide extended warranty services in the model construction, but in real life, manufacturers are also concerned about fairness or not. Therefore, in future studies of extended warranty service supply chains, situations where manufacturers have fair concern behaviour could be included.

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