

# FOF and Risk Parity Model

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**Abstract:** FOF has a history of decades of operation and development in the United States, and has a relatively mature market. However, it is a newly emerging thing in China. Due to its low risk nature, FOF is already the main target of pension investment. With the increasing downward pressure on the global economy, the demand and interest of individual and institutional investors in FOF are also increasing. The development of FOF in China has not been smooth, and many issues cannot be separated from the core issue of FOF, which is asset allocation. From the perspective of asset allocation, this article selects the trading data of three indices in the global stock market in the past four years for empirical analysis. Through the selected performance evaluation indicators, the risk parity model, equal volatility model, and equal weight model are visually compared in terms of risk and return, proving that the risk parity model has good risk diversification effect and returns ability on this basis. At the end of the article, a summary of the entire article is provided and suggestions for the application of the risk parity model in FOF are proposed.

**Keywords:** FOF, Risk Parity Model, Performance evaluation indicators.

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## 1. Introduction

### 1.1. Research Background

FOF, also known as Fund of Funds, is a special form of fund that does not invest in traditional investment targets such as stocks and bonds, but indirectly invests in underlying assets by investing in other funds. According to the nature of investment objects, FOFs can be roughly divided into three types. One type is FOFs that invest in hedge funds (FOHF). The earliest FOF was Leveraged Capital Holdings launched by the Rothschild family in the late 1960s, which belongs to FOHF; One type is FOF (PEFOF), which invests in private equity funds. It first emerged in the United States in the 1970s, while the first private FOF in China was China Merchants Fund Treasure launched by China Merchants Securities in 2005; One type is FOFs that invest in public funds. The world's first truly public FOFs were launched by the Pioneer Fund in the United States, while China did not receive the first batch of public FOFs until September 2017. FOF has four modes: internal manager/self owned fund, internal manager/market fund, external manager/self owned fund, and external manager/market fund. The first model is often adopted by companies with large scale, rich product types, and strong operational management capabilities. Under this model, FOF issuing companies manage and invest in their own fund products, represented by Pioneer Fund Company. Due to the fact that both the operational management team and investment targets belong to their own companies and have lower costs, Pioneer Fund even implements zero fees for the master fund, which to a considerable extent solves the inherent defect of FOF's dual fees. The second model has more choices in selecting investment targets compared to the first model, which to some extent avoids the moral risks brought by internal operations. However, it also has insufficient advantages in avoiding double fees and reduces its attractiveness to investors. Fund companies adopting this model mostly have a high-quality and stable customer base, represented by John Hancock Fund Company. The third model has significant advantages in complementing external managers and issuing companies, taking into account the

professionalism of external managers and good risk aversion. It is currently the mainstream model, and a large number of companies have adopted this model, represented by Fidelity Fund in the United States. The fourth model has lower requirements for issuing companies and does not impose restrictions on the investment targets of external managers. Institutions that adopt this model often have prominent channel advantages, typically represented by banks and insurance. FOF, due to its inherent nature of secondary diversification of investment risks, naturally has a low risk nature. PEFOF helps small and medium-sized investors bypass the high threshold of private equity funds, and the 401K plan in the United States has led to a surge in investment demand for pension funds. Chinese regulatory authorities are also pushing public funds to be included in the investment scope of personal tax deferred commercial pension funds, which are strong driving forces for the rapid development of FOF.

China's FOF issuance was once cold, with a total initial fundraising of 16.636 billion yuan for the first batch of six FOFs. However, at the beginning of 2019, the scale had shrunk to 6.2 billion yuan, a decrease of over 60%. After a year of development, the number and scale of FOF products have increased, with a total of 91 FOFs reaching 25.993 billion yuan, of which pension target funds account for nearly 70%, and the average annualized return rate is close to 6%. China's FOF started and developed relatively late, and the cultivation and operation of the FOF investment management team is not as mature as the United States. There is still a significant gap between investment decision-making, asset allocation, and government regulation compared to the United States. One important reason for the previous setbacks in FOF development is excessive stability. Overallocation of monetary funds and double fees have led to poor early performance of FOF, which is a typical manifestation of improper asset allocation. The allocation of underlying assets is the core of FOF. Reasonable allocation of underlying assets can prevent the risk of the asset portfolio from being concentrated on a particular asset within the portfolio, and minimize the risk by diversifying the risk at a given expected

return level, or pursuing maximum returns at a given risk level, greatly improving the performance of the asset portfolio and enhancing its attractiveness. With the increasing trade frictions between China and the United States and the increasing downward pressure on the global economy, various assets around the world are inevitably more volatile. In the situation where the offensive and defensive nature of asset portfolios configured according to traditional methods is increasingly unsatisfactory, the exploration of a new and more stable and profitable asset allocation method becomes more reasonable and urgent.

## 1.2. Research Background

This article first introduces the relevant background of FOF and the development process of asset allocation methods. Based on the actual situation of global capital markets, risk parity portfolios, equal weight portfolios, and equal volatility portfolios are constructed. Using the data of each portfolio for quantitative analysis, comparable performance evaluation indicators are obtained to explore whether risk parity portfolios can achieve better performance, and then discuss the advantages and disadvantages of risk parity portfolios. Finally, this article discusses the improvement direction and development prospects of risk parity portfolios in FOF based on their performance. This article consists of four parts:

The first part introduces the main categories and four operational models of FOF, and briefly explains the development drivers of FOF. Although FOF has received increasing attention from the Chinese academic community recently, and more and more institutions have increased their exploration of FOF, there is still a long way to go in terms of the development of FOF in China and research on asset allocation methods. Afterwards, a brief introduction was given to the current research status both domestically and internationally, focusing mainly on the development process and empirical research of asset allocation methods. Finally, the research content and significance of this article were introduced.

The second part is mainly divided into two sections. The first section introduces the theoretical basis of the three asset allocation methods used in the empirical analysis of this article, namely the equal weight model, equal volatility model, and risk parity model. The focus is on the theoretical introduction and calculation process of the risk parity model, and a brief analysis of the advantages and disadvantages of the three models. The second section introduces the performance evaluation indicators used for performance comparison of investment portfolios constructed using three models in the empirical analysis section, including volatility, cumulative return, annualized return, maximum pullback, and Karma ratio.

