

# Capital Allocation Analysis based on Markowitz Model and Index Model

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**Abstract:** In finance, the question of how best to invest money to make more money is not an easy one. Some academics and hands-on practitioners have studied into this field for decades. This article delves into this issue, with a special focus on a detailed analysis of various investment plans. To get a better idea of which plan is better in a given situation, this paper dug through a bunch of data that included ten different company stocks from various industries, as well as daily revenue data for the S&P 500 over a 20-year history. This paper mainly looked at two key investment plans: the Markowitz model and the Index model. This paper did some simple math and theoretical research, and then this paper used the capabilities of Excel to help us find the best solution. And the analysis specially adopted a monthly summary method, minimized the complexity of the data on a horizon. And the study critically evaluating investment strategies within real-world constraints such as Regulation T, arbitrary "box" constraints, and scenarios involving no short positions or exclusion of broad indices. The finding shows that the performance of the assets chosen by Markowitz is better than Index Model under the given circumstances, which indicates the more complex model and addition data could probably provide a better result. This dissertation aims to contribute significantly to the field of finance by enhancing the understanding of portfolio management strategies, thus aiding in informed decision-making in the ever-evolving financial landscape.

**Keywords:** Markowitz Model, Index Model, Portfolio Optimization.

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## 1. Introduction

In the dynamic realm of finance, the pursuit of optimizing investment portfolios remains a cornerstone of both academic study and practical application. This paper mainly analyzes the investment strategy, especially the optimization of the investment portfolio. The aim of the study is to combine theoretical knowledge with practical cases to gain insight into the complex factors that control financial markets.

The core of the article focuses on two important investment models: the Markowitz model and the index model. These two models, which form the basis of portfolio optimization theory, have been studied in depth under different constraints, such as T-rule constraints, various "box" constraints, unlimited cases, no shorting, and exclusion of broad base indices, in order to better understand their applicability and performance in various financial environments. The aim of the study was not only to demonstrate these models in action, but also to compare their effectiveness under different regulatory and operating conditions. In a constantly changing and challenging financial market, such research is very important and can provide certain decision support for financial practitioners and analysts.

## 2. Literature Review

The literature review provides different research on portfolio construction and optimization. Which includes traditional basic models in portfolio management, advanced statistical models, and modern machine learning techniques.

Gültekin, Shohfi, and Guerard provide an extensive analysis of efficient portfolio construction during the time period from 1999 to 2017. They dug particularly into the 3-factor risk model and stressed out the superiority of statistical models. The multi-factor model is better than the fundamental

models in generating significant excess returns and improving Sharpe Ratios. It also underscores the advantages of Markowitz portfolio construction and illustrates its limitations like complicated data and laborious computing process. This research claims that some academic models are somehow impractical in real-world settings and the emergence of robo-advisors pushes forward the advancement of traditional optimization approaches [1].

Zhang had his research on a deep learning algorithm called autoencoder. The edge of his finding is in tracking stock indexes, especially the CSI 300 index. The autoencoder is very effective at figuring out the complex relationships between the index components. It could use that information to build an ever-changing tracking portfolio. This new approach performs better than the traditional approach in some aspects, especially for the small portfolio in stocks, because it can efficiently encode large amounts of information for better tracking. The article also highlights that these algorithms have unparalleled superiority in identifying underlying assets and help people make timely investment decisions. As the technology in AI areas gets more mature, deep learning algorithms have become increasingly widely used in the financial sectors. Which could benefit regions including index tracking for complex systems such as stock markets [2].

Dimitriu discusses the application of the Efficient Market Hypothesis in the context of portfolio construction during the period from 2007 to 2012, particularly focusing on the impact of the economic crisis. It explores the selection of optimal portfolios based on risk and return considerations, taking into account various companies from different sectors. The analysis reveals that certain portfolios offer higher returns at the cost of increased risk, while others prioritize risk mitigation. Notably, the study identifies the prominent role of Bat Bascov S.A. in efficient portfolios and observes that risk

tends to decrease with a higher number of assets [3].

