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Synthesis of Research on Joint Replenishment Problems and Cost Sharing

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Abstract: Among all existing inventory replenishment models, this research was dedicated to the Joint Replenishment Problem (JRP), which consists in the replenishment of multiple items simultaneously, aiming total cost reduction. Literature has presented several optimal and approximated solutions to this problem, with different applications and techniques, which results in a large quantity of solution proposals. Therefore, This paper organizes the related concepts and model studies of joint replenishment problem, and collects the solution algorithms of joint replenishment problem in different contexts, which provides reference for the research trend of joint replenishment afterwards. The cost sharing problem arising from the joint replenishment problem is also studied in the relevant literature to provide reference for the subsequent design of cost sharing rules.

Keywords: Inventory management, joint replenishment problem, cost sharing.

1. Introduction

In order to generate higher profits, third-party overseas warehouses increase the frequency of delivery of goods by setting storage rates that increase with storage time. To reduce the costs incurred by using third-party overseas warehouses, small cross-border e-commerce retailers form alliances to jointly use overseas warehouses for replenishment. Involving more retailers in joint replenishment will help save more costs. When the fixed cost is very high, the average cost saving rate after cooperation can reach 50%. The problem of joint replenishment is to purchase multiple items in groups from a single supplier, thereby solving the inventory problem of multiple item storage and thus reducing the associated purchasing costs. In reality, when multiple items are supplied from a single supplier or central warehouse, or transported by the same means of transportation, there will be a scale effect on joint replenishment purchases and transportation, and not only this, but also reduce the cost of inventory holding, backorders, sales losses, etc. Therefore, the applicability of joint replenishment is widely recognized.

Although joint replenishment has obvious advantages and many successful cases, there are also many resistance to cooperation, for example, some enterprises will focus on immediate benefits, ignoring the development of enterprises in the long-term strategic height, so that each other in the sharing of information and cooperation is not willing to make their own information transparent, it is difficult to achieve information sharing. Poor communication and coordination between joint replenishment members can also become a resistance to cooperation, resulting in replenishment timing and conditions cannot match individual needs. In addition, enterprises are to maximize their own interests as the ultimate goal, how to fairly and reasonably distribute benefits is also the key to cooperation. Shanghai Jia Lian Lian Ying Purchasing Co., Ltd. is a supermarket joint replenishment alliance formed by Hunan BBK and four other enterprises, but after the establishment of the annual purchasing volume of only 4 million yuan, in contrast to the total annual purchasing volume of more than 10 billion yuan of the various supermarkets, and the purchasing alliance was established with a significant gap compared to the expectations. There are many reasons for such a gap, first of all, the alliance parties need to negotiate communication and information sharing, the four enterprises from Hunan, Shandong, Hubei, Guangxi, respectively, the distance between each other, communication is not convenient, in-formation is difficult to coordinate. The other second is that in some substantive interests in the handling of the issue, the mechanisms are not perfect, such as cost sharing, the distribution of substantive benefits. The location of the supplier to the alliance enterprise distance is not the same, or transportation costs are not the same, resulting in relatively low logistics costs of the alliance enterprise will not be willing to bear the cost for the allies. Therefore, a reasonable cost-sharing mechanism is also quite important.

2. Joint Replenishment Model

Over the decades, as the demand for products increases, cost reduction has become an important goal, and more and more research has been conducted on the inventory replenishment problem. From a mathematical point of view, many scholars have proposed a large number of solutions through optimization models with the aim of reducing costs and meeting demand[1]. One of the main themes is the Joint Replenishment Problem (JRP), which consists of replenishing different types of items in the same order, usually from the same supplier, with the aim of reducing the total inventory cost[2].

JRP is considered NP-hard because of its combinatorial nature. Therefore, recent research has focused on finding more efficient and faster solution algorithms that take into account the real-world environment in which the problem resides, such as meta-heuristics, rather than just generating exact solutions, which can lead to significant computational time. Porras & Dekker (2008) [3]asserted that compared to an economic order quantity model with a single-item approach that takes into account a set of 20 products, a joint replenishment strategy can save up to 13% of the total cost.

