

Financing Mechanism Design of Small, Medium and Micro Manufacturing Enterprises with Capital Constraints in Different Life Cycle Stages

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Abstract: In order to solve the problem that manufacturers have sufficient funds when producing traditional products and lack of funds when producing green products, this paper considers three financing strategies: no financing, bank loan financing and internal financing, and constructs centralized decision-making and manufacturer-led Stackelberg game model. It is found that the wholesale price, production input, green input level and supply chain profit of the three financing methods are not optimal under the manufacturer-led decision-making. On this basis, aiming at the overall profit under centralized decision-making, this paper designs the combined contract of "revenue sharing contract + two-part cost + green cost sharing" to coordinate the supply chain, and puts forward the scope of optimal coordination of the combined contract under three financing models.

Keywords: Financing optimization, Capital constraints, Green supply chain.

1. Introduction

Due to the global awareness of environmental deterioration and energy exhaustion [1], in recent years, in order to protect the environment, save resources and build a sustainable green economic system, countries have successively introduced relevant policies to promote the rapid development of the green economic system. In December 2019, the European Commission announced the European Green Deal to combat climate change and promote sustainable development. In February 2021, The State Council issued the Guiding Opinions on Accelerating the Establishment of a sound Green low-carbon Circular Development economic System, pointing out that it is necessary to establish a sound green low-carbon circular development economic system, ensure the realization of carbon peak and carbon neutrality goals, and promote China's green development to a new level. In February 2022, the U.S. Department of Energy released a Strategy to Protect Supply Chains for a Clean Energy Transition, including increasing domestic raw material supply to expand domestic manufacturing capacity, increasing clean energy deployment, attracting skilled U.S. clean energy workforce, and enhancing supply chain information and decision-making capabilities. In this context, more and more enterprises are slowly shifting from the traditional supply chain to the green supply chain, and constantly increasing the level of product green related technology investment, in order to improve their competitive advantage. For example, GAC Honda Automobile Co., Ltd launched the "green service" project, requiring dealers to adopt new equipment, new materials and new processes that are conducive to environmental protection [2]. H&M, a well-known fast fashion retailer, applies carbon emission reduction technology to reduce pollutant emissions in the manufacturing process and improve its competitiveness [3]. All these indicate that the implementation of green supply chain management by enterprises has become an inevitable trend [2].

However, in the process of implementing green supply chain management, enterprises, on the one hand, face

financial constraints due to the high investment in technology related to the production of green products; on the other hand, enterprises will increase additional costs when implementing green supply chain, which leads to a certain conflict of interest distribution between upstream and downstream enterprises [4]. Therefore, for the green supply chain composed of manufacturers and retailers, it is of great practical significance to study the coordination contract mechanism under different financing decisions.

The research related to this paper mainly involves two aspects: (1) The research on green supply chain financing. Zhang Daru et al. [5] constructed a green supply chain consisting of a single financially constrained manufacturer and a single retailer, and studied the impact of trade credit loan and bank loan on pricing decisions and profits of manufacturers and retailers. Jin Yaqi et al. [6] studied the green product investment and financing decision of suppliers under the single financing channel of bank credit and the mixed financing channel of retailers paying part of the loan in advance and bank credit. Wang Wenli and Zhang Ruizhen [7] studied the optimal decision and profit of both sides of the supply chain under green credit and prepayment financing, considering the situation of suppliers with financial constraints and risk of R&D failure to conduct green product R&D. An et al. [8] constructed a supply chain consisting of a manufacturer with limited capital and a supplier with sufficient capital facing uncertain demand, and studied how the supply chain chooses green credit financing and trade credit financing under the hard constraint of carbon emission. Wu et al. [9] studied the optimal pricing, optimal emission reduction and ordering decision under commercial credit and bank credit respectively in the green supply chain with financial constraints of retailers. (2) Related research on green supply chain coordination. Sun Jiayi [10] and other scholars have studied the decision-making and coordination problems of low-carbon closed-loop supply chain with government subsidies and equity cooperation. Yu Nana and Yuan Shuaipeng [11] designed a joint contract consisting of revenue sharing and cost sharing, and studied the