On the other hand, Beste focuses on the portfolio optimization model proposed by Markowitz in 1952. This model is based on the mean - variance analysis, assuming that investors tend to choose there are both high return and low risk investment portfolio. In Markowitz's theoretical framework, each portfolio can be represented as a point on the plane of risk and return and form a special curve (hyperbola). This model uses equal mean and equal variance curves to find the optimal portfolio. However, applying this method to more than three types of securities are challenging due to geometric complexity. Beste also explores how adding new securities to the portfolio improves the hyperbola, questioning the usual practice of choosing only those with high expected returns and low risk [4]. Pandey presents an analysis of optimal portfolio formation using real-world data and two distinct constraint sets. It employs the Markowitz portfolio analysis framework, which focuses on creating efficient portfolios that balance risk and return. The primary output of this analysis is the identification of an efficient frontier, representing portfolios that achieve the highest return for a given level of risk. The study illustrates how investors can minimize risk and maximize returns, guiding them on when and why to invest in specific portfolios. The Markowitz model is utilized to calculate standard deviations and returns for feasible portfolios, identifying the efficient frontier as the boundary for portfolios with increasing returns [5].

It can be seen that the above key topics fully demonstrate the effectiveness of statistical models and the control of risks, and further combine with deep learning algorithms to improve the efficiency of portfolio construction. However, the research did not have a thorough look at which model is the more efficient and reliable for the investors. Considering Markowitz model and Index model are both famous models used by multiple financial institution for many years, putting a glance at which model is better is an attracting topic. This paper also explores the application of the efficient market hypothesis in the contemporary economic context, through which it fully demonstrates how to balance risk and return in portfolio selection.

### 3. Methodology

#### 3.1. Model Introduction

##### 3.1.1. Markowitz Model

The Markowitz model is a tool to help investors make better portfolio decisions, developed by Harry Markowitz in 1952. The core idea of this model is to obtain the maximum expected return as far as possible under a certain level of risk, or to reduce the risk as far as possible under a certain income target.

Steps of Markowitz Model in assets allocation are as follows [6]:

Using historical data to calculate the variance and covariance of the return of stock prices

Changing weights allocated to the different assets to draw a figure with volatility  $\sigma$  as the X-axis and return rate  $r$  as the Y-axis which contains all possible portfolios in the markets

Investors can find the efficient frontier and some important portfolios such as Minimal Variance Portfolio and Tangency Portfolio which has the highest Sharpe Ratio

##### 3.1.2. Index Model

In layman's terms, the index model is a tool for analyzing

stock returns, and it takes into account two main types of risk. The first is systemic risk, that is, market risk. This risk comes from big events that affect the entire stock market, such as economic crises, policy changes or global financial turmoil. These are external factors beyond the individual investor's control that affect all stocks in the market. Simply put, when the economic situation is bad or the market is unstable, the price of most stocks may fall, which is systemic risk. And the second type of risk is Specific risk, also known as unsystematic risk, is the risk specific to an individual security that can be diversified away in this well-constructed portfolio.

The advantage of Index model is that only linear regression is required and only two factors – systematic risk and specific risk are included. The model incorporates the use of a market index, typically representing the overall market's performance, as a proxy for systematic risk. In its simplest form, the Index Model expresses the return on an individual security as a function of its sensitivity to market movements (beta), the expected return of the market, and a unique risk component [7]:

$$R_i = \alpha_i + \beta_i R_m + \varepsilon_i$$

Where:

$R_i$  is the return on the individual security.

$\alpha_i$  is the security's alpha, representing the expected excess return not explained by market movements.

$\beta_i$  is the beta coefficient, indicating the security's sensitivity to market returns.

$R_m$  is the return on the market index.

$\varepsilon_i$  is the specific risk component, representing idiosyncratic or company-specific risk.

The Index Model is valuable in portfolio management and security analysis as it helps investors understand the risk and return characteristics of individual securities in the context of the broader market. It provides insights into how much of a security's risk and return can be attributed to market movements versus company-specific factors[8].

#### 3.2. Assumption

Considering that the market has different participants, to analyze the optimal decisions for different investors, this paper separate them into 5 groups whose investment behaviors are constrained by the terms below:

1. Regulation T by FINRA, allows broker-dealers to allow their customers to have positions, 50% or more of which are funded by the customer's account equity.

2. Some arbitrary "box" constraints on weights, which may be provided by the client.

3. Don't have any additional optimization constraints.

4. Typical limitations existing in the U.S. mutual fund industry: a U.S. open-ended mutual fund is not allowed to have any short positions,

5. To check if the inclusion of the broad index into the portfolio has positive or negative effect, remove the investment on SPX index.