When the interdependence between different product groups in the same order from a single supplier is taken into account in the inventory replenishment process, it emerges that the JRP objective is to optimize the total replenishment cost by reducing the inventory ordering and holding costs. Thus, joint replenishment has two advantages: discounts can be obtained from suppliers when ordering large quantities of multiple items; and the (ordering) fixed cost per item as well as transportation costs can be reduced[4, 5].

Joint replenishment is the process by which multiple retailers order products or multiple items from the same supplier and reduce costs by combining orders to create economies of scale. The joint replenishment problem has been studied by many scholars as early as the 1960s in the literature of Starr and Miller (1962)[6]. Khouja and Goyal (2008)[7] and Bastos et al. (2017)[8] reviewed the joint replenishment literature from 1989 to 2005 and from 2006 to 2015, respectively. The assumptions of the classical joint replenishment model are similar to those of the EOQ model; i.e., deterministic demand, linear holding costs, no stock-outs, and immediate replenishment. The joint replenishment problem is a special case of multiple retailers in a single warehouse, in which the supplier incurs a primary replenishment cost when it replenishes to the public warehouse and a secondary replenishment cost when the retailer replenishes its own inventory from the public warehouse. The supplier-managed public warehouse is a staging point, but the product is not in it. The goal of joint replenishment is usually to minimize the total cost per unit of time by determining the replenishment cycle Ding and Kaminsky (2020)[9]. When there are no quantity discounts, the purchase price of a product is usually known and the holding cost rate (per unit of time/second) is usually constant in a joint replenishment problem. However, in a joint replenishment problem for a single product, different retailers usually have different holding cost rates, and in a joint replenishment problem for multiple products, the holding cost rate varies from product to product. For perishable products, the holding cost can be set to infinity when the holding time exceeds the shelf life.[10] Although there is a rich literature on inventory models that consider different holding cost rates, such as age-dependent holding costs, constant holding cost rates have been widely used in joint replenishment problems[12].

JRP models have a wide range of applications. Since the problem is considered an NP problem, it is more difficult to find a solution within an acceptable computational time and these solutions are more practical for a company's decision-making process compared to single-item inventory replenishment models[13]. In the literature, the model considers two main solution strategies for JRP[16]: Direct Grouping Strategy (DGS)-products are grouped by groups and items in each group follow the same cycle time and Indirect Grouping Strategy (IGS)-each product has its own cycle time, i.e., a multiple of the base cycle time[18].

There is still a large body of literature on the joint replenishment problem with deterministic demand even though the stochastic and dynamic demand in the joint replenishment problem is considered closer to reality. One possible reason for studying deterministic demands is the difficulty of solving joint replenishment problems. Arkin et al. (1989)[20] proved that the joint replenishment problem without capacity constraints and deterministic demands in a finite range is NP-complete, and Cohen and Yedidison (2018)[21] proved that joint replenishment without capacity constraints and determinism in an infinite range is NP-hard. The solution of joint replenishment problems with stochastic or dynamic demand is usually more difficult. This also means that polynomial-time algorithms for finding optimal solutions to classical joint replenishment are still unknown, and for joint replenishment problems with a large number of items, it usually takes a long time to obtain a solution using dynamic programming (Boctor et al., 2004)[22], branch partitioning, and other exact algorithms. Therefore, some scholars have worked on designing approximation algorithms that can quickly obtain an approximate solution with a constant worst-case ratio. For example, Roundy (1985)[23] reported that the optimal power of two strategies produced a solution with an error of 2% of the joint replenishment problem over an infinite range. Levi et al. (2006) presented an approximate solution to the joint replenishment problem for a single item over a finite range and proved that the cost is at most twice the optimal cost. Thereafter, Levi et al. (2008) proposed an approximate solution to the joint replenishment problem with a cost no more than 1.8 times the optimal cost. In addition, both (Levi et al., 2006) and (Levi et al., 2008) consider non-static holding costs. Nonnerand and Souza (2009) consider joint replenishment with a horizon, where the holding cost is infinite if the holding time exceeds the horizon, and provide algorithms with a cost that does not exceed 1.6 times the optimal cost. Segev (2014)[24] devised a $(1 + \varepsilon)$ algorithm to compute the joint replenishment problem. However, the search for such approximate algorithms has slowed down significantly in recent years.