The third part is empirical analysis, which uses recent global financial market index data to explain the construction process and conduct data processing. Performance evaluation indicators are used to compare and examine the performance of risk parity portfolios, and based on their performance, a brief discussion is conducted on the advantages and disadvantages of risk parity portfolios and the other two asset allocation models, and a brief analysis is conducted to address the issues involved.

The fourth part is the conclusion of this article. Based on the previous discussion, the total price and evaluation are conducted, and it is concluded that the risk parity model has good results in asset allocation. At the same time, the future

development direction of the application of the risk parity model in FOF is briefly explained.

With the increasing downward pressure on economies in various countries, the volatility of global financial assets has increased to varying degrees, and the effectiveness of old asset allocation strategies is increasingly being questioned. In this context and environment, the theory of risk parity has emerged, and empirical analysis of its effectiveness is necessary. Due to various reasons such as external regulatory support and internal operational management, FOF has not fully utilized its inherent advantages in China, and its development is not smooth. Ultimately, it is a problem of asset allocation. This article takes FOF as the theme, conducts empirical analysis on the risk parity method, discusses the improvement direction of the risk parity method in practical application through the performance of the risk parity combination, and summarizes the inspiration drawn from it.

## 2. Literature Review

The early asset allocation methods commonly used by people were mainly based on equal weight combinations and the 40-60 rule. The equal weight combination, as the name suggests, refers to the allocation of the same weight to each asset in the asset portfolio, while the 40-60 rule refers to stocks accounting for 60% and bonds accounting for 40% of the asset portfolio. This asset allocation method originated from the modern portfolio theory that became popular in the late 1950s, and both methods belong to constant mixing strategies. Although the constant mixed strategy has a certain degree of risk diversification, both profitability and stability are not satisfactory. Although the operation is convenient, it is possible that most of the risk is contributed by a certain asset concentration, and the risk diversification effect is not satisfactory. In 1952, Markowitz proposed the mean variance method, marking a huge leap in asset allocation. Investment portfolios can be constructed based on quantitative analysis. The mean variance theory treats the price change of an investment portfolio as a random variable, measures returns based on its mean, measures risks based on its variance, and takes the proportion of various securities in the investment portfolio as a variable. Therefore, the problem of finding the minimum risk investment portfolio with a certain return is classified as a quadratic programming problem under linear constraints. The mean variance method is an important foundation of modern asset allocation theory, but users need to be able to continuously and accurately estimate the expected returns, risks, and correlation coefficients of assets, and undergo extensive calculations, which is not practical enough. On the basis of the mean variance method, Sharp, Trino, and Mosin et al. proposed the Capital Asset Pricing Model (CAPM) in 1964. CAPM uses a simple linear relationship table to express the theoretical relationship between the expected return of an asset and its risk, which considers the relationship between the expected return of an asset and the sensitivity of the asset to risk  $\beta$ . There is a positive correlation between values. The B-L model proposed by Fisher Black and Robert Litterman in 1992 further improved Markowitz's mean variance method, introducing subjective considerations from investors. The historical data method used in the model is more widely used than the overly subjective scenario analysis method. However, in the practical application of asset allocation, the mean variance method still has a significant flaw, as it cannot guarantee sufficient diversification of investment portfolio risks. In the

1990s, the all-weather strategy of Qiaoshui Fund involved the strategic idea of risk parity. In 2005, Dr. Qian Enping of Pan'an Fund proposed the risk parity theory for the first time, which introduced the concept of risk contribution and attempted to make the risk contribution of various assets equal in order to achieve sufficient risk diversification. In 2011, Denis Chaves et al. analyzed and proposed that the advantage of the leveraged risk parity model over the mean variance model is that investors do not need to form investment portfolios based on their return assumptions, which can to some extent reduce the risk of asset allocation decision bias caused by uncertainty.

In China, in 2014, based on the mean variance method, Li Jinxin, Tu Wei, Wang Zhiguo, and Zou Hengfu et al. compared investment portfolios under equal weight strategy, mean variance strategy, Bayesian strategy, and minimum variance strategy through empirical research based on monthly data from the Chinese stock market. The results showed that parameter uncertainty would affect the performance of mean variance strategy, and among them, equal weight strategy performed the best, It can be considered as a performance comparison benchmark for asset allocation in the Chinese market. In 2018, Zhang Li applied modern asset allocation models to FOFs. Through extensive empirical analysis, it was found that in the absence of leverage, the minimum variance model performed the best in the equal weight model, minimum variance model, and risk parity model, but the returns were too low. After adding leverage, the annualized returns of the investment portfolio constructed by the risk parity model also continued to improve, relatively speaking, performing the best. In 2019, Zheng Xia, Wang Jin, Li Yue, and Cui Yujie et al. proposed a quantitative analysis method for FOF fund investment using the risk parity model. Based on the construction of investment portfolios and empirical analysis, they believed that although the annualized return of the risk parity model was lower than that of the equal weight model, the annualized volatility and Sharpe ratio of the risk parity model were better than those of the equal weight model, This indicates that the risk parity model performs well in dispersing risk but has poor profitability.

### 3. Asset Allocation Methods and Performance Evaluation Indicators

#### 3.1. Asset allocation methods

The three asset allocation models used in the empirical analysis section of this article are the equal weight model, the equal volatility model, and the risk parity model. This section aims to clarify the construction methods of each model and provide a brief analysis, mainly introducing the risk parity combination.

##### 3.1.1. Equal weight model

The equal weight model is the most convenient and feasible asset allocation method for investors. The equal weight model, also known as the constant proportion investment method, directly assigns the same weight to each underlying asset without considering issues such as risk and expected return. The threshold for understanding and application is very small, and the calculation and operating costs are extremely low. The formula for calculating the weight is as follows:

$$\omega_i = \frac{1}{n} \quad (1)$$

Where  $n$  is the total number of types of underlying assets,  $\omega_i$  is the weight of any underlying asset. The equal weight model and the 40-60 rule are collectively referred to as constant mixed strategies. Although their advantages are obvious, their disadvantages are equally prominent. They are rigid, making it difficult to make timely adjustments and reactions when the market environment undergoes significant changes. Even if there are many types of underlying assets, they may not be able to fully disperse risks, and asset portfolios are difficult to achieve both offensive and defensive qualities.