For better visualization, 3 frontiers are introduced - minimal risk frontier, minimal return frontier and efficient frontier. The minimal risk frontier is constructed by the portfolios with the lowest standard deviation when the return level is settled. The minimal return frontier and efficient frontier are constructed by the portfolios that generate the lowest/highest return under a given risk level. The intersection point of the two frontiers presents the minimal risk portfolio. And the efficient frontier is often used to get the Capital Market Line which is helpful in the generating the

efficient risky portfolio.

Minimal Risk or Variance Frontier:

$$\begin{cases} \sigma(\vec{w}) = \min \\ \text{subject to: } r(\vec{w}) = \text{const} \end{cases}$$

Minimal Return Frontier:

$$\begin{cases} r(\vec{w}) = \min \\ \text{subject to: } \sigma(\vec{w}) = \text{const} \end{cases}$$

Efficient Frontier:

$$\begin{cases} r(\vec{w}) = \min \\ \text{subject to: } \sigma(\vec{w}) = \text{const} \end{cases}$$

### 3.3. Descriptive analysis

For practical purposes, this paper selected 10 stocks from the four sectors of technology, energy, consumer defense, and consumer cycle between May 11, 2001, and May 12, 2021,

for a total of 5,030 trading days.

To have a preliminary study on the stock data, the study needs to focus especially on the historical average return and standard deviation which measure the payoff and risk of the portfolio. And for Markowitz model, the covariance matrix is provided for a direct look at the correlation relationship between different assets and computation of the portfolio risk.

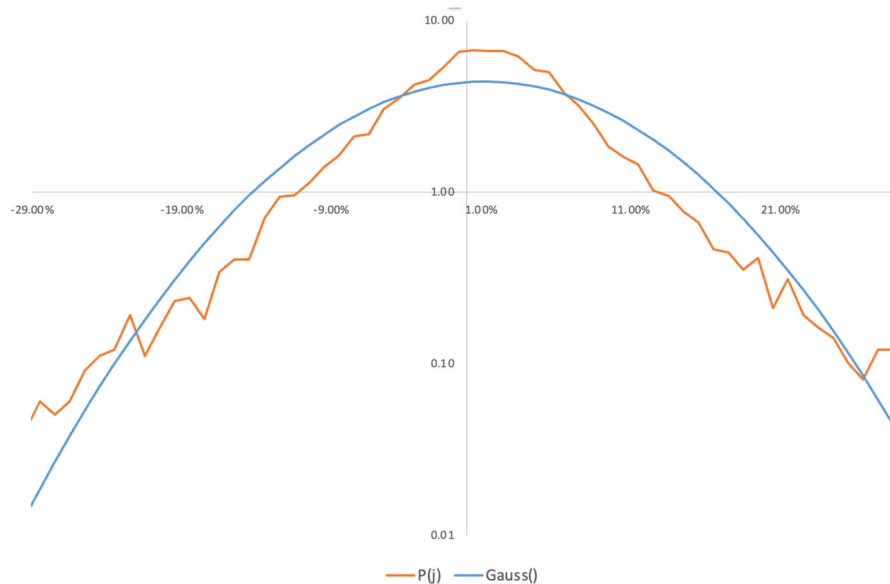
The summary data of the stocks are provided in the two tables followed. AKAM has the highest annual average return at 28.1%. The stock with the lowest annual return is XOM at 5.4%. AKAM also has a high annual standard deviation of 63.1%, indicating higher volatility. The beta values range from 0.53 for PEP to 1.65 for AKAM, suggesting AKAM is more sensitive to market movements. Alpha values, which represent the specific risk, are all at a low level. Aside from the basic data of the stocks, the value of kurtosis and skewness is worth looking as well.

**Table 1.** Descriptive statistics of the stocks

	SPX	QCOM	AKAM	ORCL	MSFT	CVX	XOM	IMO	KO	PEP	MCD
<b>Annual Average Return</b>	7.5%	13.1%	28.1%	11.1%	13.1%	8.8%	5.4%	10.9%	7.0%	7.9%	13.5%
<b>Annual StDev</b>	14.9%	33.3%	63.1%	27.8%	23.3%	22.3%	20.8%	30.5%	16.3%	15.1%	18.7%
<b>beta</b>	1.0000	1.2492	1.6550	1.0220	1.0022	0.9206	0.7944	1.0722	0.5378	0.5300	0.6754
<b>alpha</b>	0.0000	0.0365	0.1565	0.0341	0.0559	0.0184	-0.0063	0.0286	0.0298	0.0389	0.0836
<b>residual Stdev</b>	0.0%	27.6%	58.1%	23.3%	17.9%	17.6%	17.1%	26.0%	14.2%	12.9%	15.7%
<b>Skewness</b>	-0.18	0.42	1.48	0.45	0.26	0.12	0.26	-0.16	-0.02	0.22	0.18
<b>Kurtosis</b>	12.37	8.57	17.27	8.22	9.36	23.42	12.14	12.76	11.74	20.34	15.55