3. Overview of Cost-sharing Studies

Multiple buyers form a cooperative alliance for joint purchasing with the goal of reducing their own costs, and the cost shared after cooperation should not be higher than the cost incurred by replenishment alone. Moreover, an unfair cost-sharing program may cause purchasers to leave the cooperative alliance, so a fair and reasonable cost-sharing rule is one of the key factors for purchasers to form a long-term and stable cooperative relationship. Based on the sharing of demand and cost information, some scholars have applied cooperative game theory to design a cost sharing scheme for purchasers' alliance, Meca and Timmerl firstly designed a proportional cost sharing rule for a joint replenishment problem with deterministic demand of a product, and proved that this sharing rule is a kernel allocation scheme[25]. Anily and Havivl gave a classical joint replenishment problem using power of two-strategy problem using the power of two strategy replenishment as the kernel allocation scheme, and give a counterexample to show that sharing the main setup cost among all members may result in an allocation scheme that is not in the kernel of the cooperative game[26]. Zhang proves that Lagrangian dyadic allocation is used as the kernel allocation scheme for the joint replenishment problem with the power of two strategy[27]. Dror and Hartma give a definition of indivisible product set[28]. further conducted a sensitivity analysis on the parameters of the joint replenishment problem with indivisible product sets, and concluded that the indivisibility of the game is more probable than concavity. Chen and Zhang constructed a linear programming model of the joint replenishment problem, and gave an allocation scheme for the dyadic solution[30]. Toriello and Uhan defined the dynamics of the cooperative game, and the Lagrangian dyadic allocation using the power of two strategy[32]. Uhan defined the kernel of dynamic allocation of cooperative game and gave the dynamic allocation scheme located in the kernel of joint replenishment game when the ordering cost function is concave[32]. Hezarkhani et al. used the shapley value to allocate the total cost of retailers' joint purchases either directly or indirectly from the manufacturer or intermediary and pointed out that the shapley-valued allocation scheme is located in the kernel due to the fact that the joint purchasing game is a concave game[33]. Elomri et al. studied the efficient coalition structure of non-super additive cooperative game, find the efficient coalition by maximizing the cost saving rate and gave the allocation scheme based on the cost rule[34]. Jouida et al. gave an algorithm for cooperative replenishment and obtained stable cooperative coalition by this algorithm[35]. Oterol and Zhang both used the shapley-valued allocation scheme, in which Zhang proved that their cooperative game is a concave game i.e., the Shapley allocation scheme is located in the kernel of the cooperative game[36]. Yinfang Ye et al. studied the interval-value game of joint ordering and gave the interval-value proportional residual allocation value and the variable-weight Shapley value allocation method respectively[37]. It can be seen that the solution in the kernel of the cooperative game is the cost sharing scheme that scholars want, because such an allocation scheme is to make the cooperative alliance stable, but there is a lot of literature that lacks the analysis of the stability of the alliance.

Incomplete sharing of demand and cost information can also lead to joint replenishment, and purchasers form a non-cooperative game, so that the solution obtained may not be optimal for the coalition or may require more stringent conditions to achieve coalitional optimality, i.e., it is usually not possible to maximize the coalition's returns. However, such joint replenishment also has some advantages in terms of the confidentiality of private information, and thus the joint replenishment problem formed in a non-cooperative form can be applied to competing purchasers.

4. Research Review

It is clear from the above literature that despite the abundance of literature on joint replenishment, fewer studies have been conducted on the cost variation of the joint replenishment problem. Compared to the rich research on joint replenishment problems of individual buyers, there is a dearth of research on joint replenishment problems of multiple retailers, e.g., joint replenishment problems of multiple buyers with controlled delivery schedules are very rare. In addition, the joint replenishment problem is that buyers replenish the products they want to purchase, but the joint replenishment problem does not study the product screening problem before purchasing. In reality, it may be necessary to update the product categories every once in a while, such as the department store retailer's seasonal products on and off the shelves, screening the newly launched products and so on.