##### 3.1.2. Equal volatility model

A commonly used approach is to measure the variance or volatility of asset or portfolio returns to measure their risk level. The corresponding risk of high volatility is also considered high. Therefore, a reasonable approach is to increase the proportion of assets with low volatility in the asset portfolio, reduce or even not hold assets with high volatility. This leads to the equivalent volatility model, and the weight calculation formula is as follows:

$$\omega_i \sigma_i = \omega_j \sigma_j \quad (2)$$

among  $\omega_i$  and  $\omega_j$  represents the weight of different assets in the asset portfolio,  $\sigma_i$  and  $\sigma_j$  represents the volatility of the corresponding asset. After simplification, it can be concluded that:

$$\omega_i = \frac{\sigma_i^{-1}}{\sum_{k=1}^n \sigma_k^{-1}} \quad (3)$$

Where  $n$  is the total number of types of underlying assets. It can be seen that the proportion of the underlying asset in the equal volatility model decreases as its volatility increases, and the weight is inversely proportional to the volatility.

##### 3.1.3. Risk Parity Model

The all-weather strategy of Qiaoshui Fund in the 1990s involved the idea of risk parity, and the true birth of risk parity theory originated from a paper published by Dr. Qian Enping of Pan'an Fund, the father of risk parity, in 2005. On the basis of analyzing the development process of asset allocation theory, he systematically proposed the risk parity theory, which does not consider the return rate of each underlying asset and only determines the investment weight by making the contribution of each asset to the portfolio risk equal, thereby controlling the total risk of the investment portfolio. Taking an investment portfolio containing  $n$  assets as an example:

$$R_p = \sum_{i=1}^n \omega_i R_i \quad (4)$$

Among them,  $R_p$  is the return on the investment portfolio, and  $R_i$  is the return on the  $i$ -th asset,  $\omega_i$  is the proportion of the  $i$ -th asset in the investment portfolio, and satisfies:

$$\sum_{i=1}^n \omega_i = 1 \quad (5)$$

In the risk parity model, the standard deviation of asset return is used to measure risk, and the risk of the investment portfolio is:

$$\sigma_p = \sqrt{\sum_{i=1}^n \sum_{j=1}^n \omega_i \omega_j \text{cov}(R_i, R_j)} \quad (6)$$

among  $\sigma_p$  is the risk of the asset portfolio, and  $MRC_i$  is the marginal risk contribution of asset  $i$  to the asset portfolio, i.e. the weight of asset  $\omega_i$  Caused by the growth of  $\sigma_p$ . The growth of  $p$  can be determined by Taking the partial derivative of  $\sigma_p$  yields:

$$MRC_i = \frac{\partial \sigma_p}{\partial \omega_i} = \frac{\sum_{j=1}^n \omega_j \text{cov}(R_i, R_j)}{\sigma_p} = \frac{\text{cov}(R_i, R_p)}{\sigma_p} \quad (7)$$

Furthermore, the total risk contribution of asset  $i$  to the investment portfolio,  $TRC_i$ , is:

$$TRC_i = \omega_i \frac{\partial \sigma_p}{\partial \omega_i} = \frac{\sum_{j=1}^n \omega_i \omega_j \text{cov}(R_i, R_j)}{\sigma_p} = \omega_i \frac{\text{cov}(R_i, R_p)}{\sigma_p} \quad (8)$$

Namely:

$$\frac{TRC_i}{\sigma_p} = \omega_i \beta_i \quad (9)$$

Among them  $\beta_i = \frac{\text{cov}(R_i, R_p)}{\sigma_p^2}$ , the value of asset  $i$   $\beta$  Coefficient, which measures the sensitivity of asset  $i$  to portfolio fluctuations. And observe  $\sigma_p$  can be obtained:

$$\sigma_p = \frac{\sum_{i=1}^n \sum_{j=1}^n \omega_i \omega_j \text{cov}(R_i, R_j)}{\sigma_p} = \sum_{i=1}^n TRC_i \quad (10)$$

From this, it can be seen that the risk contribution weight of asset  $i$  to the investment portfolio is not equal to  $\omega_i$ , but rather the ratio of  $\omega_i$  to asset  $i$   $\beta$  The product of coefficients. When there are too many types of assets in the portfolio, the above equation may not be able to obtain an analytical solution. Maillard proposed the following method in 2010 to solve this problem:

$$\left\{ \begin{array}{l} x^* = \arg \min f(x) \\ \text{s. t. } \begin{cases} \sum_i x_i = 1 \\ 0 \leq x_i \leq 1 \end{cases} \\ f(x) = \sum_{i=1}^n \sum_{j=1}^n (TRC_i - TRC_j)^2 \end{array} \right. \quad (11)$$

### 3.1.4. comparative analysis

From the above, it can be seen that different asset allocation models have different starting points and different methods for determining asset allocation weights, as listed below:

Equal weight model:

Equal volatility model:

Risk Parity Model:

The three asset allocation models all focus on risk diversification, but their methods are different, so the performance of the final investment portfolio will also vary.

## 3.2. Performance evaluation indicators

The performance of investment portfolios needs to be rationally evaluated through a series of performance evaluation indicators. This article mainly uses five commonly used indicators to evaluate fund performance, namely volatility, cumulative return, annualized return, maximum pullback, and Karma ratio.