In the process of deriving the numerical characteristics of the stock prices, this paper can also find an interesting phenomenon on the distribution of the stock price - the classic

“high kurtosis and fat-tailed” shape. It is a common phenomenon which could be observed on the financial market.



**Figure 2.** The distribution of stock price compared to the gaussian distribution

Fat tails are often observed in financial markets due to various factors, including:

**Market Sentiment:** Behavioral factors and market sentiment can lead to rapid and exaggerated movements in stock prices.

**Unforeseen Events:** Unexpected news, geopolitical events, or other unforeseen factors can trigger significant price changes.

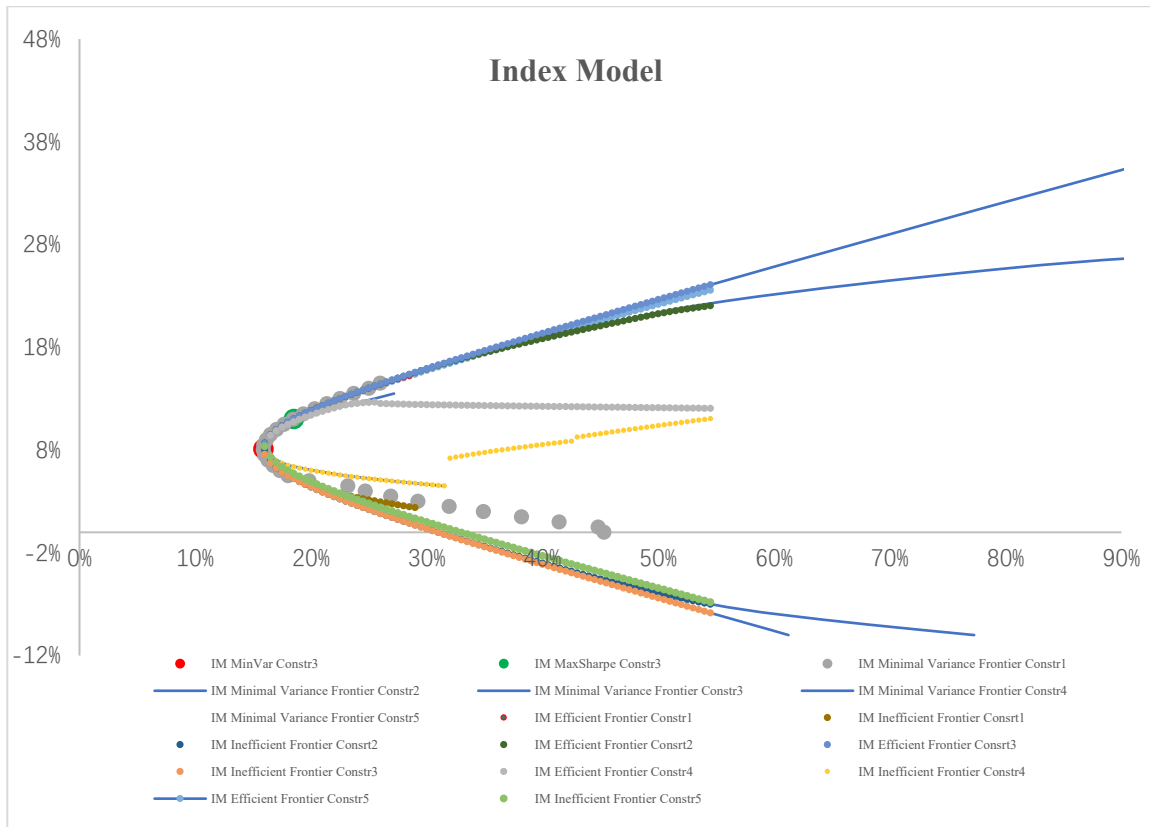
**Liquidity Shocks:** Sudden changes in liquidity conditions

can exacerbate price movements, especially in less liquid markets.

