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Research on Tail Risk Hedging in the Digital Asset Market

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Abstract: In recent years, the digital asset market has experienced rapid growth, but at the same time, its implicit tail risks have gradually raised concerns, especially amidst the global economic downturn caused by the pandemic. This paper aims to study tail risks and propose hedging strategies. Based on the GARCH and VaR models, this paper uses weekly return data from July 2017 to December 2021 for Bitcoin digital cryptocurrency and traditional non-digital asset market indices such as stocks, bonds, gold, and commodities. Utilizing EViews and Excel, the study explores tail risk measurement and portfolio diversification effects in the digital asset market. The main findings are as follows: (1) During the sample period, cryptocurrency returns exhibited significant volatility, with weak correlations with returns from traditional non-digital assets. Further analysis revealed strong hedging effects of digital assets against the bond market but weaker effects against the gold and stock markets, as well as commodity markets; (2) Cryptocurrency in traditional investment portfolios can enhance the risk-return ratio by adding it to the asset mix.

Keywords: Digital Assets; Tail Risk; Investment Portfolio; GARCH; VaR.

1. Introduction

1.1. Background and Significance

In today's world, with the wave of digitalization sweeping across the globe, the digital economy has witnessed positive development worldwide. Over the past decade, the market value of digital assets and the market capitalization of digital cryptocurrencies have experienced significant growth across various types [1]. From the perspective of China's actual situation, due to the development of the market economy and the continuous improvement in people's living standards, there is an increasing awareness and willingness among people to invest, coupled with a deeper understanding of finance. In order to increase wealth, people are keen on asset allocation and investment. At the G20 Summit held in Hangzhou, a partnership was established through G20 cooperation. President Xi mentioned at the 2018 APEC Summit that the digital economy is the way forward for global development. This indicates that the country places digital economic development in an important position and provides infrastructure and institutional guarantees for it. In this process, the United States quickly recognized the importance of the digital economy. Coupled with its overall economic size and rapid technological development, the size of the U.S. digital economy reached $13 trillion in 2019. As the world's second-largest economy, China's scale is less than half that of the United States. Nowadays, using blockchain technology for online transactions can connect values and facilitate transactions between buyers and sellers. It has also spawned digital assets such as blockchain-based cryptocurrencies. Since its inception, Bitcoin has flourished and gradually evolved into a global digital cryptocurrency market, attracting attention and recognition [2]. During the pandemic, many cryptocurrencies experienced soaring prices and trading volumes, temporarily surpassing the market capitalization of the US dollar. The attractiveness of digital cryptocurrencies in global investment portfolios is increasing. It has become an asset. However, behind the soaring market capitalization of digital assets lies risks. Essentially, digital assets, like other commodities, are influenced to a certain extent by supply and demand. When supply exceeds demand, prices fall, and when supply is less than demand, prices rise. Supply and demand can be said to be the most fundamental factors causing currency price fluctuations. Some currencies experience sharp declines or fail to appreciate due to oversupply, where supply exceeds demand, making it naturally difficult to appreciate. Therefore, an issue worth noting is how to measure the tail risk of digital assets and allocate and diversify portfolios of digital assets with both academic and practical value. The application value lies in utilizing the special properties of digital cryptocurrencies to diversify and hedge risks in financial practice by studying the allocation and combination of digital assets. As a new asset, digital cryptocurrency remains a mystery. Research on digital cryptocurrencies is predominant. Expanding portfolio theory to the field of digital assets not only enhances portfolio theory in the digital environment but also helps improve digital asset pricing and other related theories [3].

1.2. Research Content

This paper aims to study the tail risk hedging in the digital asset market. Based on this, the main research content of this paper is designed as follows:

(1) Descriptive statistics and correlation analysis of digital assets and related variables. Bitcoin, Shanghai Stock Exchange Index, SSE Corporate Bond Index, Shenzhen Bulk Commodity Index, Gold Futures. Descriptive statistical analysis of weekly returns from July 2, 2017, to December 26, 2021, to determine if the GARCH model is suitable. At the same time, this paper also conducted an overall correlation analysis of various indicators to examine the relationship between different indicators and analyze their correlation with various asset classes.

(2) This paper discusses the tail risk and hedging effect of digital assets. The digital asset market will be represented by Bitcoin in digital cryptocurrencies, with the Shanghai Stock Exchange Index as the benchmark for the stock market, the

SSE Corporate Bond Index as the reference for the bond market, and the Shenzhen Bulk Commodity Index and Gold Futures Index as representatives for the commodity market. The GARCH model is used for correlation analysis. By calculating the VaR value of Bitcoin using t-statistics and the GARCH model (as Bitcoin occupies a large market share in the digital asset market, with Bitcoin's market value reaching 41.15% of the total market value of digital assets as of April 2022, almost half of the share).