coordination problem of closed-loop supply chain with differential pricing, considering the carbon emission reduction implemented by manufacturers and the low carbon preference of consumers. Under the carbon cap-and-trade mechanism, Tai Xianghui and Li Fang [12] discussed the relationship between carbon emission reduction rate, sales price, supply chain profit and consumers' low-carbon preference under decentralized and centralized decision-making, and coordinated the supply chain through contracts. Pan Chen et al. [13] studied the selection of different cost-sharing contracts in a green supply chain consisting of a manufacturer and a retailer. Peng et al. [14] studied the optimal decision-making of supply chain production, pricing and carbon emission reduction under decentralized and centralized decision-making, and the research results showed that quantity discount contracts can effectively coordinate low-carbon supply chains.

Through the above literature review, it can be found that the research on green supply chain financing and the research on green supply chain coordination rarely focus on medium and micro manufacturing enterprises. However, in reality, medium and micro manufacturing enterprises are universal, and are an important pillar of the national economy. Therefore, this paper constructs a two-level supply chain consisting of a single financially constrained manufacturer and a single retailer. Among them, manufacturers can choose to produce traditional products or green products, the production of traditional products have sufficient funds without financing, but the production of green products face financial constraints, manufacturers can choose bank loans or internal financing financing methods to solve their own shortage of funds. Based on the above background, under the three models of no financing, bank loan and internal financing, this paper studies the optimal decision of product green input level, production input amount and supply chain profit under manufacturer-led decentralized decision making. On this basis, this paper takes the overall profit under the centralized decision-making as the goal, and designs a combined contract of 'revenue sharing contract + two costs + green cost sharing' to coordinate the supply chain.

2. Problem Description and Model Assumptions

2.1. Problem Description

Consider a two-level supply chain consisting of a manufacturer and a retailer. In a two-level supply chain, the manufacturer wholesales the product to the retailer at the price p , while the retailer sells the product directly to the market at the price w . The manufacturer holds the decision to produce the product, and can choose to produce traditional products or green products during production. When manufacturers choose to produce traditional products, they have sufficient funds and do not need financing, but when manufacturers choose to produce green products, they will face a shortage of funds. At this time, manufacturers can choose bank loans or internal financing of retailers to solve the problem of financial constraints. In this case, this paper considers the financing optimization mechanism design of small, medium and micro manufacturing enterprises under the non-financing model, bank loan model and internal financing model.

2.2. Model Assumptions

In this paper, π_i^{jg} represents the profit of supply chain member i when it chooses decision mode j under model g , where $i \in \{M, R, S\}$ represents manufacturer, retailer and supply chain system respectively; $j \in \{A, B, C\}$ represents no financing model, bank loan model and internal financing model; $g \in \{I, D, N\}$ represents centralized decision-making, decentralized decision-making and combined contract decision-making respectively. In addition, based on the actual situation, without losing the premise of generality, this paper puts forward the following hypothesis:

(1) The price for products is affected by the randomness of the production input, immediate factor of output, green input level, and market price, that is, $p_t = a - (q_t + \varepsilon) + e_t + \xi$. Here, $t \in \{1, 2, 3\}$ represents chain 1 (no financing), chain 2 (bank loan), chain 3 (internal financing), the mean and variance of ε are u_1 and δ^2 , the mean and variance of ξ are u_2 and 0, respectively; a is the scale of market demand; q is the production input; e is the level of green; and the green input cost is $C(e) = e^2$.

(2) Under decentralized decision-making, the manufacturer is the leader of the Stackelberg game and the retailer is the follower.

(3) The interest rate paid by the manufacturer for internal financing with the retailer is the same as the interest rate paid by the manufacturer for choosing a bank loan.

(4) Market information is symmetrical and complete, and all decision-making members of the supply chain are risk-neutral decision-makers.