### 3.2.1. Volatility

Volatility is used to describe the stability of financial asset prices, measure the uncertainty of asset returns, and reflect the

risk level of asset value. The higher the volatility, the greater the volatility of financial asset prices, and the greater the uncertainty of asset returns; The lower the volatility, the smaller the volatility of financial asset prices, and the more deterministic the asset yield. The formula is as follows:

$$\sigma_p = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (R_i - \bar{R}_p)^2} \quad (12)$$

Where  $n$  is the total number of trading days,  $R_i$  is the yield of the underlying asset on day  $i$ , and  $\bar{R}_p$  is the average daily frequency yield of the underlying asset within the selected area. Assuming that the daily return on assets is independently and identically distributed, according to the rule of 'uncertainty is proportional to time length', The standard deviation of the T-day return is T times the root of the daily return standard deviation. The yield data used in this article is the daily frequency yield. As people are more concerned about the actual trading days, and the annual trading days are about 252 days, the annualized volatility calculation formula is as follows:

$$\sigma_p = \sqrt{\frac{252}{n-1} \sum_{i=1}^n (R_i - \bar{R}_p)^2} \quad (13)$$

### 3.2.2. Accumulated Yield

Among all performance evaluation indicators, cumulative rate of return is the most intuitive and understandable evaluation indicator. The cumulative rate of return refers to the cumulative return from the beginning of portfolio construction to the end of operational management, considering only the final relative initial return of the portfolio. The formula is as follows:

$$R = \frac{P_n}{P_0} - 1 \quad (14)$$

Among them,  $P_n$  is the value of the investment portfolio at the end of the selected interval, and  $P_0$  is the initial value of the selected interval. At the same time, due to the different investment durations of different assets or the different observation perspectives of different investors, the cumulative rate of return alone sometimes cannot be directly used to compare the returns of different assets. Therefore, the cumulative rate of return is often converted into an annualized rate of return  $R_y$  for intuitive comparison. It is a theoretical return rather than an actual return, representing the theoretical return of the underlying asset when the investment period is one year. The calculation formula is as follows:

$$R_y = \left[ \left( \frac{\text{Internal investment income}}{\text{principal}} \right) / \text{Investment days} \right] * 365 \times 100\% \quad (15)$$

Or:

$$R_y = \sqrt[t]{1 + R} - 1 \quad (16)$$

Among them,  $R$  is the cumulative rate of return, and  $t$  is the cumulative investment time calculated in years. This form of annualized rate of return calculation formula implies the assumption of compound interest per year.

### 3.2.3. Maximum Drawdown

The meaning of the maximum pullback rate is the maximum value of the yield pullback range when the net

value of the product drops from this point to the lowest point at any time point within the target cycle. Simply put, it is the maximum proportion of losses that investors may experience during this period. The maximum pullback is used to measure the worst-case scenario that may occur after buying the underlying asset, and is an important risk indicator. In the field of hedge funds and quantitative trading, the maximum pullback is often more important than volatility. The maximum pullback may not necessarily be the difference between the highest and lowest points of the portfolio value divided by the highest point. If an investor purchases when the underlying asset price drops, the maximum pullback may also occur, as follows:

$$U = \max \left( \frac{D_i - D_j}{D_i} \right) \quad (17)$$

Among them,  $i$  is any day, and  $D_i$  is the net value on the  $i$ -th day. During operation, it can be essentially understood as evaluating the pullback rate at each time point and taking the maximum value. In Excel, the daily net worth can be subtracted from the lowest net worth on that day and subsequent trading days, and then divided by the daily net worth to obtain the daily withdrawal rate. Finally, the maximum daily withdrawal rate is taken as the maximum withdrawal rate.

#### 3.2.4. Karma ratio

The Karma ratio is a performance evaluation indicator first proposed by Terry Young in 1991, mainly studying the relationship between the maximum pullback rate and the return rate. The calculation formula is as follows:

$$C = \frac{R}{U} \quad (18)$$

Among them,  $C$  represents the Karma ratio,  $U$  represents the historical maximum pullback,  $R$  is the annualized return of the underlying asset, and a higher Karma ratio indicates a higher return for the fund when it bears the maximum loss per unit. It can be seen that the performance and risk level of the underlying asset cannot be solely viewed from the level of maximum pullback. Assets with a high Karma ratio may still obtain greater returns while bearing smaller risks.

### 3.3. Overview of this chapter

This chapter consists of two sections. The first section introduces the three asset allocation models adopted in this article and focuses on analyzing the theoretical basis for solving the asset allocation weights of each model. It also briefly analyzes the characteristics and advantages and disadvantages of each model. The second section explains and introduces the performance evaluation indicators adopted in this article and the theoretical basis for calculating each indicator. On top of this, it briefly analyzes the characteristics and inspection aspects of each indicator.

## 4. Research Design and Empirical Analysis

### 4.1. Data Selection

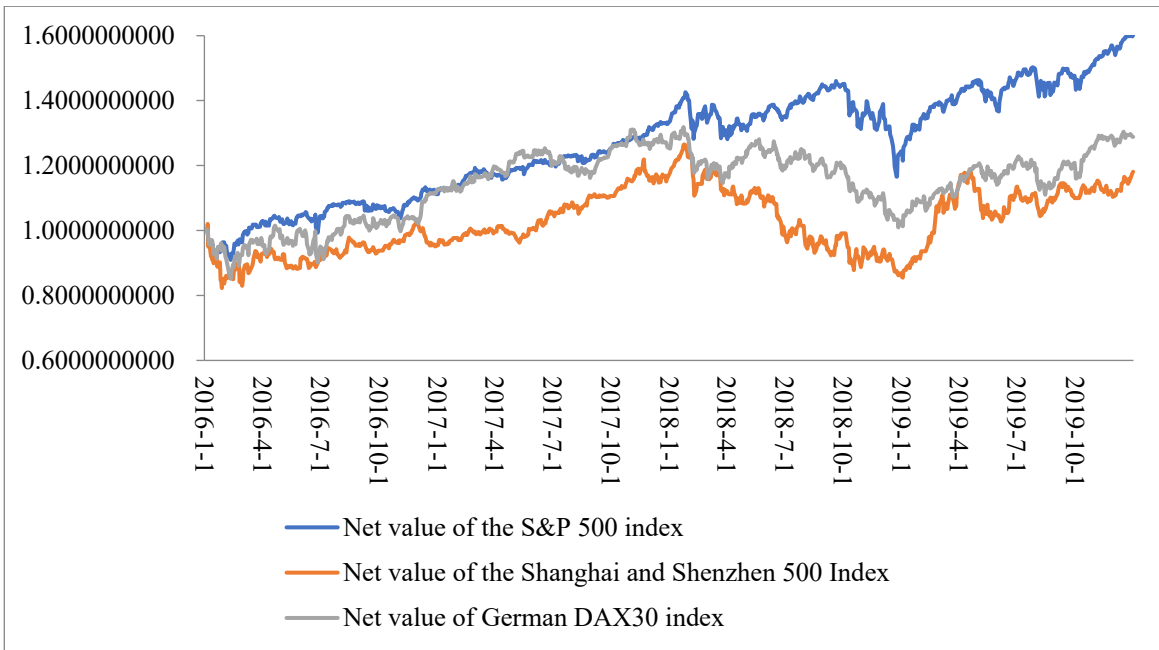
Due to the background of FOF in this article, and the core of FOF lies in asset allocation rather than the selection of