(3) Study on the risk diversification effect of digital assets [4]. Using the Markowitz efficient frontier model and Excel's solver function, the paper maximizes hedging for digital assets (Bitcoin) through asset allocation and portfolio combination.

1.3. Research Methods

Based on the above research content, this paper mainly adopts the following research methods:

Qualitative and quantitative research methods are used. Qualitative analysis is to analyze the research objects from the perspective of "quality". Specifically, it is to process the obtained materials through methods such as induction, deduction, analysis and synthesis, abstraction, and generalization, in order to distinguish between true and false, to connect causes and effects, to explore from surface to essence, and to achieve the purpose of understanding the essence of things and revealing internal laws. Based on the literature on digital cryptocurrencies at home and abroad, this paper qualitatively defines digital assets and digital cryptocurrencies based on their functions, categorizing them as a type of digital asset [5]. Quantitative analysis is a method used in scientific research to accurately understand the research objects by quantitative means, thereby better revealing laws, grasping essences, clarifying relationships, and predicting future trends. Based on the data of the objects themselves, this paper statistically analyzes and evaluates them, establishes corresponding mathematical models, and evaluates them. Firstly, the data of each variable is processed and analyzed, mainly based on the variables selected by domestic and foreign scholars when studying related issues, and analyzed according to the actual situation. Reasonable time periods are selected, irrelevant data is filtered out, empirical analysis is conducted, and simple descriptive statistics are performed, etc.

1.4. Innovations and Limitations

(1) Innovative research perspectives. Currently, there is limited research on digital assets. This paper conducts research from the perspective of digital assets and calculates the tail risk of digital assets using the VaR method, providing insights into its proportion in investment portfolios.

(2) Innovative research methods. This paper uses representative asset indicators to represent various asset classes and employs the GARCH model and VaR model to study and analyze the tail risk hedging and diversification effects of digital assets [6].

However, this study has the following limitations:

(1) This paper only empirically studies Bitcoin, the most representative digital cryptocurrency in digital assets. However, due to the limited sample size, the conclusions may be biased.

(2) Due to the limitations of the model and the selection and quantity of data resources, the sample size and time range selected for the study are limited, resulting in less significant research results.

2. Concept Definitions, Theoretical Framework and Literature Review

2.1. Concept Definitions

2.1.1. Digital Assets

The term "digital asset" is primarily used in the modern context, first introduced with the emergence of blockchain technology and cryptocurrencies. Coined by Kud in 2009, the birth of the first cryptocurrency, Bitcoin, became a new phenomenon in the field of economics. Simultaneously, the development of blockchain technology, a distributed ledger, constructed transaction blocks in sequential chains according to certain rules, with each subsequent block containing information from the previous one. This blockchain operation ensures the security and transparency of transactions and processes, leading to its widespread application in various fields. These trends have fueled the rapid development of the digital economy and sparked significant attention to the phenomenon of digital assets. However, due to vague interpretations of the definition of digital assets and the lack of basic standards, some entities are labeled as "digital assets" when they are not actually digital assets [7].

2.1.2. Digital Cryptocurrencies

Cryptocurrency is one of the developments of blockchain technology, which is often used as decentralized digital currency. The term "cryptocurrency" refers to a type of virtual currency without physical form, and "cryptocurrency" also implies that the transaction currency is invisible and secure. This digital currency has many types, such as Bitcoin, Ethereum, Litecoin, Monero, and many others. Although it has no physical form, its function is similar to traditional currencies, and it has exchange rates. The fluctuation of cryptocurrency exchange rates often leads to unexpected events, which traders often exploit. The form of cryptocurrency trading involves transferring from one person to another online, allowing for direct transactions without intermediaries [8]. Each technology has its advantages and disadvantages besides efficiency and convenience. The drawback of cryptocurrency is the lack of an authoritative institution responsible for handling all issues that arise in transactions, and money laundering crimes frequently occur, presenting a challenge in utilizing cryptocurrency and blockchain technology in the current era of globalization. From a conceptual perspective, we can generally classify digital currencies into two categories: non-legal digital currencies and central bank digital currencies. Non-legal digital currencies mainly include encrypted digital currencies, namely Tokens and Coins. Digital currencies can be roughly divided into two stages based on their time of emergence: the first stage is the emergence of non-legal digital currencies, mostly consisting of encrypted digital currencies. The earliest ones include Bitcoin, Ethereum, and Litecoin. Due to the instability of their value, stablecoin emerged. The second stage is the birth of legal digital currencies. Due to the significant problems arising from non-legal digital currencies and the threat they pose to the status of sovereign currencies, central bank digital currencies emerged [9]. Cryptocurrency, or Crypto-Currency, has only gained attention in the last decade, primarily accompanying the rapid appreciation of mainstream encrypted assets such as Bitcoin, and is a digital currency or virtual currency built on encryption algorithms and encryption technology. Cryptocurrencies are not entirely based on blockchain technology. For example, DigiCash, released in the early 1990s, is one form of encrypted electronic payment, but it is not based on blockchain technology.