3. No Financing (Model A)

Under the no-financing model, manufacturers only produce traditional products and have sufficient funds, the expected profit functions of manufacturers, retailers and supply chains can be obtained as follows:

$$\pi_M^A = (w_1 - c)(q_1 + u_1) \quad (1)$$

$$\pi_R^A = (a - q_1 + u_2 - w_1)(q_1 + u_1) - u_1 q_1 - u_1^2 - \delta^2 \quad (2)$$

$$\pi_S^A = (a - q_1 + u_2 - w_1)(q_1 + u_1) + (w_1 - c)(q_1 + u_1) - u_1 q_1 - u_1^2 - \delta^2 \quad (3)$$

3.1. Decentralized Decision-making

Under the decentralized decision-making, the Stackelberg game is played between manufacturer and retailer. The order of the manufacturing supply chain decision is as follows: First, the manufacturer decides the wholesale price w_1 of the product to the retailer; Secondly, the retailer decides its production input q_1 according to the manufacturer's decision, and solves equations (1) and (2) by backward derivation, the optimal wholesale price w_1^{D*} and optimal production input q_1^{D*} of the manufacturer and retailer can be obtained as follows:

$$\begin{cases} w_1^{D*} = \frac{1}{2}(a + c + u_2) \\ q_1^{D*} = \frac{1}{4}(a - c - 4u_1 + u_2) \end{cases}$$

By substituting the optimal wholesale price w_1^{D*} and the optimal production input q_1^{D*} into (1) and (2), the optimal expected profit of the manufacturer π_M^{AD*} and the optimal expected profit of the retailer π_R^{AD*} are respectively:

$$\begin{cases} \pi_M^{AD*} = \frac{1}{8}(a - c + u_2)^2 \\ \pi_R^{AD*} = \frac{1}{16}[(a - c + u_2)^2 - 16\delta^2] \end{cases}$$

3.2. Centralized Decision-making

Under the centralized decision-making, the manufacturer and the retailer belong to a unified whole, and the profit of the whole supply chain system is maximized by determining the optimal production input q_1^{I*} . At this time, the expected profit of the supply chain is expressed as (3). The first-order condition of production input q_1 is obtained by equation (3), and the optimal production input q_1^{I*} is obtained as follows :

$$q_1^{I*} = \frac{1}{2}(a - c - 2u_1 + u_2)$$

By substituting the optimal production q_1^{I*} input into (3), the optimal expected profit π_S^{AI*} of the supply chain is obtained as follows:

$$\pi_S^{AI*} = \frac{1}{4}(a - c + u_2)^2 - \delta^2$$

Proposition 1: Under the non-financing model, compared with the decentralized decision-making, the centralized decision-making can promote the increase of production input, that is $q_1^{D*} < q_1^{I*}$. So decentralized decision-making cannot coordinate the supply chain.

3.3. Combined Contract Decision-making

It can be seen from proposition 1 that under the decentralized decision making, the production input of the manufacturer cannot reach the optimal level under the centralized decision making. Therefore, this section constructs a combined contract of "Revenue sharing contract + two-part fee + green cost sharing" to coordinate the supply chain, which is marked $(\varphi, k, T, \lambda, w)$. Specifically: (1) the manufacturer will share the φ ($0 < \varphi < 1$) times of its own income with the retailer, and the retailer will also share the green production cost of the manufacturer k ($0 < k < 1$) times, in order to encourage the manufacturer to improve the green level of the product, and the retailer will also provide financial support T for the manufacturer to improve the production equipment and other costs; (2) λ ($0 < \lambda < 1$) is a multiple of the expected profit in the supply chain, and w is the manufacturer's wholesale price per unit of product.

Under this mechanism, the expected profit function of the manufacturer and retailer in the non-financing model is as follows:

$$\pi_M^{AN} = (1 - \phi)(w_1 - c)(q_1 + u_1) + T$$

$$\begin{aligned} \pi_R^{AN} &= [(a - q_1 + u_2 - w_1)(q_1 + u_1) - u_1 q_1 \\ &\quad - u_1^2 - \delta^2] + \phi(w_1 - c)(q_1 + u_1) - T \end{aligned}$$

Under the combined contract decision-making, the first derivation of production input q_1 is obtained. Let $\frac{\partial \pi_R^{AN}}{\partial q_1} = 0$, and the optimal production input q_1^{N*} is:

$$q_1^{N*} = \frac{1}{2}(a - 2u_1 + u_2 - w_1 - c\phi + w_1\phi)$$

Proposition 2: Under the combined contract, if the contract parameters meet the following conditions, the combined contract can effectively coordinate the studied supply chain, that is, $\varphi = 1$.

