

Research on Pricing Model of Chinese Convertible Bonds

-- Based on The Nature of Convertible Bonds Themselves

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Abstract: This paper aims to create a new model for convertible bond pricing based on convertible bond rules and convertible bond strategies. And based on the concept of value investment, this paper analyzes and deduces various situations of convertible bonds by using future cash flow discount thinking. In addition, two new options that achieve the conditional call price and options that exceed the conditional call price are presented, and a more flexible formulation of the model for computing the price of convertible bonds is derived. However, this model requires the user to have some experience with convertible bonds. This might make a bit of sense for academic development and could help people better understand and participate in the convertible bond market.

Keywords: Financial Market, Pricing of Convertible Bonds, Convertible Arbitrage.

1. Introduction

1.1. Research background

In 2017, to encourage the issuance of convertible bonds, the China Securities Regulatory Commission revised the new rules for convertible bonds in September of the same year, changing the fund subscription to a credit subscription. Since then, the scale of the convertible bond market has continued to expand, the richness of the issuer industry has increased, and the convertible bond has attracted more and more attention from a tiny variety, and the investor structure has also changed a lot, and the investment strategy of convertible bond has become more diversified. The size of the convertible bond market continued to grow from 34.4 billion yuan at the end of 2016 to 647.1 billion yuan at the end of 2021 with a total turnover of 15.2 trillion yuan, showing strong liquidity and ample vitality. In terms of financing size, it grew from 21.252 billion yuan in 2016 to 277.167 billion yuan in 2021, with an average annual growth rate of more than 160 percent. In the same period of 2021, the total financing of the A-share market was 1,674.3 billion yuan, and convertible bonds accounted for 16.55% of the total quota financing of the A-share market, which became an essential part of the financing of the A-share market, and also an essential part of China's financial market. Therefore, studying the pricing of convertible bonds will help investors better participate in the investment of the convertible bond market, thus attracting more people and funds to pay attention to the convertible bond market and promote the healthy development of the convertible bond market, which can also promote the further development of China's financial market, provide a better financial market environment for listed companies, and thus promote the better development of China's economy.

1.2. Research overview

There have been more than 20 years of research on the pricing of convertible bonds in China, which are mostly based

on the theory of option pricing. For example, Zheng Zhenlong and Lin Hai's research on the pricing of Chinese convertible bonds is based on the Black-Scholes model as the basic formula. [1] They assumed that the equity conversion in China's convertible bonds would not be executed in advance, so the value of the convertible bonds W was roughly expressed as:

Normal European call option value + maturity date bond value present value + duration bond interest present value

This model, proposed by American economists Fischer Black and Myron Scholes in 1973. [2] Which is a mathematical model for pricing options contracts. The model is based on a series of assumptions and is primarily applicable to European-style options, i.e. options that can only be exercised on the expiration date. However, in fact, the conversion of convertible bonds to equity in China is executed in advance, and the Black-Scholes model is also set based on a series of assumptions, so the actual situation of the market often deviates. Another example is Jiang Dianchun and Zhang Xin's research on the pricing of convertible corporate bonds. [3] It uses a binary tree model to price Chinese convertible bonds, demonstrates in detail the irrationality of Black-Scholes model, and demonstrates the applicability of the binary tree model to the pricing of Chinese convertible bonds. However, the binary tree model also has its limitations, such as the approximate error caused by discretization and the assumption of market volatility. In addition, Zhang Weiguo, Shi Qingsheng and Xu Wenkun study convertible bond pricing based on full least squares quasi-Monte Carlo method. [4] By using stochastic Faure sequence and variance reduction techniques, the least square Monte Carlo simulation pricing method is used to price convertible bonds. However, for complex financial products such as convertible bonds, the selection and fitting of the return function may involve some subjective judgments, which need to be carefully considered and verified. Therefore, most of the existing research on the pricing of convertible bonds is derived from the research on options. However, as a financial product containing complex and exotic options,