2.2. Theoretical Framework

The theoretical foundation of this thesis primarily lies in the theory of risk management, encompassing two branches: portfolio theory and risk measurement theory.

2.2.1. Portfolio Theory

Portfolio theory can be broadly categorized into two types: narrow and broad. The narrow portfolio theory typically refers to the widely recognized Markowitz Portfolio Theory (1952), which is a comprehensive theory of portfolio construction. In a broader sense, portfolio theory also includes capital market theory, which is based on asset allocation.

2.2.2. Risk Measurement Theory

The development of risk measurement theory can be divided into three periods: the first period is the traditional risk measurement stage, which uses measures like variance and risk factors as standards; the second period is the modern risk measurement stage, where Value at Risk (VaR) is the internationally accepted risk measurement tool; and the last period is the consistent risk measurement stage, represented by Expected Shortfall (ES) [10].

VaR is currently the international standard for risk management tools, initially developed by JP Morgan to address the risk needs of its banking operations and quickly adopted as an industry standard. VaR quantifies and measures financial risk using probability theory and mathematical statistics. Its major advantage is providing a one-dimensional approximation of multidimensional risks, which can be used to measure different risks in different markets and represented numerically, thus having broad applicability.

2.3. Literature Review

2.3.1. Foreign Studies

Foreign scholars have shown a significant interest in research concerning asset allocation and systemic risk. Early works, such as Cremers & Kritzman (2003), primarily utilized matrix values to predict and utilize investment portfolios [11]. Based on a quasi-utility function, they relied on theory rather than empirical return distributions. Capie & Wood (2005) conducted research on hedging gold prices, analyzing the gold price over the past 30 years using gold prices, pound sterling to US dollar, and Japanese yen to US dollar as benchmarks. The study revealed a negative, typical non-elastic relationship between gold prices and exchange rates, but the strength of this relationship varied over time. Baur & Lucey (2009) initiated research on capital flight, studying securities to bonds, then from stocks to bonds. They proposed defining the financial system and examining its impact. Through empirical research on eight developed countries including the United States, the United Kingdom, Germany, and Japan, they found that capital flight does exist, and it is not unique to a single country but occurs simultaneously in multiple countries. Baur & Lucey (2010) subsequently studied the hedging effect of gold, observing whether gold has a hedging effect on stocks and bonds. This is of significant importance regarding whether gold can serve as a hedge asset for Bitcoin. Ratner & Chiu explored the potential risk reduction benefits of Credit Default Swaps (CDS). This study utilized GARCH dynamic conditional correlation coefficients from the US stock market sector from 2004 to 2011 to verify the effectiveness of CDS in various stock markets. The research found that CDS is a relatively safe asset in both stock market volatility and financial crises. Girardi & Ergün (2013) investigated not only the relationship between institutional contributions to systemic risk but also their characteristics. Mainik & Schaanning (2014) conducted a thorough analysis of CoVaR and other systemic risk measures' dependence consistency. In recent years, foreign scholars have conducted more in-depth analyses of tail risks and portfolio analysis issues related to digital assets, especially the increasing research on Bitcoin. Bouoiyour, etc [12]. (2014) studied the pricing of Bitcoin, exploring Granger causality, namely the value and trading of Bitcoin, affirming Bitcoin's extreme speculative nature while also acknowledging its economic role. Brière, etc. (2015) studied diversified Bitcoin investment portfolios, including traditional assets (global stocks, bonds, hard currencies) and alternative investments (commodities, hedge funds, real estate). Dyhrberg (2015) conducted risk analysis on Bitcoin by selecting macro variables such as gold, USD to GBP, USD to EUR, and the UK FTSE index. This indicates that Bitcoin has similarities with gold and the USD, making it an effective investment tool. Dyhrberg (2016) subsequently analyzed the financial asset capabilities of Bitcoin using the GARCH model. The preliminary model showed some similarities with gold and the USD, demonstrating advantages in hedging and trading. Asymmetric GARCH also showed that Bitcoin can be used for risk management and is suitable for risk-averse investors expecting negative market impacts. Furthermore, it is believed that due to Bitcoin's categorization as a commodity that falls between gold and the US dollar, it holds a certain position in both financial markets and portfolio management fields [13]. Ciaian, etc. (201X) studied Bitcoin prices, considering not only traditional currency price determinants such as market forces of supply and demand but also Bitcoin's attractiveness to investors and users. They used daily data from a five-year period (2009-2015) and employed time-series analysis methods. The results showed that market forces and Bitcoin's attractiveness to investors and users significantly influence Bitcoin prices, but these influences change over time. The conclusion drawn was that the development of macro finance plays a significant role in promoting Bitcoin prices. Peter, etc. (2017) used dynamic conditional correlation models to test whether Bitcoin has the potential to act as a hedge or safe haven for world major stock indices, bonds, oil, gold, and common stocks. Stavroyiannis & Babalos (2017) studied the dynamic characteristics of Bitcoin and the S&P 500 index, employing various econometric methods including univariate and multivariate GARCH models and vector autoregressive specifications. Additionally, they explored whether Bitcoin could be classified as a possible hedging, diversifying, or safe-haven asset in the US market, and whether it possesses any attributes similar to gold. Catania, etc [14]. (2018) utilized four cryptocurrencies with the highest trading volume—Bitcoin, Ethereum, Litecoin, and Ripple—to conduct time-series forecasting of cryptocurrency volatility. Ardia, etc. (2019) described the risk through Bitcoin GARCH volatility dynamics' mechanism changes. Bouri & Gupta (2019) aimed to examine Bitcoin's price levels and volatility and their impact on structural breakdowns. They used parametric and semi-parametric techniques and found strong evidence in the level series that Bitcoin lacks mean reversion.