**Market Dynamics:** Interactions between market participants, algorithmic trading, and complex financial instruments can contribute to nonlinear price behavior.

The second table presents the correlation coefficients between the stocks and S&P 500 index is used as a benchmark. The correlations between SPX and other stocks vary, with the highest correlation being 73.3% with MSFT, and the lowest





**Figure 4.** The Permissible Portfolios Region under Index Model

## 4.2. Minimum Variance Portfolio

Investors with high risk aversion typically exhibit a demand for investment options that balance the desire for returns with a preference for lower risk. Seeking the minimum

variance portfolio is important for this group. To find this specific portfolio for different groups, this paper finds out the weights assigned to different assets with minimal variance under the assumption of different constraints. Which are already showed in the figure 3 and 4.

**Table 3.** Weights of min variance portfolio calculated by Markowitz Model

	SPX	QCOM	AKAM	ORCL	MSFT	CVX	XOM	IMO	KO	PEP	MCD
Constr1	28.03%	-1.09%	-1.48%	-0.57%	-2.69%	-10.58%	3.54%	6.43%	26.93%	29.98%	21.50%
Constr2	28.01%	-1.10%	-1.48%	-0.57%	-2.68%	-10.57%	3.55%	6.42%	26.93%	29.98%	21.50%
Constr3	28.03%	-1.09%	-1.48%	-0.57%	-2.69%	-10.58%	3.54%	6.43%	26.93%	29.98%	21.50%
Constr4	15.70%	0.15%	0.00%	0.00%	0.05%	0.00%	0.23%	4.01%	27.67%	30.64%	21.56%
Constr5	0.00%	1.12%	-0.40%	1.86%	2.06%	-7.29%	6.84%	7.19%	30.19%	33.71%	24.71%

**Table 4.** Weights of min variance portfolio calculated by Index Model

	SPX	QCOM	AKAM	ORCL	MSFT	CVX	XOM	IMO	KO	PEP	MCD
Constr1	-2.07%	-11.23%	0.00%	-11.53%	-11.12%	-9.19%	-4.86%	28.24%	24.89%	26.18%	70.69%
Constr2	30.12%	-2.66%	-2.68%	-2.02%	-1.79%	-0.60%	1.95%	2.35%	28.26%	28.92%	18.15%
Constr3	30.11%	-2.68%	-2.67%	-2.01%	-1.82%	-0.58%	1.96%	2.33%	28.27%	28.94%	18.15%
Constr4	19.13%	0.00%	0.00%	0.00%	0.00%	0.00%	1.99%	2.37%	28.71%	29.39%	18.41%
Constr5	0.00%	-0.70%	-1.72%	0.39%	1.33%	2.57%	5.45%	4.44%	32.89%	33.59%	21.75%

On the Weight assignment, except for the scenario of constraint 1, the two modes have similar distribution on the assets, indicating that even under different model, the

minimum variance portfolios are likely to be similar based on the same dataset. The return, standard deviation and Sharpe Ratio under different constraints are shown below:

**Table 5.** Return, risk, and sharpe ratio of min variance portfolio

	Markowitz Model			Index Model		
	Return	StDev	Sharpe	Return	StDev	Sharpe
Constr1	8.30%	15.76%	0.526	10.51%	22.64%	0.464
Constr2	8.30%	15.76%	0.526	8.12%	15.88%	0.511
Constr3	8.30%	15.76%	0.526	8.12%	15.88%	0.511
Constr4	8.59%	15.85%	0.542	8.36%	16.00%	0.523
Constr5	8.62%	15.90%	0.542	8.37%	16.11%	0.520

The standard deviation given by the two models shown above do not have a large difference. The Sharpe ratios of minimum variance portfolios given by Markowitz Model are higher than Index Model.

### 4.3. Maximum Sharpe Ratio Portfolio

The Sharpe Ratio is crucial for evaluating a portfolio's risk-adjusted performance. By considering both return and

volatility, it provides a single metric for comparing investment opportunities. A higher Sharpe Ratio indicates better risk-adjusted returns, aiding investors in making informed decisions to optimize their portfolios [10]. By adding different constraints to the assumption, weights assigned to different stocks are derived to get the maximum Sharpe ratio.