At this time, the expected profits of manufacturers and retailers are :

$$\begin{cases} \pi_M^{AN*} = \lambda \pi_S^{AI*} + T = T \\ \pi_R^{AN*} = (1 - \lambda) \pi_S^{AI*} - T = \pi_S^{AI*} - T \end{cases}$$

Proposition 3: In the case of supply chain coordination, if the financial support meets the following conditions, the expected profits of manufacturers and retailers will be improved by Pareto.

$$\frac{1}{8}(a - c + u_2)^2 \leq T \leq \frac{3}{16}(a - c + u_2)^2$$

Certificate

(1) That is, $\varphi = 1$ is substituted into q_1^{N*} , it can be found that $q_1^{N*} = q_1^{I*}$, so the supply chain is coordinated

(2) The constraint conditions for manufacturers and retailers to participate in combined contracts are $\pi_M^{AN*} > \pi_M^{AD*}, \pi_R^{AN*} > \pi_R^{AD*}$ respectively, and the result of proposition 2 can be obtained by combining the expected profits of manufacturers and retailers under decentralized decision-making.

4. Bank Loan (Model B)

Under the bank loan model, the manufacturer only produces green products, and the manufacturer is faced with financial constraints. The manufacturer solves the problem of insufficient funds by borrowing from the bank, and the expected profit function of the manufacturer, retailer and supply chain can be obtained as follows:

$$\pi_M^B = (w_1 - c)(q_1 + u_1) - (1 + r)e_2^2 \quad (4)$$

$$\begin{aligned} \pi_R^B &= (a - q_2 + e_2 + u_2 - w_2)(q_2 + u_1) \\ &\quad - u_1 q_2 - u_1^2 - \delta^2 \end{aligned} \quad (5)$$

$$\begin{aligned} \pi_S^B &= (w_2 - c)(q_2 + u_1) - (1 + r)e_2^2 + \\ &\quad (a - q_2 + e_2 + u_2 - w_2)(q_2 + u_1) - u_1 q_2 - u_1^2 - \delta^2 \end{aligned} \quad (6)$$

4.1. Decentralized Decision-Making

Under decentralized decision-making, the Stackelberg game is played between manufacturer and retailer. The order of the manufacturing supply chain decision is as follows: First, the manufacturer decides the wholesale price w_2 and the green input level e_2 ; Secondly, the retailer decides its production input q_3 according to the manufacturer's decision, and then uses the backward derivation method to solve equations (4) and (5), the optimal wholesale price w_2^{D*} , the optimal green input level e_2^{D*} and the optimal production input q_2^{D*} of the manufacturer and retailer can be obtained as follows:

$$\begin{cases} w_2^{D*} = \frac{4a(1+r) + c(3+4r) + 4(1+r)u_2}{7+8r} \\ e_2^{D*} = \frac{a-c+u_2}{7+8r} \\ q_2^{D*} = \frac{2(a-c+u_2)(1+r) - (7+8r)u_1}{7+8r} \end{cases}$$

By substituting the optimal wholesale price w_2^{D*} , the optimal green input level e_2^{D*} and the optimal production input q_2^{D*} into (4) and (5), the optimal expected profit of the manufacturer π_M^{BD*} and the optimal expected profit of the retailer π_R^{BD*} are respectively:

$$\begin{cases} \pi_M^{BD*} = \frac{(1+r)(a-c+u_2)^2}{7+8r} \\ \pi_R^{BD*} = \frac{4(a-c+u_2)(1+r)^2 - (7\delta+8r\delta)^2}{(7+8r)^2} \end{cases}$$