2.3.2. Domestic Studies

Compared to foreign academia, domestic academia has conducted relatively fewer analyses and studies on issues such as digital asset allocation and portfolio management. Tu Xinshu and Wang Chunfeng (2002) proposed VAR estimation and introduced new issues. By assuming a given acceptable VaR, they determined the maximum return on a portfolio investment given a set of securities while simultaneously satisfying VaR constraints. Li Xiang and Liu Shaobo (2015) conducted optimal decision analysis of asset allocation using Monte Carlo simulation methods to determine the value of cryptocurrency asset allocation and their respective proportions in a large asset class portfolio [15]. Li Jihong, etc. (2016), using Bitcoin as an example, compared Bitcoin with traditional currencies and digital currencies planned to be issued by central banks worldwide, revealing their basic characteristics and focusing on their "decentralization" features. Niu Yuemin (2017) studied the long-term stability and short-term lead-lag relationships between Bitcoin prices and five major macroeconomic indicators (Dow Jones Index, US dollar, crude oil, federal funds rate, and gold prices), exploring whether Bitcoin has become a mature alternative investment tool and whether there is interaction between Bitcoin price volatility and other investment instruments. Yan Fangling, etc. (2018), discussed the reasons for changes in Bitcoin prices and theoretically analyzed the impact of China's monetary policy and domestic market demand on Bitcoin price changes. The article also empirically studied factors such as exchange rates, gold, stocks, and the prices of investment tools such as Bitcoin, gold, stock indices, and exchange rates. Fu Zhongliang (2019) conducted a comprehensive analysis using eight assets including cryptocurrency index, Shanghai Stock Exchange index, S&P 500 index, US Treasury index, US corporate bond index, commodity index, gold index, and US dollar index, and used the DCC-GARCH model to hedge other assets [16].

2.3.3. Conclusive Review

From the literature review both domestically and internationally, it is evident that cryptocurrencies and other digital assets have garnered widespread attention in academia, with research on tail risk in the digital asset market and portfolio analysis issues being increasingly prominent. Existing domestic research mainly focuses on exploring the value and mechanisms of cryptocurrencies, with limited studies on systematically analyzing tail risk of digital assets and portfolio analysis issues. This calls for innovative research approaches to delve deeper into systematically studying tail risk in the digital asset market and establishing portfolio analysis frameworks by measuring risks and implementing hedging and diversification strategies [17].

3. Model Selection

3.1. VaR-GARCH Model

3.1.1 The Value at Risk (VaR) model quantifies the maximum expected loss of a financial asset or portfolio within a specified time horizon at a certain confidence level during normal market fluctuations. VaR can be expressed as:

$$\mathrm{Prob}\{\Delta V（\Delta t, \Delta x）\leqslant \mathrm{VaR}\}=1-\alpha \tag{1}$$

Where: ΔV is the total change in value of a financial asset or portfolio over a specific time period; Δt is the holding period; Δx is the risk factor; VaR is the maximum loss limit; Prob represents the probability that the actual loss of asset value is less than the maximum expected loss; α is the confidence level.

The expected return distribution function of VaR needs to be fitted appropriately by statistical tools to accurately describe the characteristics of the "fat-tailed" return distribution of financial assets. In this study, the variance-covariance method is used to measure asset VaR values.