**Table 6.** Weights of maximum Sharpe ratio portfolio calculated by Markowitz Model

	SPX	QCOM	AKAM	ORCL	MSFT	CVX	XOM	IMO	KO	PEP	MCD
Constr1	-10.47%	0.00%	2.08%	-0.59%	22.67%	14.11%	-38.93%	9.04%	8.88%	33.17%	60.06%
Constr2	-30.24%	0.94%	3.27%	-1.81%	28.81%	22.89%	-46.96%	10.54%	11.27%	36.85%	64.43%
Constr3	-30.24%	0.95%	3.27%	-1.81%	28.81%	22.89%	-46.96%	10.54%	11.27%	36.85%	64.43%
Constr4	0.00%	0.00%	0.53%	0.00%	16.23%	0.00%	0.00%	0.00%	1.69%	25.44%	56.11%
Constr5	0.00%	-1.30%	2.05%	-4.44%	23.42%	18.85%	-49.52%	9.68%	8.18%	32.82%	60.26%

**Table 7.** This paperights of maximum sharpe ratio portfolio calculated by Markowitz Model

	SPX	QCOM	AKAM	ORCL	MSFT	CVX	XOM	IMO	KO	PEP	MCD
Constr1	-2.70%	0.40%	1.91%	0.34%	13.65%	-0.94%	-12.84%	1.30%	20.53%	29.71%	48.62%
Constr2	-2.96%	0.45%	1.93%	0.35%	13.61%	-0.75%	-12.77%	1.28%	20.48%	29.70%	48.67%
Constr3	-2.70%	0.40%	1.90%	0.34%	13.65%	-0.93%	-12.84%	1.30%	20.53%	29.71%	48.62%
Constr4	0.00%	0.00%	1.13%	0.00%	10.81%	0.00%	0.00%	0.00%	17.52%	25.59%	44.96%
Constr5	0.00%	0.22%	1.81%	0.13%	13.34%	-1.21%	-13.11%	1.12%	20.15%	29.31%	48.23%

The tables shown above indicate the weights assigned to different assets according to certain constraint and model. The main difference of Markowitz model and Index Model is that Markowitz model chooses to have a relatively large short

position on the SPX and XOM. They both choose to have a large long position on the PEP and MCD.

The return, risk and Sharpe ratio are shown below:

**Table 8.** Return, risk, and Sharpe ratio of min variance portfolio

	Markowitz Model			Index Model		
	Return	StDev	Sharpe	Return	StDev	Sharpe
<b>Constr1</b>	12.82%	19.14%	0.670	11.03%	18.51%	0.596
<b>Constr2</b>	13.57%	20.15%	0.673	11.03%	18.52%	0.596
<b>Constr3</b>	13.57%	20.15%	0.673	11.03%	18.51%	0.596
<b>Constr4</b>	11.17%	17.87%	0.625	10.31%	17.66%	0.584
<b>Constr5</b>	13.14%	19.64%	0.669	11.00%	18.47%	0.596

As this paper can tell from table 6, the Markowitz Model generates higher return and Sharpe Ratio compared to the Index Model. The performance of Markowitz model is better than Index Model in this case.

In conclusion, in the process of deriving the optimal portfolio under different assumptions, Markowitz Model prevailed over Index Model and seems to be preferred by the investors in real financial market.

## 5. Conclusion

To gain deeper insights into the better plan for a given financial scenario, this paper discreetly analyzed a substantial dataset. This dataset comprised ten distinct company stocks crossing various industries and S&P 500 index over a comprehensive 20-year historical span. The primary focus is put on evaluating two pivotal investment plans: the Markowitz model and the Index model.

Through a combination of mathematical analysis and theoretical exploration, this paper sought to find out the best solution. Using the computational prowess of Excel and R, this paper tried to handle the difficulties of this intricate financial landscape. Our analysis adopted a concise monthly summary approach, simplifying the data for ease of

comprehension. Furthermore, our study took into account the real-world constraints which are imposed by regulations, such as Regulation T, arbitrary constraints denoted as "box" constraints, and scenarios devoid of short positions or the exclusion of broad indices.

The results of our investigation revealed that the performance of assets chosen by the Markowitz model surpassed that of the Index Model under different constraints. This outcome suggests that a more intricate model, coupled with additional data, has the potential to yield superior results. In essence, this dissertation aspires to make a significant contribution to the field of finance by enhancing the comprehension of portfolio management strategies in this industry, which could enable individuals to make well-informed decisions in their own investment.

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