convertible bond is not only a single option, so many pricing models are difficult to explain the specific composition of various options and prices. Therefore, it is difficult to judge the actual conditions of convertible bonds. And convertible bonds have a very strong part of its bond nature, which is the basic nature can not be ignored. Therefore, based on the nature of bonds, I will analyze the price of convertible bonds by combining various constituent options, and derive a pricing model that is exclusive to convertible bonds and can be used for price analysis of convertible bonds under specific conditions from the nature and regulations of convertible bonds. In addition, I bring the mathematical symbol "v", which is rarely used in economic analysis, into the formula in order to show all future possibilities at one point in time, thus assisting investors in making better decisions.

2. Research on Pricing of Convertible Bonds

2.1. Assumed condition

Assumption1: There is no speculation on the convertible bond itself in the market.

Reasoning: Because in the convertible bond market there will be some speculation on the convertible bond itself. In these cases, the price of convertible bonds does not depend on the value of convertible bonds nor on the value of equity swaps, but is far from the hype of the value of the bonds themselves. For example, this situation often occurs in the newly listed convertible bond market, because the lock-up period of major shareholders leads to the relatively small amount of convertible bonds in circulation, which will attract the attention of hot money for speculation. In addition to this situation, other possible situations such as the small size of the convertible bond or some other special factors may also cause speculation. Under such circumstances, it is very dangerous to participate in the transaction of convertible bonds, so it is not recommended to participate in the transaction of convertible bonds under such circumstances. In this case, it is difficult to estimate the price, so it is assumed that there is no speculation on the convertible bond itself in the market.

Assumption2: The optimal strategy of China's convertible bond issuers is to encourage investors to convert convertible bonds into company shares as early as possible and at the highest possible conversion price.

Demonstration: See Zheng Zhenlong and Lin Hai [5]

Assumption3: Suppose the interest on a convertible bond is paid once a year.

Reasoning: Because interest payments on Chinese convertible bonds are usually paid once a year.

2.2. Pricing model derivation

As for the derivation of the convertible bond model, we start with the most basic part of the bond value, the conversion value and the resale value.

The bond value is calculated using the formula that the future cash flow is converted to the current value. The future cash flow conversion formula is as follows:

$$PV = (CF_1)/(1+r)^1 + (CF_2)/(1+r)^2 + \dots + (CF_n)/(1+r)^n \quad (1)$$

PV is the present value or discounted value. CF_i is the cash flow for Phase i. r is the discount rate, which represents the opportunity cost or discount rate of each period of funds. In

the calculation of bond value, r is generally the risk-free rate of return. But, in the case of convertible bonds, r needs to make a judgment based on its bond grade and risk. The formula takes into account the concept of time value, where the value of money in the future is lower than the value of money in the present.

Based on this, we derive that the bond value v :

$$v = \frac{I_1}{(1+r)^1} + \frac{I_2}{(1+r)^2} + \dots + \frac{I_n + F + F*(r_b - 1)*(1-t_r)}{(1+r)^n} \quad (2)$$

Here, I_n stands for the interest of each period, F stands for the face value of the bond, and r_b stands for the ratio of the after-tax repurchase price to the face value. The repurchase price is generally used to compensate the creditor for the lower interest charged on the loan funds. In many cases in China, the repurchase price includes the last interest, so I_N can be regarded as 0 and only the cash flow of repurchase can be calculated. t_r represents the tax rate. Because Chinese individuals and institutions participate in the convertible bond market, the interest tax rate is different, 20% for individuals and 10% for institutions. Moreover, institutions are more specialized in the market, have larger capital scale and more financial instruments, so it is suggested that the tax rate should be calculated based on the tax rate of institutions. The formula for calculating I_n should be:

$$I_n = F * r_n * (1 - t_r) \quad (3)$$

r_n is the interest rate per period.