Assuming the return rate R follows a normal distribution N（μ，σ^2）, the VaR value of the return rate over t days at a given confidence level α is calculated as:

$$r^*=Z_\alpha \sigma \sqrt{t} \tag{2}$$

The daily VaR value of financial asset prices is calculated as:

$$\mathrm{VaR}=Z_\alpha \sigma P \tag{3}$$

The variance-covariance method leads to the derivation of the dynamic variance-covariance method, which assumes that the volatility of the return distribution changes over time, further improving the fitting accuracy of financial data volatility characteristics [18].

3.1.2 GARCH Model

Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model is an important extension of the ARCH model. Assuming that the return rate r_t follows a normal distribution N（μ_1，σ_1）, the general form of the GARCH (p,q) model is expressed as:

$$\varepsilon_t=\sigma_t u_t \tag{4}$$

$$\sigma_t^2=\alpha_0+\alpha_1\varepsilon_{t-1}^2+\alpha_2\varepsilon_{t-2}^2+\cdots+\alpha_p\varepsilon_{t-p}^2+\beta_1\sigma_{t-1}^2+\beta_2\sigma_{t-2}^2+\ldots\beta_q\sigma_{t-q}^2 \tag{5}$$

Where the terms in the equation are explained as follows:

μ_1 represents the independent and identically distributed white noise sequence; p represents the p-th order lag of historical return rates; q represents the q-th order lag of previous variances [19].

$$\alpha_0\geqslant 0，\alpha_i\geqslant 0，$$

$$\beta_j\geqslant 0; 0<\textstyle\sum_{i=1}^{p}\alpha_i+\sum_{j=1}^{q}\beta_j<1。$$

The GARCH model describes the variation of conditional variance over time, effectively demonstrating volatility and explaining the phenomenon of volatility clustering. Calculation methods for GARCH models exist for distributions following the t-distribution and the GED distribution.

4. Variable Selection and Data Description

4.1. Variable Selection

This paper conducts an empirical study on the historical weekly returns of five major stock indices from July 2, 2017, to December 26, 2021. The selection of this period's data is based on several considerations: Firstly, it spans five years, providing timely market information. Secondly, this period witnessed significant fluctuations in the digital asset market, making it suitable for studying digital asset allocation under various market conditions. Additionally, due to different trading hours in various markets, weekly data from the same period is uniformly adopted for ease of statistical analysis [20]. Finally, five variables are derived, representing 235 weeks of returns. These five indices include Bitcoin, Shanghai Stock Exchange Index, SSE Corporate Bond Index, Shenzhen Bulk Commodity Index, and Gold Futures. Bitcoin is currently the most popular digital cryptocurrency and digital asset. The Shanghai Stock Exchange Index, SSE Corporate Bond Index, Gold Index, and Shenzhen Bulk Commodity Index represent the stock market, bond market, commodity market, respectively. When the risk-return ratio of a portfolio increases, it serves as a risk distribution tool; otherwise, it does not have a risk distribution function. The table below shows the names and symbols of the variables.

Variable Name and Symbol

Variable Name	Variable Symbol
Bitcoin	BTC
Shanghai Stock Exchange Index	SSEC
SSE Corporate Bond Index	SSEEBI
Shenzhen Bulk Commodity Index	SZCPI
Gold Futures	ZGM2

4.2. Data Description

4.2.1. Data Source

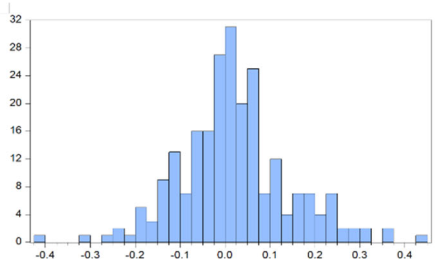
The historical weekly returns data for Bitcoin, Shanghai Stock Exchange Index, SSE Corporate Bond Index, Shenzhen Bulk Commodity Index, and Gold Futures from July 2, 2017, to December 26, 2021, is sourced from cn.investing.com. The dataset includes 235 weeks of returns, excluding weekend data for all indices except Bitcoin [21].