underlying assets, this article uses stock indices as the underlying assets. In order to reduce the possibility of the underlying assets being jointly affected by the same factors, such as domestic policy factors, changes in domestic market environment, or changes in interest rates and inflation levels, this article chooses representative stock market indices in the global financial market, such as the S&P 500 Index, the Shanghai Shenzhen 300 Index, and the German DAX30 Index, from the United States, China, and Germany. In addition, due to the fact that the longer the interval selected for empirical analysis is not necessarily the better, as data from too long ago may not have much reference value for current investment decisions, and selecting an interval that is too short is also not good, which cannot reflect the true operating level, return situation, and the impact of excluding short-term contingency factors. Therefore, this article selects the closing price data of the three major stock indices from July 1, 2015 to December 31, 2019, which is a total of four and a half years. In addition, due to differences in social culture, policy systems, and other factors in different countries, the trading days of stock markets vary. The approach adopted in this article is that as long as one country's stock market is trading on a certain day, and if there is no trading data in the two countries, the trading data from the previous day should be extended to ensure the completeness and ease of processing of the data. After adjustment, there were a total of 1172 trading days of data, of which 132 trading days from July 1, 2015 to December 31, 2015 were only used for initial calculations and not for other purposes. The actual data used for empirical analysis was 1040 trading days from January 4, 2016 to December 31, 2019.

### 4.2. Data statistics

After selecting the data as mentioned earlier, this section conducted a brief statistical analysis on the original stock index data and obtained the following chart (See Figure 1):

From the above stock index net value trend chart, it can be seen that all three major stock indices have roughly gone through a process of first rising, then falling, and then rising. The decline process of the S&P 500 index occurred in a relatively short period of time and quickly recovered, while compared to Germany's DAX30 and Shanghai Shenzhen 300, the change curve is more gentle, and the market recovery also took more time. From the graph, it can be intuitively seen that the S&P 500 index performed the best and achieved high returns over a four-year period, with a significant increase in the net value of the index. Compared to this, the DAX30 index in Germany performed second, with growth performance similar to the S&P 500 in the first two years of the four years, and significantly lower than the S&P 500 in the latter two years, with a significant difference. The net value growth of the index was average in the four years. The worst performing index is the Shanghai and Shenzhen 300 Index, which has performed significantly lower than the first two indices in the first three years of the four years. During this period, the net value only approached Germany's DAX30 for a short period of time, and then fell significantly behind in the subsequent decline process. In the last year, the net worth gradually approached the German DAX30 index, and the final net worth growth was the lowest. The detailed statistical indicators are shown in the table below (See Table 1) :



**Figure 1.** Three major stock index net worth trend charts

**Table 1.** Three major stock index net worth trend charts

	Mean	Annualized volatility	Min	Max	Accumulated Yield	Annualized return	Maximum Drawdown	Karma ratio	sharpe ratio
S&P 500	0.047%	12.726%	-4.182%	5.052%	60.113%	12.488%	20.178%	0.619	0.845
CSI 300	0.016%	18.464%	-7.020%	5.948%	18.089%	4.244%	32.462%	0.131	0.083
Germany DAX30	0.025%	15.472%	-6.823%	3.506%	28.838%	6.540%	23.438%	0.279	0.427

From the table, it can be seen that the cumulative returns of the three major stock indices are all positive, with the S&P 500 performing best at 60% and the Shanghai and Shenzhen 300 performing worst. In terms of maximum pullback, the S&P 500 has the highest risk, with a possible loss ratio of up to 20%. The German DAX30 index has the lowest risk. From the perspective of volatility, the S&P 500 has the best stability, while the Shanghai and Shenzhen 300 Index has the worst stability. Based on other indicators, it can be concluded that the S&P 500 index has the best performance among the three major stock indices and is significantly better than the other two. Except for the relatively high correlation between the S&P 500 and the German DAX30, the correlation between the S&P 500 and the Shanghai Shenzhen 300, as well as the Shanghai Shenzhen 300 and the German DAX30, is relatively low, indicating that risk diversification is possible and necessary.

### 4.3. Model construction

#### 4.3.1. Preparation for building the model

Before conducting performance comparative analysis, the specific construction process is as follows:

1. Data range: From July 1, 2015 to December 31, 2019, the data from July 1, 2015 to December 31, 2015 is only used for weight calculation on January 1, 2016, and the actual observation period is from January 1, 2016 to December 31, 2019.

2. Frequency of warehouse adjustment: The weight is adjusted every three months, that is, on January 1st, April 1st, July 1st, and October 1st each year.

3. Sample interval for position adjustment: Calculate new asset allocation weights based on the transaction data from the first half of each position adjustment.