According to the simple relationship between the face value, the conversion price and the present value of the stock, an accurate calculation formula for the conversion value of the bond can be directly obtained:

$$v_t = \frac{F}{P_z} * P_s \quad (4)$$

Where v_t is the conversion value, P_z is the conversion stock price, and P_s is the current sale price of the stock.

Regarding the repurchase price, according to the current accrued interest formula of the convertible bond:

$$IA = B * i * \frac{t}{365} \quad (5)$$

IA stands for interest accrued during the period; B refers to the total face amount of the convertible bonds held by the holders of the convertible bonds issued this time; i refers to the current coupon rate of the convertible bond; t refers to the number of interest accruing days, that is, the actual number of calendar days from the last interest payment date to the redemption date of the current interest accruing year (counting the beginning and not the end).

We can get the resale price should be:

$$V_r = F + F * r_n * \frac{t}{365} \quad (6)$$

t is the number of days that interest accrues and r_n is the current interest.

Next, we begin to deduce the market price, because the establishment of the price of the value part of the convertible bond needs to be based on the company's ability to repay the

convertible bond. Therefore, when judging whether the partial value of convertible bonds is established, various indicators such as current ratio, quick ratio, cash ratio, interest cover multiple, debt-to-total asset ratio, long-term debt ratio and short-term debt ratio should be used to judge whether the company has default risk in combination with the company's financial statements and operating conditions. When we judge that the company is at risk of default and that the company is unable to repay the convertible debt, the underlying value of the convertible debt is only the conversion value. The market price shall be the conversion value plus the option value. The option value can be divided into the option of the new convertible bond value after the revision of the game convertible bond and the option of the game company debt restructuring. And only when the expected value is greater than the conversion value of the convertible bond will the price of the option part be higher than the conversion value of the convertible bond. So now we can express the bond price P as:

$$P = v_t + [(R * v_{t1}) \vee (R * V_r)] - v_t = \left(R * \frac{F}{P_z} * P_{s1}\right) \vee [R * (F + F * r_n * \frac{t}{365})] \quad (7)$$

Here:

$$[(R * v_{t1}) \vee (R * V_r)] - v_t \quad (8)$$

Is the partial price of the option. Because the waiting time of this option game is generally short, the future cash flow is not converted.

$$R * v_{t1} - v_t \quad (9)$$

It is the option price on the downwardly revised tranche of the convertible bond. v_{t1} is the expected conversion value of the convertible bonds after the downward revision, P_{z1} is the expected conversion price after the downward revision, and P_{s1} is the estimated sale price of the shares after the downward revision.

$$R * V_r - v_t \quad (10)$$

It is the option price of the convertible bond game restructuring redemption or other means of recovering the face value. R represents the probability of the event, which can be estimated through the accumulation of long trading experience in the convertible bond market. For example, Zhengbang convertible bond (convertible bond code 128114) plummeted in May 2023 due to the company's loss and the risk of delisting and default of the convertible bond. However, after the intention of 13 companies to participate in the restructuring of its convertible bond restructuring possibility to increase the price of convertible bonds to return, after the restructuring of investors to determine the strength of the great Great Korong Holding Co., Ltd. to participate in its restructuring, successful part of the bond face value payment success rate of close to 100 percent, so its price recovered to near 100 yuan. " \vee " is the mathematical symbol "or".