4.2.2. Descriptive Analysis

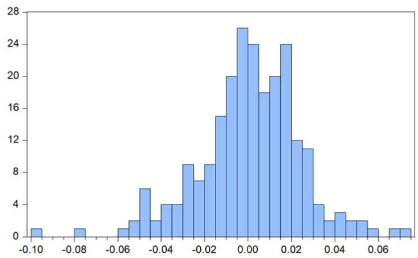
Variable	Mean	Max	Min	Std.	Skewness	Kurtosis	J-B Statistic	p-value
BTC	2.0022%	43.6200%	-41.6900%	0.121549	0.248988	4.089466	14.05022	0.000889
SSEC	0.0843%	7.3100%	-9.6000%	0.023586	-0.412568	4.471364	27.50903	0.000001
SSEEBI	0.0862%	2.6000%	-0.1500%	0.000540	-0.046932	5.036241	40.16585	0.000000
SZCPI	0.1254%	20.5500%	-11.2200%	0.033306	0.016172	3.534951	2.764477	0.251016
ZGM2	0.1605%	10.6300%	-9.4200%	0.019892	0.085375	8.452739	291.4148	0.000000

Note: Mean, maximum value, minimum value are presented in percentage format with four decimal places, while standard deviation, kurtosis, skewness, J-B statistic, and p-value are presented with six decimal places.

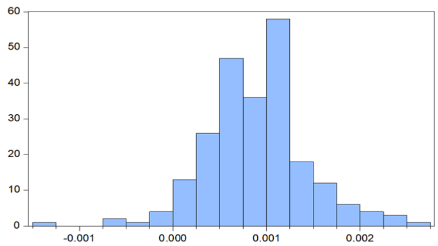
BTC



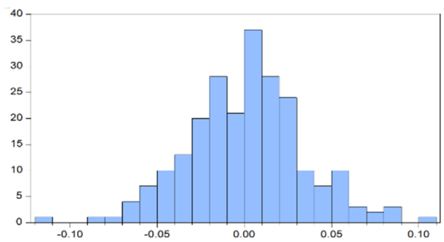
SSEC



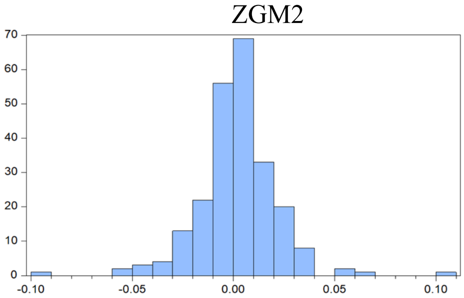
SSEEBI



SZCPI



ZGM2



From the table, it can be observed that during the period from July 2, 2017, to December 26, 2021, the highest weekly return rate among these two variables is for Bitcoin, at 2.0022%, while for the Shanghai Stock Exchange Index, it is 0.0843%. All assets have positive average return rates. Looking at historical data, the price of cryptocurrency has experienced several significant fluctuations [22]. For instance, on July 16, 2017, its price surged by 43.62%, while by 2020, it had dropped by 41.69%. In terms of volatility, Bitcoin exhibits the highest volatility, indicating significant fluctuations in its returns, which is consistent with reality. Many people have limited knowledge of digital assets, making it difficult to decide on the quantity to purchase and how to select high-quality cryptocurrencies. For cryptocurrencies like Bitcoin, short-term fluctuations, both upward and downward, are substantial. Moreover, trading can be conducted at any time, and the threshold for investors is low. Many view this as a short-term, speculative asset.

From the figures, it can be seen that the logarithmic return distribution of the Shanghai Stock Exchange Index and the SSE Corporate Bond Index is left-skewed, while the skewness of the other three indicators is above 0, indicating right-skewed return distributions [23]. In terms of kurtosis, except for the Shenzhen Bulk Commodity Index, all other variables have kurtosis values exceeding 3, indicating leptokurtic tails. Looking at the Jarque-Bera statistic and p-values, except for the Shenzhen Bulk Commodity Index, all variables are significant, rejecting the null hypothesis of conformity to a standard normal distribution, thus not following a standard normal distribution.

5. Empirical Results Analysis

5.1. Risk Measurement Analysis

5.1.1. Stationarity Analysis

The Augmented Dickey-Fuller (ADF) unit root test method was employed to test the stationarity of the time series data. The test results are shown in Table 5.1.

ADF Unit Root Test Results for Each Variable

Variable	T-statistic	P-value
BTC	-15.29805	0.0000
SSEC	-16.49246	0.0000
SSEEBI	-6.184721	0.0000
SZCPI	-17.02380	0.0000
ZGM2	-18.12789	0.0000

Note: When the confidence level is 1%, the critical value of the t-statistic is -3.46; when the confidence level is 5%, the critical value of the t-statistic is -2.87; when the confidence level is 10%, the critical value of the t-statistic is -2.57 [24].

From the table, it can be seen that the t-statistic values of each variable are much smaller than -3.14. The critical t-statistic values at the 1%, 5%, and 10% confidence levels are -3.46, -2.87, and -2.57, respectively, indicating that at a 1% confidence level, all variables are significant. The results indicate that all variables pass the ADF test and are stable time series, laying a solid foundation for further research on ARCH effects.