And since the allocation rule at the core of the cooperative game is a relatively fair and reasonable allocation scheme, the core of the cooperative game is usually not easy to find or may not even exist. In addition, the periodic joint replenishment problem is an NP problem both in the finite and infinite range, which means that finding the optimal solution is not easy. Heuristics and intelligent algorithms are most commonly used for joint replenishment problems. Researchers have verified through numerical experiments that they are fast and effective methods, but there is no guarantee of the quality of the solution in the worst case scenario, which may result in less savings for the major coalition than for the minor coalition, and thus no guarantee of coalition stability. Approximation algorithms can predict the worst-case scenario, and retailers use this prediction to determine whether to participate in joint replenishment. However, there has been little research on approximation algorithms that can guarantee the worst-case scenario in joint replenishment problems.

In the current selection of algorithms for solving the joint replenishment problem, me-ta-heuristic algorithms such as differential evolutionary algorithms and genetic algorithms have been used extensively, and it can be shown that there are a variety of algorithms that are all applicable to the computation of the joint replenishment cost model. Differential evolutionary algorithms are constantly being used in different contexts and are improving; while the search for approximation algorithms has slowed down significantly. Therefore approximation algorithms may have more room for development and be able to be used in more contexts.

Joint replenishment of multiple purchasers involves a benefit distribution problem, and even in the same context as the joint replenishment problem for a single product requires the design of a reasonable benefit distribution rule in order to incentivize purchasers to participate in the cooperation. No single benefit distribution rule can be applied to every situation. There are many solution concepts in cooperative game theory, but they all have their own advantages and disadvantages, and it is worth exploring which solution can be applied to the corresponding joint replenishment problem or whether it is necessary to design a special allocation rule.

References

[1] Saracoglu I, Topaloglu S, Keskinturk T. A genetic algorithm approach for multi-product multi-period continuous review inventory models. Expert Systems with Applications, 2014, 41(18), 8189-8202.

[2] Khouja M & Goyal S K. A review of the joint replenishment problem literature: 1989-2005. European Journal of Operational Research, 2008, 186(1), 1-16.

[3] Porras E, Dekker R. An efficient optimal solution method for the joint replenishment problem with minimum order quantities. European Journal of Operational Research, 2006, 174(3), 1595-1615.

[4] Wang L, He J & Zeng, Y R. A differential evolution algorithm for joint replenishment problem using direct grouping and its application. Expert Systems: International Journal of Knowledge Engineering and Neural Networks, 2012, 29(5), 429-441.

[5] Salameh M K, et al. Joint replenishment model with substitution. Applied Mathematical Modelling, 2014, 38(14), 3662-3671.

[6] Wang L, He J, Wu, D & Zeng, Y.-R. A novel differential evolution algorithm for joint replenishment problem under interdependence and its application. International Journal of Production Economics, 2012, 135(1), 190-198.

[7] Qu H, Wang L & Liu R. A contrastive study of the stochastic location-inventory problem with joint replenishment and independent replenishment. Expert Systems with Applications, 2015, 42(4), 2061-2072.

[8] Wang L, Shi Y & Liu S. An improved fruit fly optimization algorithm and its application to joint replenishment problems. Expert Systems with Applications, 2015, 42(9), 4310-4323.

[9] Ding S, Kaminsky PM. Centralized and decentralized warehouse logistics collaboration. Manuf Serv Oper Manag, 2020, 22(4):812–831

[10] Levi R, Sviridenko M. Improved approximation algorithm for the one-warehouse multi-retailer problem, Approximation, Randomization, and Combinatorial Optimization. Algorithms and Techniques, 9th International workshop on approximation algorithms for combinatorial optimization problems, APPROX 2006 and 10th international workshop on randomization and comput, Springer-Verlag, New York, 2006, 188–199

[11] Levi R, Roundy RO, Shmoys DB, Sviridenko MI. A constant approximation algorithm for the one-warehouse multiretailer problem. Manag Sci, 2008, 54(4):763–776

[12] Barron Y. A Replenishment Inventory Model with a Stock Dependent Demand and Age Stock Dependent Cost Functions in a Random Environment. Asia Pacific J Oper Res, 2021:2150035

[13] Federgruen A, Wang M. Inventory models with shelf-age and delay-dependent inventory costs. Oper Res, 2015, 63(3):701–715