When the expected value is equal to or less than the conversion value of the convertible bond at this time, the conversion bond price is only represented by the conversion price at this time, for example, in July 2023, Blue Shield shares enter the delisting process. Blue Shield convertible bond (convertible bond code 123015) after analysis, there are no such conditions as downward revision, restructuring, promotion of conversion, back sale, redemption, etc. So its price has been fluctuating around its conversion price. At this

time, the price formula of the convertible bond is:

$$P = v_t = \frac{F}{P_z} * P_s \quad (11)$$

To sum up, we can summarize the formula when the value of the bond part of the convertible bond is not established as follows:

$$P = v_t \vee (R * v_{t1}) \vee (R * V_r) = \left(\frac{F}{P_z} * P_s\right) \vee \left(R * \frac{F}{P_{z1}} * P_{s1}\right) \vee \left[R * \left(F + F * r_n * \frac{t}{365}\right)\right] \quad (12)$$

Next, we begin to discuss the value of convertible bonds when part of the value of convertible bonds is established. First, the base value of a convertible bond can be easily expressed as the discounted present value of the bond and the conversion value. If the value of this part is denoted by the letter B_v , the equation should be:

$$B_v = v \vee v_t = \left[\frac{I_1}{(1+r)^1} + \frac{I_2}{(1+r)^2} + \dots + \frac{I_n + F + F * (r_b - 1) * (1 - t_r)}{(1+r)^n}\right] \vee \left(\frac{F}{P_z} * P_s\right) \quad (13)$$

However, in ordinary times, it is seriously inconsistent with the actual situation to measure the market price of convertible bonds only by the basic value in the financial market, because as a complex financial product, convertible bonds still have many options to measure. However, relevant studies usually only talk about these options in general and regard them as the same factor, which makes it difficult to adjust the price estimate of convertible bonds according to changes in specific relevant information of convertible bonds. Or by looking at the specific forms of these options one by one, this does not simplify the complex issue or is often limited to part of the overall integrity. Therefore, the following will put forward the value of options concretized fresh thinking.

First, if the market price of the bond is only the discounted face value of the bond or the conversion price, then the convertible bond is only represented by its underlying value. If the market price of the convertible bond is higher than the above two base values, then the higher part is the option price. In terms of option prices, we divide into options that achieve the conditional call price and options that exceed the conditional call price. What is a conditional call option price? For example, in the listing notice of Huaibei Mining (stock code 600985) Huai 22 convertible bonds (convertible bond code 110088), the conditional redemption clause of the issuance terms mentions that "during the convertible bond conversion period of this issue, when either of the following two situations occurs, The board of directors of the company has the right to decide to redeem all or part of the convertible bonds that have not been converted at the price of the face value of the bonds plus the accrued interest for the current period: ① During the convertible bond conversion period, if the closing price of the company's A-shares on at least 15 of the 30 consecutive trading days is not less than 130% of the current conversion price (including 130%) ". So the face value of the bond multiplied by 130% or \$130 is the conditional call price mentioned earlier. According to the research of Zheng Zhenlong and Lin Hai [5], the optimal strategy for Chinese convertible bond issuers is to encourage investors to convert convertible bonds into company shares at the highest possible conversion price as soon as possible. When the price of the convertible bond reaches the

conditional redemption price of the convertible bond for a certain period of time, the board of directors of the company has the right to decide to redeem all or part of the convertible bond at the price of the face value of the bond plus the current accrued interest. If the board of directors were to redeem the bonds, creditors would face a loss of interest from the current conditional redemption price to the face value of the bonds plus the current accrued interest, thereby facilitating the conversion of the convertible bonds to holders. In turn, the creditors of convertible bonds have reason to expect that the issuer will be motivated to promote the conversion of convertible bonds, so that the price of convertible bonds will reach the conditional redemption price in the future. This section is the option that reaches the conditional call price. But different companies face different market conditions and may reach conditional call price at different times. For example, some companies reach the callable condition price in advance due to the increase in the value of shares before the listing of bonds, and may directly enter the stage of options that exceed the conditional call price. Some companies like to carry out convertible debt to equity to achieve the tradition of financing purposes, can be estimated based on the past situation to reach the redemption condition price. Others, because they are in a fast-growing or hot sector of the market, can estimate how long it will take them to reach a callable price based on their rise. Thus set the option value O_i that reaches the conditional call price to:

$$O_i = \frac{M}{(1+r)^n} - B_v \quad (14)$$

Where M denotes the price at which the convertible bond reaches the conditional redemption, n denotes the estimated time to reach the conditional redemption price, and r is the same as the previous r .