5.1.2. ARCH Effect Test

Before using the GARCH model, it is necessary to test the ARCH effect of each sequence and then test the heteroskedasticity of the residuals of each sequence's mean equation. First, an autoregressive model from lag 1 to lag 36 was established using EViews software to obtain autocorrelation and partial autocorrelation. Tables 5.2-9 show the autocorrelation and partial autocorrelation of each variable in stages 1-5.

Autocorrelation and partial autocorrelation coefficients of Bitcoin

Lag Period	AC	PAC	Q-Stat	Prob
1	-0.005	-0.005	0.0058	0.940
2	0.045	0.045	0.5005	0.779
3	0.088	0.089	2.3732	0.499
4	-0.033	-0.034	2.6295	0.622
5	0.033	0.025	2.9007	0.715

Autocorrelation and partial autocorrelation coefficients of the Shanghai Stock Exchange Index

Lag Period	AC	PAC	Q-Stat	Prob
1	-0.066	-0.066	1.0317	0.310
2	-0.009	-0.013	1.0501	0.592
3	-0.057	-0.058	1.8088	0.613
4	0.023	0.015	1.9330	0.748
5	-0.070	-0.070	3.1079	0.683

Autocorrelation and partial autocorrelation coefficients of the SSE Corporate Bond Index

Lag Period	AC	PAC	Q-Stat	Prob
1	0.434	0.434	44.170	0.000
2	0.408	0.271	83.512	0.000
3	0.339	0.123	110.81	0.000
4	0.285	0.055	130.12	0.000
5	0.235	0.020	143.38	0.000

Autocorrelation and partial autocorrelation coefficients of the Shenzhen Bulk Commodity Index

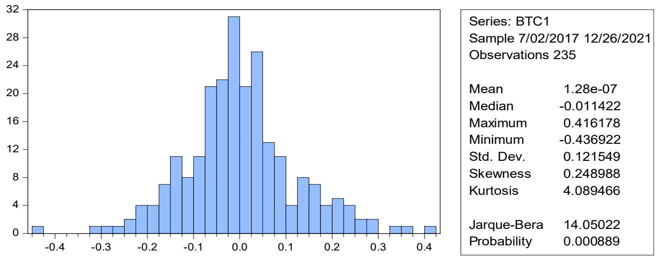
Lag Period	AC	PAC	Q-Stat	Prob
1	-0.112	-0.112	2.9194	0.088
2	0.088	0.076	4.7366	0.094
3	-0.066	-0.050	5.7794	0.123
4	-0.019	-0.038	5.8619	0.210
5	-0.021	-0.018	5.9689	0.309

Autocorrelation and partial autocorrelation coefficients of Gold Futures

Lag Period	AC	PAC	Q-Stat	Prob
1	-0.169	-0.169	6.7979	0.009
2	0.071	0.044	8.0028	0.018
3	-0.057	-0.039	8.7799	0.032
4	-0.098	-0.120	11.098	0.025
5	0.120	0.095	14.604	0.012

From the autocorrelation and partial correlation coefficients of each lag variable, it can be seen that the Q-statistic values for Bitcoin, the Shanghai Stock Exchange Index, and the Shenzhen Bulk Commodity Index all have p-values exceeding 10% confidence level, indicating no rejection of the null hypothesis, meaning that there is no significant time series correlation in each sequence's variable [25]. However, for the SSE Corporate Bond Index and Gold Futures, the p-values are less than the 10% confidence level, so the null hypothesis cannot be rejected, indicating sequence correlation. Therefore, for Bitcoin, the mean model is set to white noise.

Model establishment: rt=πt+εt, each variable is de-meaned to obtain a new variable BTC1.



The LM test (Lagrange Multiplier test) was then used to test the ARCH effect.

Lag Period	AC	PAC	Q-Stat	Prob
1	0.183	0.183	7.9642	0.005
2	0.174	0.146	15.238	0.000
3	0.168	0.120	21.996	0.000
4	0.140	0.078	26.705	0.000
5	0.104	0.036	29.321	0.000

Through the graph, it can be seen that there is autocorrelation in the sequence, indicating the presence of an ARCH effect.

Commonly used GARCH models include GARCH(1,1), GARCH(1,2), and GARCH(2,1). We model with multiple models separately [26].

GARCH(1,1)

Akaike info criterion	-1.455204
Schwarz criterion	-1.411039

GARCH(2,1)

Akaike info criterion	-1.448624
Schwarz criterion	-1.389737

GARCH(1,2)

Akaike info criterion	-1.458363
Schwarz criterion	-1.399477

Based on the comparison of SC and AIC, GARCH(1,1) has the best performance as all coefficients pass the t-test.

Then, through the model, a t-value of 6.578689 is obtained, and the var value of BTC is calculated. Below is the var value for Bitcoin.