4. Underlying assets: S&P 500 Index, Shanghai and Shenzhen 300 Index, and German DAX30 Index.

5. Model selection: equal weight model, equal volatility model, and risk parity model.

#### 4.3.2. Calculation issues related to asset allocation models

The core formula of the risk parity model was explained earlier:

$$TRC_i = TRC_j \quad (19)$$

For any  $i, j$ , both hold true. Therefore, in the calculation of the risk parity model in this article  $TRC_1 = TRC_2 = TRC_3$ , it can be simplified as follows:

$$\omega_1 \frac{\text{cov}(R_1, R_p)}{\sigma_p} = \omega_2 \frac{\text{cov}(R_2, R_p)}{\sigma_p} = \omega_3 \frac{\text{cov}(R_3, R_p)}{\sigma_p} \quad (20)$$

Namely

$$\begin{aligned} & \omega_1 \text{cov}(R_1, \omega_1 R_1 + \omega_2 R_2 + \omega_3 R_3) \\ &= \omega_2 \text{cov}(R_2, \omega_1 R_1 + \omega_2 R_2 + \omega_3 R_3) \\ &= \omega_3 \text{cov}(R_3, \omega_1 R_1 + \omega_2 R_2 + \omega_3 R_3) \end{aligned} \quad (21)$$

Namely

$$\begin{cases} \omega_1^2 D_1 + \omega_1 \omega_3 \text{cov}(R_1, R_3) = \omega_2^2 D_2 + \omega_2 \omega_3 \text{cov}(R_2, R_3) \\ \omega_1^2 D_1 + \omega_1 \omega_2 \text{cov}(R_1, R_2) = \omega_3^2 D_3 + \omega_2 \omega_3 \text{cov}(R_2, R_3) \\ \omega_1 + \omega_2 + \omega_3 = 1 \end{cases} \quad (22)$$

By combining the  $\omega_1 = \omega_2 = \omega_3$  of the equal weight model and the  $\omega_1 \sigma_1 = \omega_2 \sigma_2 = \omega_3 \sigma_3$  of the equal volatility

model mentioned earlier, the investment weights of each asset for each portfolio adjustment can be calculated. The weight of each asset in the equal weight model is always 1/3, so there will be no further illustration or discussion here. Firstly, the weight calculation results of a total of 16 position adjustments using the equal volatility model over four years are shown in the following (See Figure 2):

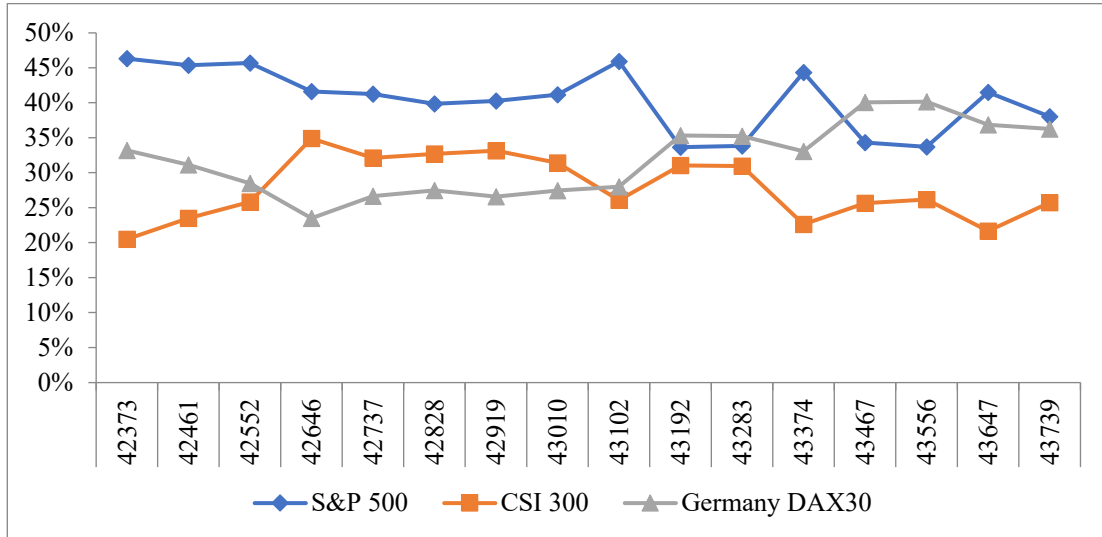


Figure 2. Figure of changes in asset weights for equal volatility models

From the graph, it can be seen that as the risk profile of the three types of assets continues to change, the weight ratios of the three major stock indices are also constantly changing. The proportion of the S&P 500 index in the first two years has slowly declined steadily, while the weight of the latter two years has fluctuated greatly, indicating that the risk of the S&P 500 index in the first two years is relatively low and stable, while the stability of the risk situation in the latter two years has deteriorated. The weight ratio of the Shanghai and Shenzhen 300 Index gradually increased in the first two years and then remained roughly stable, while it decreased in volatility in the latter two years. The situation of the German DAX30 Index is exactly the opposite, that is, it decreased in the first two years and then remained stable, and rose in

volatility in the latter two years. Overall, throughout the entire data range, the risk of the S&P 500 index has increased and the stability of the risk profile has deteriorated. The risk level of the Shanghai and Shenzhen 300 index has been fluctuating, while the risk profile of the German DAX30 index has decreased despite fluctuations.

In addition, since this article does not consider the special situation of short selling for comparative analysis of three asset allocation portfolios, there is an additional condition in the weight calculation of the risk parity model, which is . Based on this, the weight calculation results of 16 positions adjustment are shown in the following (See Figure 3):