4.2. Centralized Decision-Making

Under centralized decision-making, the manufacturers and the retailers belong to a unified whole and maximize the profits of the entire supply chain system by determining the optimal green input level e_2^{I*} and the optimal production input q_2^{I*} . At this time, the expected profit of the supply chain is expressed as formula (6), and the first-order and second-order conditions of green input level e_2 and production input q_2 are obtained by formula (6). It can be seen that π_S^B is the joint concave function of (e_2, q_2) . By solving $\frac{\partial \pi_S^B}{\partial e_2} = 0$ and $\frac{\partial \pi_S^B}{\partial q_2} = 0$ jointly, the optimal green input level e_2^{I*} and production input q_2^{I*} of the manufacturer can be obtained as follows:

$$\begin{cases} e_2^{I*} = \frac{a-c+u_2}{3+4r} \\ q_2^{I*} = \frac{2(a-c+u_2)(1+r) - (3+4r)u_1}{3+4r} \end{cases}$$

By substituting the optimal green input level e_2^{I*} and the optimal production q_2^{I*} input into (6), the optimal expected profit π_S^{BI*} of the supply chain is obtained as follows:

$$\pi_S^{BI*} = \frac{(a-c+u_2)^2(1+r) + ru_2^2 - (3+4r)\delta^2}{3+4r}$$

Proposition 4: Under the bank loan model, compared with the decentralized decision-making, the centralized decision-making can increase the production input and the green input level, that is, $e_2^{D*} < e_2^{I*}, q_2^{D*} < q_2^{I*}$. So decentralized decision-making cannot coordinate the supply chain.

4.3. Combined Contract Decision-Making

It can be seen from proposition 4 that under the decentralized decision making, the production input and green input level of the manufacturer cannot reach the optimal level under the centralized decision making. Thus, like section 3.3 composite contract decisions, this section coordinates the supply chain by building composite contract decisions. Under this mechanism, the expected profit function of the manufacturer and retailer in the bank loan model is as follows:

$$\begin{aligned} \pi_M^{BN} &= (1-\phi)[(w_2-c)(q_2+u_1) \\ &\quad - (1+r)e_2^2] + k(1+r)e_2^2 + T \end{aligned}$$

$$\begin{aligned} \pi_R^{BN} &= (a-q_2+e_2+u_2-w_2)(q_2+u_1) - u_1q_2 - u_1^2 - \delta^2 + \\ &\quad \phi[(w_2-c)(q_2+u_1) - (1+r)e_2^2] - k(1+r)e_2^2 - T \end{aligned}$$

Under the combined contract decision-making, the first-order and second-order conditions of green input level e_2 and production input q_2 are obtained, and it can be seen that π_R^{BN} is the joint concave function of (e_2, q_2) . By solving the sum jointly, let $\frac{\partial \pi_R^{BN}}{\partial e_2} = 0, \frac{\partial \pi_R^{BN}}{\partial q_2} = 0$, the optimal green input level e_2^{N*} and production input q_2^{N*} of the manufacturer can be obtained as follows:

$$\begin{cases} e_2^{N*} = \frac{a+u_2-w_2-c\phi+w_2\phi}{-1+4k(1+r)+4(1+r)\phi} \\ \quad u_1 - 4u_1(k-\phi)(1+r) + 2(1+r)(k+\phi) \\ q_2^{N*} = \frac{[a+u_2+w_2(-1+\phi) - c\phi]}{-1+4(k+\phi)(1+r)} \end{cases}$$

Proposition 5: Under the combined contract, if the contract parameters meet the following conditions, the combined contract can effectively coordinate the studied supply chain, that is, $\phi = 1, k = 0$.

At this time, the expected profits of manufacturers and retailers are :

$$\begin{cases} \pi_M^{BN*} = \lambda \pi_S^{BI*} + T = T \\ \pi_R^{BN*} = (1-\lambda)\pi_S^{BI*} - T = \pi_S^{BI*} - T \end{cases}$$

Proposition 6: In the case of supply chain coordination, if the financial support meets the following conditions, the expected profits of manufacturers and retailers will be improved by Pareto.

$$\begin{aligned} \frac{(1+r)(a-c+u_2)^2}{7+8r} &\leq T \leq \\ \frac{(37+121r+132r^2+48r^3)(a-c+u_2)^2}{(3+4r)(7+8r)^2} & \end{aligned}$$