Mean	0.226738
Std	0.055116

5.2. Risk Diversification Analysis

Below, we conduct a tail analysis and hedge for Bitcoin using the Markowitz model in Excel.

First, we calculate the covariance and correlation coefficients using Excel's formula feature.

Correlation Coefficients	BTC	SSEC	SSEEBI	SZCPI	ZGM2
BTC	1	0.079048	-0.09295722	0.148319857	0.14195121
SSEC		1	-0.00475576	0.792627147	0.204856395
SSEEBI			1	0.013582587	0.103614018
SZCPI				1	0.177346886
ZGM2					1

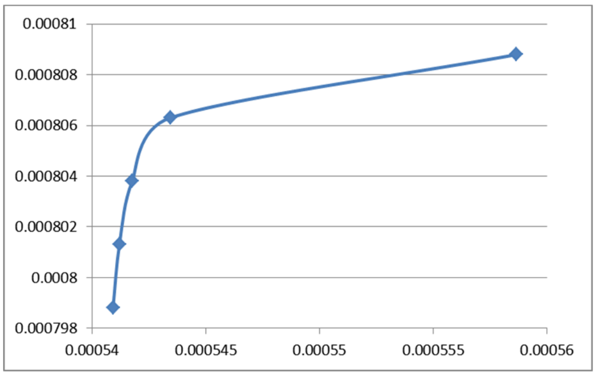
Covariance	BTC	SSEC	SSEEBI	SZCPI	ZGM2
BTC	0.014711353	0.000227	-6.1101E-06	0.000601966	0.000341752
SSEC		0.000554	-6.0278E-08	0.000621305	9.62526E-05
SSEEBI			2.90027E-07	2.42269E-07	1.11399E-06
SZCPI				0.001104485	0.000117904
ZGM2					0.000393995

Then, we use Excel's Solver to calculate the minimum variance portfolio and the optimal portfolio.

	1	2	3	4	5
Mean Excess	0.000798817	0.000801312	0.000803806	0.000806301	0.000808796
Mean	0.000868817	0.000871312	0.000873807	0.000876301	0.000878797
Sd	0.000540929	0.00054121	0.000541776	0.000543454	0.000558663
slope	1.47674911	1.480592862	1.454652625	1.48365938	1.447735443
BTC	0.000352225	0.000482445	0.000478949	0.000742836	0.000710275
SSEC	3.36687E-05	3.329E-05	3.32896E-05	3.36637E-05	3.36654E-05
SSEEBI	0.999614106	0.999484265	0.995373361	0.9992235	0.994344811
SZCPI	0	0	0.001388222	0	0.001484334
ZGM2	0	0	0.002726179	0	0.003426915
VaR	0.000330	0.000289	0.000267	0.000209	0.000222

From the analysis in the table, it can be inferred that to hedge against the significant risk of Bitcoin, the asset allocation of the portfolio will allocate the majority of funds to the Shanghai Corporate Bond Index, with a weight exceeding 99%.

The following graph illustrates the efficient frontier based on portfolio investment under the Markowitz theory.



6. Research Conclusion and Implications

6.1. Research Conclusion

Based on the GARCH and VaR models, this study utilized weekly return rate data of digital cryptocurrencies such as Bitcoin and traditional non-digital assets market indices including stocks, bonds, gold, and commodities from July 2017 to December 2021. By using EViews and Excel software, the study investigated the issues of tail risk in digital assets and portfolio analysis from the perspectives of tail risk measurement and hedging diversification effects. The main research conclusions are as follows:

(1) During the sample period, the volatility of cryptocurrency returns was significant, and there existed weak correlations with returns of non-digital traditional assets. Further analysis indicated that stock and bond markets have hedging effects on digital assets, while gold and commodity markets have weaker hedging effects.

(2) Adding traditional investment portfolios to digital cryptocurrencies can enhance portfolio returns and risk-return ratios, indicating that certain traditional assets can hedge against the tail risk of digital assets to some extent.

6.2. Implications

In recent years, digital technology has continuously progressed and been applied in various fields, leading to the booming development of digital assets such as cryptocurrencies and digital currency investments worldwide. Combining the research results mentioned above, the following implications are proposed:

(1) It is essential to approach and invest in digital assets rationally. As a new type of asset, the returns of digital assets indeed surpass many traditional investment targets. However, they also come with significant risks, often leading to losses exceeding risk tolerance for those with limited understanding. Therefore, it is crucial to invest appropriately in digital cryptocurrencies under full understanding and capacity to bear risks.

(2) Selectively invest in high-quality digital assets to reduce or even avoid investment risks associated with investing in other asset classes. Despite the significant tail risk of digital assets mentioned in the research, it is indeed possible to substantially reduce the risk of digital assets through portfolio investment, thereby bringing substantial returns.

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