Options that exceed the conditional call price lose the value of the purchase above the call price as the stock falls. This is similar to a call option on a stock, and this part of the price after the direct purchase of the stock bullish price is consistent. So it's essentially the same as buying shares directly. Therefore, it is generally not recommended to participate in this type of convertible bond transaction. Its option value is O_o :

$$O_o = 0 \quad (15)$$

When the conditional redemption price is exceeded, the convertible bond is priced as follows:

$$P = v_t \quad (16)$$

In addition to the two options mentioned above that have an impact on the price, there are many options that can affect the price of a convertible bond. But these terms can be grouped into two categories: call and sell-back, but since they are calculated in the same way, we can view the bond price in this case as follows:

$$P = R * V_r = R * \left(F + F * r_n * \frac{t}{365} \right) \quad (17)$$

Because there is also the possibility of a downward revision because the stock price is going down. Therefore, to sum up, the bond price formula when the partial value of the convertible bond is established should be as follows:

$$P = v \vee v_t \vee (B_v + O_o) \vee (R * V_r) \vee (R * v_{t1}) =$$

$$\left[\frac{I_1}{(1+r)^1} + \frac{I_2}{(1+r)^2} + \dots + \frac{I_n + F + F * (r_b - 1) * (1 - t_r)}{(1+r)^n} \right] \vee \left(\frac{F}{P_z} * P_s \right) \vee \left[\frac{M}{(1+r)^n} \right] \vee \left[R * \left(F + F * r_n * \frac{t}{365} \right) \right] \vee \left(R * \frac{F}{P_{z1}} * P_{s1} \right) \quad (18)$$

The value of the bond part of the comprehensive convertible bond is established and not established. The price formula for convertible bonds can be summarized as follows:

$$P = v_t \vee (R * v_{t1}) \vee (R * V_r) \vee v \vee (B_v + O_o) \quad (19)$$

By the above deduction, we can find that the final decision of the convertible bond price comes from only two parts: the bond value and the equity transfer value. It's just that there are various factors that are affecting these two things that are causing the price of the bond to change. As an old Chinese saying goes, "All changes remain the same." Therefore, no matter what kind of rule changes will occur in the convertible bond market in the future, as long as the study of its final impact on the bond value and equity transfer value is clear, it can help market participants judge the real value of convertible bonds, and more convenient for market participants to make decisions. Thus, an abstract formula is given to measure the value of a convertible bond:

$$S = (A \dots) \vee (E \dots) \quad (20)$$

S represents the value of the convertible bond, $A \dots$ represents the various possibilities of its bond value portion, and $E \dots$ represents the various possibilities of the conversion value.

3. Conclusion

Through the summary of the rules of convertible bonds and empirical research, this study takes value investment as the guidance and deduces a new pricing model of convertible bonds according to the discount formula of future cash flow, which is different from most studies. The paper analyzes and summarizes the value of option, and creatively puts forward two kinds of option forms: the option that reaches the conditional call price and the option that exceeds the conditional call price. Finally, the models that can be used to calculate the price of convertible bonds under various conditions are obtained:

$$P = v_t \vee (R * v_{t1}) \vee (R * V_r) \vee v \vee (B_v + O_o) \quad (21)$$

This is a combination of strategy research and pricing research on convertible bonds. It is also the further study of convertible bond option. While it may have some significance for academic development, it can also help participants in the convertible bond market better understand and participate in the convertible bond market. The advantage of this model is that it can estimate the value of convertible bonds well according to various specific changes in the actual situation. At the same time, this model also has a difficult problem to solve. That is, the calculation of this model requires users to have some experience in convertible bonds. For example, the discount rate R , the time to reach the redemption price n and the probability of various situations R need to be judged by investors.

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