Certificate

(1) That is, $\varnothing = 1, k = 1$ is substituted into e_2^{N*} and q_2^{N*} , it can be found that $e_2^{N*} = e_2^{I*}, q_2^{N*} = q_2^{I*}$, so the supply chain is coordinated

(2) The constraint conditions for manufacturers and retailers to participate in combined contracts are $\pi_M^{BN*} > \pi_M^{BD*}, \pi_R^{BN*} > \pi_R^{BD*}$ respectively, and the result of proposition 4 can be obtained by combining the expected profits of manufacturers and retailers under decentralized decision-making.

5. Internal Financing (Model C)

Under the internal financing model, the manufacturer only produces green products and faces the problem of fund shortage. The manufacturer solves the problem of fund constraint by borrowing from retailers (internal financing). In this case, the expected profit function of the manufacturer, retailer and supply chain can be obtained as follows:

$$\pi_M^C = (w_3 - c)(q_3 + u_1) - (1+r)e_3^2 \quad (7)$$

$$\pi_R^C = (a - q_3 + e_3 + u_2 - w_3)(q_3 + u_1) - u_1 q_3 - u_1^2 - \delta^2 + r e_3^2 \quad (8)$$

$$\pi_S^C = (w_3 - c)(q_3 + u_1) - u_1 q_3 - u_1^2 - \delta^2 - e_3^2 + (a - q_3 + e_3 + u_2 - w_3)(q_3 + u_1) \quad (9)$$

5.1. Decentralized Decision-making

Under decentralized decision-making, the Stackelberg game is played between manufacturer and retailer. The order of the manufacturing supply chain decision is as follows: First, the manufacturer decides the wholesale price w_3 and the green input level e_3 ; Secondly, the retailer decides its production input q_3 according to the manufacturer's decision, and then uses the backward derivation method to solve equations (7) and (8), the optimal wholesale price w_3^{D*} , the optimal green input level e_3^{D*} and the optimal production input q_3^{D*} of the manufacturer and retailer can be obtained as follows:

$$\begin{cases} w_3^{D*} = \frac{4a(1+r) + c(3+4r) + 4(1+r)u_2}{7+8r} \\ e_3^{D*} = \frac{a-c+u_2}{7+8r} \\ q_3^{D*} = \frac{2(a-c+u_2)(1+r) - (7+8r)u_1}{7+8r} \end{cases}$$

By substituting the optimal wholesale price w_3^{D*} , the optimal green input level e_3^{D*} and the optimal production input q_3^{D*} into (7) and (8), the optimal expected profit of the manufacturer π_M^{CD*} and the optimal expected profit of the retailer π_R^{CD*} are respectively:

$$\begin{cases} \pi_M^{CD*} = \frac{(1+r)(a-c+u_2)^2}{7+8r} \\ \pi_R^{CD*} = \frac{(a-c+u_2)^2(4+9r+4r^2) - (7\delta+8r\delta)^2}{(7+8r)^2} \end{cases}$$

5.2. Centralized Decision-Making

Under centralized decision-making, the manufacturers and the retailers belong to a unified whole and maximize the profits of the entire supply chain system by determining the optimal green input level e_3^{I*} and the optimal production input q_3^{I*} . At this time, the expected profit of the supply chain is expressed as formula (9), and the first-order and second-order conditions of green input level e_3 and production input q_3 are obtained by formula (9). It can be seen that π_S^C is the joint concave function of (e_3, q_3) . By solving $\frac{\partial \pi_S^C}{\partial e_3} = 0$ and $\frac{\partial \pi_S^C}{\partial q_3} = 0$ jointly, the optimal green input level e_3^{I*} and production input q_3^{I*} of the manufacturer can be obtained as follows:

$$\begin{cases} e_3^{I*} = \frac{1}{3}(a-c+u_2) \\ q_3^{I*} = \frac{1}{3}(2a-2c-3u_1+2u_2) \end{cases}$$

By substituting the optimal green input level e_3^{I*} and the optimal production q_3^{I*} input into (9), the optimal expected profit π_S^{CI*} of the supply chain is obtained as follows:

$$\pi_S^{CI*} = \frac{1}{3}[(a-c+u_2)^2 - 3\delta^2]$$

Proposition 7: Under the internal financing model, compared with the decentralized decision-making, the centralized decision-making can increase the production input and the green input level, that is, $e_3^{D*} < e_3^{I*}, q_3^{D*} < q_3^{I*}$. So decentralized decision-making cannot coordinate the supply chain.