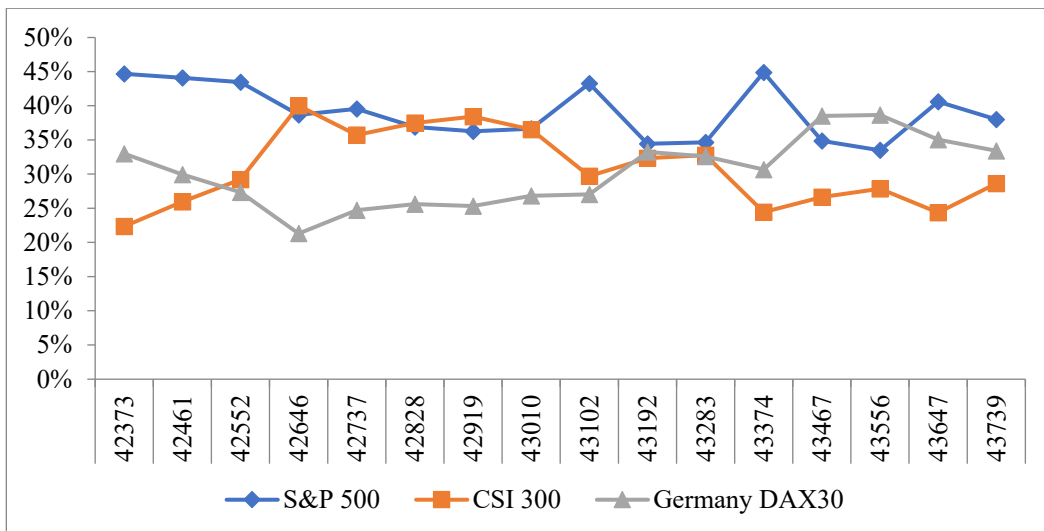


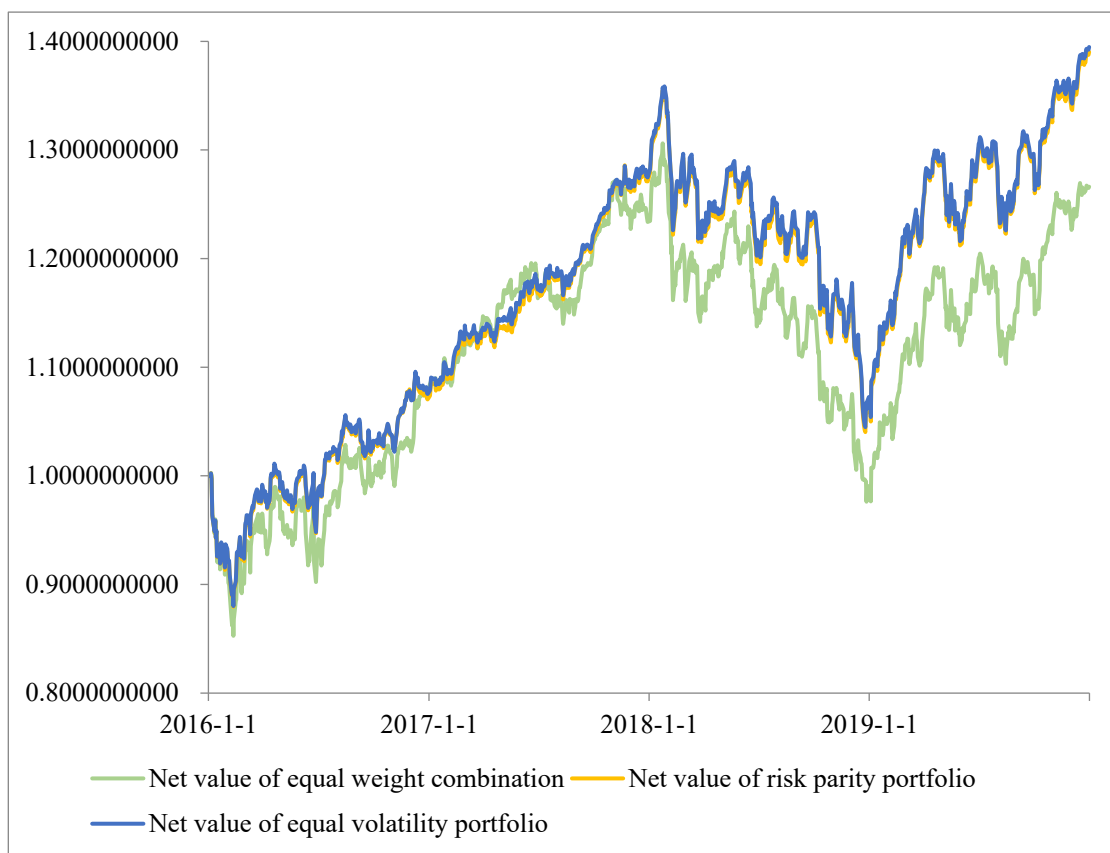
Figure 3. Risk Parity Model Underlying Asset Weight Change Chart

Firstly, it can be seen from the graph that the changes in the weight ratios of the three major stock indices in the risk parity model are relatively similar to those in the equi volatility model. The S&P 500 index first stabilized and then fluctuated, with an overall decline. The German DAX30 index first fell, then stabilized, and then rose amidst fluctuations. However, in the first two years, with the decline of the S&P 500 index and the increase of the Shanghai and Shenzhen 300 index, their proportions were similar for a period of time, and occasionally the proportion of the Shanghai and Shenzhen 300 index even exceeded that of the S&P 500 index. Overall, in both models, the average weight of the S&P 500 index is the largest, and the weights are generally maintained at high levels. This is due to the good returns and risk levels of the

S&P 500 index itself. The weight change charts of both asset allocation models show that the risk level of the Shanghai and Shenzhen 300 index is gradually increasing, while the risk level of the German DAX30 index is gradually decreasing.

#### 4.3.3. Performance Comparison Analysis

This article assumes that the initial value of each asset allocation portfolio is 1 for comparison, making it more intuitive to examine the rise and fall of returns. The return of asset allocation portfolios is calculated by weighting the returns of each stock index. Therefore, the daily net worth changes of equal weight portfolios, equal volatility portfolios, and risk parity portfolios are shown in the following (See Figure 4):