[14] Braglia M, Castellano D, Gallo M. An extension of the stochastic joint-replenishment problem under the class of cyclic policies. Oper Res Lett, 2016, 44(2):278–284

[15] Cui L, Deng J, Liu R et al. A stochastic multi-item replenishment and delivery problem with lead-time reduction initiatives and the solving methodologies. Appl Math Comput, 2020, 374:125055

[16] Noh JS, Kim JS, Sarkar B. Stochastic joint replenishment problem with quantity discounts and minimum order constraints. Oper Res Int J, 2019, 19(1):151–178

[17] Marcello B, Davide C, Liberatina S, Dongping S. Controlling lead times and minor ordering costs in the joint replenishment problem with stochastic demands under the class of cyclic policies. Int Trans Oper Res, 2021, 28(1):376–400

[18] Yang YH, Kim JS. An adaptive joint replenishment policy for items with non-stationary demands. Oper Res Int J, 2020, 20(3):1665–1684

[19] Pasandideh SHR, Niaki STA, Abdollahi R. Modeling and solving a bi-objective joint replen-ishment-location problem under incremental discount: MOHSA and NSGA-II. Oper Res Int J, 2020, 20(4):2365–2396

[20] Arkin E, Joneja D, Roundy R. Computational complexity of uncapacitated multi-echelon production planning problems. Oper Res Lett, 1989, 8(2):61–66

[21] Cohen T, Yedidsion L. The periodic joint replenishment problem is strongly np-hard. Math Oper Res, 2018, 43(4):1269–1289

[22] Boctor FF, Laporte G, Renaud J. algorithms for the dynamic-demand joint replenishment prob-lem. Int J Prod Res, 2004, 42(13):2667–2678

[23] Roundy R. 98%-effective integer-ratio lot-sizing for one-warehouse multi-retailer systems. Manag Sci, 1985, 31(11):1416–1430

[24] Segev D. An approximate dynamic-programming approach to the joint replenishment problem. Math Oper Res, 2014, 39(2):432–444

[25] Meca A, Timmer J, Ignacio García-Jurado, et al. Inventory games. European Journal of Operational Research, 2004, 156(1): 127-139.

[26] Anily S, Haviv A M. The Cost Allocation Problem for the First Order Interaction Joint Replenishment Model. Operations Research, 2007, 55(2): 292-302

[27] Zhang J. Cost allocation for joint replenishment models. Operations Research, 2009, 57 (1): 146-156.

[28] Dror M, Hartma B C. Shipment consolidation: Who pays for it and how much?. Management Science, 2007, 53(1): 78-87.

[29] Dror M, Hartman B C, Chan W. The cost allocation issue in joint replenishment. International Journal of Production Economics, 2012, 135(1): 242-254.

[30] Chen X, Zhang J W. A Stochastic Programming Duality Approach to Inventory Centralization Games. Operations Research, 2009, 57(4): 840-851.

[31] Zhang C J. A Stochastic Programming Duality Approach to Inventory Centralization Games. Operations Research, 2007, 57(4): 840-851.

[32] Toriello A, Uhan N A. Dynamic cost allocation for economic lot sizing games. Operations Research Letters, 2014, 42 (1): 82-84.

[33] Hezarkhani B, Slikker M, Van Woensel T. Collaborative replenishment in the presence of intermediaries. European Journal of Operational Research, 2018, 266(1): 135-146.

[34] Elomri A, Ghaffari A, Jemai Z, et al. Coalition Formation and Cost Allocation for Joint Replenishment Systems. Production & Operations Management, 2012, 21(6): 1015-1027.

[35] Jouida S B, Krichen S, Klibi W. Coalition-formation problem for sourcing contract design in supply networks. European Journal of Operational Research, 2017, 257: 539-558.

[36] Otero-Palencia C, Amaya–Mier R, Yie-Pinedo R. A stochastic joint replenishment problem considering transportation and warehouse constraints with gain sharing by Shapley Value allocation. International Journal of Production Research, 2019, 57(10): 3036-3059.

[37] Zhang J. Cost allocation for joint replenishment models. Operations Research, 2009, 57 (1): 146-156.