5.3. Combined Contract Decision-making

It can be seen from proposition 7 that under the decentralized decision making, the production input and green input level of the manufacturer cannot reach the optimal level under the centralized decision making. Thus, like section 3.3 composite contract decisions, this section coordinates the supply chain by building composite contract decisions. Under this mechanism, the expected profit function of the manufacturer and retailer in the bank loan model is as follows:

$$\begin{aligned} \pi_M^{CN} &= (1-\phi)[(w_3 - c)(q_3 + u_1) - (1+r)e_3^2] \\ &\quad + k(1+r)e_3^2 + T \end{aligned}$$

$$\begin{aligned} \pi_R^{CN} &= (a - q_3 + e_3 + u_2 - w_3)(q_3 + u_1) - u_1 q_3 - u_1^2 - \delta^2 \\ &\quad + r e_3^2 + \phi[(w_3 - c)(q_3 + u_1) - (1+r)e_3^2] - k(1+r)e_3^2 - T \end{aligned}$$

Under the combined contract decision-making, the first-order and second-order conditions of green input level e_3 and production input q_3 are obtained, and it can be seen that π_R^{CN} is the joint concave function of (e_3, q_3) . By solving the sum jointly, let $\frac{\partial \pi_R^{CN}}{\partial e_3} = 0, \frac{\partial \pi_R^{CN}}{\partial q_3} = 0$, the optimal green input level e_3^{N*} and production input q_3^{N*} of the manufacturer can be obtained as follows:

$$\begin{cases} e_3^{N*} = \frac{a + u_2 - w_3 - c\phi + w_3\phi}{-1 + 4k(1+r) + 4r(-1+\phi) + 4\phi} \\ q_3^{N*} = \frac{2(a - u_2 + w_3 - w_3\phi + c\phi)[(k+\phi)(1+r) - r]}{-1 + 4(k+\phi)(1+r) - 4r} - u_1 \end{cases}$$

Proposition 8: Under the combined contract, if the contract parameters meet the following conditions, the combined contract can effectively coordinate the studied supply chain, that is, $\phi = 1, k = 0$.

At this time, the expected profits of manufacturers and retailers are :

$$\begin{cases} \pi_M^{CN*} = \lambda\pi_S^{CI*} + T = T \\ \pi_R^{CN*} = (1-\lambda)\pi_S^{CI*} - T = \pi_S^{CI*} - T \end{cases}$$

Proposition 9: In the case of supply chain coordination, if the financial support meets the following conditions, the expected profits of manufacturers and retailers will be improved by Pareto.

$$\frac{(1+r)(a-c+u_2)^2}{7+8r} \leq T \leq \frac{(37+85r+52r^2)(a-c+u_2)^2}{3(7+8r)^2}$$

Certificate

(1) That is, $\phi = 1, k = 1$ is substituted into e_3^{N*} and q_3^{N*} , it can be found that $e_3^{N*} = e_3^{I*}, q_3^{N*} = q_3^{I*}$, so the supply chain is coordinated

(2) The constraint conditions for manufacturers and retailers to participate in combined contracts are $\pi_M^{CN*} > \pi_M^{CD*}, \pi_R^{CN*} > \pi_R^{CD*}$ respectively, and the result of proposition 6 can be obtained by combining the expected profits of manufacturers and retailers under decentralized decision-making.