**Figure 4.** Trend chart of changes in net asset value of asset portfolio

From the graph, it can be seen that the most intuitive manifestation is that the net value change of the risk parity combination highly coincides with the net value change of the equal volatility combination, with only a slight separation in 2017. In the first two years, the net worth of risk parity and equal volatility portfolios did not have a significant advantage over equal weight portfolios. For a short period of time, the net worth of medium weight portfolios even slightly exceeded the other two asset allocation portfolios. However, in the following two years, the net worth of risk parity and equal volatility portfolios showed a separation trend from equal weight portfolios, significantly leading them to equal weight portfolios. The equal weight combination has been showing a fluctuating upward trend in the first two years, reaching its peak in net worth around the beginning of 2018, and then

decreasing in volatility. It reached its minimum net worth in the following two years around the end of 2018, and although it rose in volatility, it did not reach its peak in early 2018. The net worth changes of the equal volatility portfolio and the risk parity portfolio are similar, but the difference is that the net worth of the investment portfolio constructed through these two models exceeded its peak around early 2018, reaching its maximum value throughout the entire observation period. Simple observation alone cannot directly determine the performance of asset portfolios, and the rise or fall in net worth alone is not the entire performance of asset portfolios. Therefore, the performance evaluation indicators obtained by processing the yield data of the three portfolios are shown in the following (See Table 2):

**Table 2.** Statistical Table of Performance Indicators for Each Combination

	Accumulated Yield	annualized return	annualized volatility	Maximum Drawdown	Karma ratio
Equal weight combination	26.616347%	6.077297%	13.173657%	25.269046%	0.240504
Risk Parity Portfolio	39.028176%	8.586468%	11.007566%	23.267185%	0.369038
Equal volatility combination	39.484293%	8.675420%	11.020867%	23.094413%	0.375650

From the table, it can be seen that the equiweighted portfolio has the lowest return rate, the highest volatility and maximum pullback mean the highest level of risk, and the lowest Karma ratio means the lowest return obtained by bearing the maximum loss per unit. Regardless of which dimension, the performance of the equiweighted portfolio is the worst. At the same time, consistent with the previous direct observation of the net worth change chart, the performance of the risk parity combination and the equal volatility combination is very close. The annualized return of the risk parity combination is 0.09% lower than the equal volatility combination, the annualized volatility is 0.02% lower than the equal volatility combination, the maximum pullback is 0.15% higher than the equal volatility combination, and the Karma ratio is 0.006 lower than the equal volatility combination. This means that the return of the risk parity combination is slightly lower than the equal volatility combination, and the risk level is also slightly lower than the equal volatility combination, but the difference is extremely limited.

#### 4.4. Summary and Analysis

Based on the previous discussion, this section mainly selects the three major stock indices as the underlying assets, selects the equal weight model, equal volatility model, and risk parity model, and conducts empirical analysis on market trading data in the past four years based on the determination and explanation of several performance evaluation indicators. From the results of empirical analysis, it can be seen that when the volatility of the underlying asset price is not high, the net worth growth of the equal weight combination may occasionally exceed that of the risk parity combination and the equal volatility combination, but the degree of exceeding is limited. Once the volatility of the underlying asset increases, the performance of the equal weight combination will deteriorate, and good market performance cannot bring better returns to investors. When the market performs poorly, it actually bears more losses. Overall, although equal weight portfolios have certain risk diversification effects, their effectiveness is not good, and as an asset allocation method, they are not the best choice for investors. The results of empirical analysis indicate that the risk parity combination indeed has a good risk diversification effect, and is a valuable asset allocation method that individuals and institutional investors can refer to and further explore. A prominent point in the empirical analysis results of this article is that the net value changes of the equal volatility portfolio and the risk parity portfolio are highly overlapping. This may be due to the insufficient number of types of selected underlying assets. The difference between the method of adjusting weights based on volatility for equal volatility portfolios and the method of adjusting weights based on risk contribution for risk parity portfolios cannot be clearly reflected. Moreover, these two methods mainly consider asset allocation from a risk perspective, with volatility slightly lower than the other two indices but significantly higher annualized and

cumulative returns than the other two indices. The S&P 500 index did not receive a good bias in asset allocation, which is also the reason why the performance of equal volatility and risk parity portfolios is very close.

## 5. Conclusion and Inspiration

This article introduces the relevant background of FOF, focusing on the core content of FOF, which is the allocation of underlying assets. By constructing equal weight combinations, equal volatility combinations, and risk parity combinations for comparative analysis, the practicality and effectiveness of risk parity theory in FOF asset allocation are examined. Through scheme planning and empirical analysis, the following inspirations and suggestions for using risk parity models are drawn:

Firstly, the rapid development and high market demand of FOF are based on its low risk foundation, and FOF's pursuit of profitability should be premised on controlling risk. The risk parity theory is a new breakthrough in modern asset portfolio theory, which determines asset allocation weights by balancing the risk contributions of various underlying assets as much as possible, which can greatly reduce the risk level of asset portfolios. Therefore, it is highly worthy of reference and further exploration by individuals and institutional investors.

Secondly, when using the risk parity method to construct investment models, it is possible to appropriately increase the types of underlying assets to improve the effectiveness of risk diversification, while also controlling the correlation between underlying assets. In addition, this article sets the adjustment period for asset allocation to three months, which actually implies that the nature of the underlying asset itself and its relationships remain unchanged for three months. However, the actual situation is that market conditions are constantly changing. Therefore, in practical application, the frequency of adjustment can be appropriately increased to achieve better risk diversification effect, or suitable performance indicators can be selected in advance to screen underlying assets to reduce the pressure of adjustment in the later stage. The former increases transaction costs, while the latter increases screening costs. The specific balance depends on the actual operation situation.

Thirdly, the risk parity model in this article directly allocates underlying assets. However, different asset portfolios are affected by common risk factors. If the common main risk factors in different assets can be extracted and the risk contribution can be balanced based on the risk factors, the risk parity model may perform better in dispersing risk.

Fourthly, FOF has an inherent flaw of double fees, which makes it undesirable to blindly pursue less risk and allocate too many assets to bond assets or monetary funds. This will make investment portfolios that have already suffered a decline in returns due to risk diversification less attractive to investors after double fees. Fund companies with sufficient scale and complete product categories can reduce the

management fees of the parent fund by limiting their investment targets to their own fund products. If the fund company's own conditions are insufficient, it can also adjust leverage to expand returns while controlling risks.

FOF started relatively late in China but has enormous potential. With the vigorous promotion of regulatory authorities and the development and improvement of the market, FOF has also received increasing attention from investors and scholars. Research on risk parity theory can help promote the development and improvement of FOF and help it go further in China.

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