6. Conclusion

This paper studies the coordination problem of the two-level supply chain consisting of a single manufacturer and a single retailer. By using the game theory method, the paper analyzes the decentralized decision-making and centralized decision-making under the three models of no financing, bank loan and internal financing, and obtains the relevant optimal solutions. Through comparison, it is found that the optimal green input level and the optimal production input volume under the decentralized decision-making cannot reach the optimal level under the centralized decision-making, so the decentralized decision-making cannot coordinate the supply chain. Therefore, this paper designs a combined contract of "revenue sharing + two-part costs + green cost sharing", that is, the manufacturer will give part of its revenue to the retailer, and the retailer will bear part of the manufacturer's green processing costs and production equipment improvement costs, so as to encourage the manufacturer to improve the level of green input and increase the production input, so as to realize the global optimization of the supply chain. It can benefit both manufacturers and retailers, which shows that the combined contract designed is very effective for the financing of small, medium and micro manufacturing enterprises with financial constraints.

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References

- [1] Jingjing Wang, Minli Xu, Yaling Deng, Pricing and coordination of the closed-loop supply chain under asymmetrical information[J].Journal of Systems Engineering, 2022, 37 (05): 617-631.
- [2] Hui Zhou, Lu Liu, Jian Liu, Green Supply Chain Decisions and Cost Sharing Contracts Based on Retailers' Overconfidence [J/OL]. Chinese Journal of Systems Science, 2024(03):110-115+130[2023-11-08].<http://kns.cnki.net/kcms/detail/14.1333.N.20230907.1038.040.html>.
- [3] Ci-wei Dong, Qing-yu Liu, bin Shen, Green Product Development Investment with Supplier Encroachment [J/OL]. Chinese Journal of Management Science: 1-13[2023-11-07].<https://doi.org/10.16381/j.cnki.issn1003-207x.2021.1263>.
- [4] Hao Zuo, Jianhua Lan, Peng Yang, et al, Research on low carbon supply chain pricing and coordination mechanism considering fairness concern under carbon trading [J/OL]. Journal of Railway Science and Engineering:1-12[2023-11-08].<http://kns.cnki.net/kcms/detail/43.1423.U.20231024.1502.003.html>.
- [5] Daru Zhang, Xinyue Nian, Tong Ye, et al, Research on Green Supply Chain Decision-making under Different Financing Models[J].Logistics Technology,2022,41(08):83-88.
- [6] Ya-qi Jin, Ji-zhou Zhan, Yan-heng Wang, Financing Strategy of Green Supply Chain with a Capital-constrained Supplier[J].Journal of Mathematics in Practice and Theory, 2022, 52(11): 21-32.
- [7] Wenli Wang, Ruizhen Zhang, Research on Financing Strategies for Green Product Suppliers Under Government Subsidies---Green Credit VS Prepayment [J/OL]. Journal of Systems & Management : 1-18[2023-11-07].<http://kns.cnki.net/kcms/detail/31.1977.N.20230525.1023.002.html>.
- [8] An S, Li B, Song D, et al, Green credit financing versus trade credit financing in a supply chain with carbon emission limits[J]. European Journal of Operational Research, 2021, 292(1): 125-142.
- [9] Wu D D, Yang L, Olson D L, Green supply chain management under capital constraint[J].International Journal of Production Economics, 2019, 215(SEP.):3-10.
- [10] Jiayi Sun, Lu Yang, Chunxian Teng, Decision and coordination of low-carbon closed-loop supply chain considering government subsidies and equity cooperation[J].Journal of Systems Engineering,2023,38(04):540-554.
- [11] Nana Yu,ShuaiPeng Yuan, Study on Coordination of Differential Price Closed-loop Supply Chain Based on Carbon Trading[J].Logistics Sci-tech,2023,46(17):106-111.
- [12] Xianghui Tai, Fang Li, Supply chain pricing and coordination considering consumers' low-carbon preferences under carbon quota trading[J].Computer Era,2023(07):91-95.
- [13] Chen Pan, Bai Yang, Helin Feng, et al, Different cost-sharing contracts choices in a green supply chain under carbon trading system[J].Journal of Systems Engineering,2023,38(04):555-576.
- [14] Peng H, Pang T, Cong J. Coordination contracts for a supply chain with yield uncertainty and low-carbon preference [J]. Journal of Cleaner Production, 2018, 205(Pt.1-1162):291-302.