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A New SVAR Model Based on Standardized Standard Asymmetric Exponential Power Distribution

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Abstract: This paper introduces a novel Structural Vector Autoregressive (SVAR) model incorporating non-Normal marginal errors described by the Standardized Standard Asymmetric Exponential Power Distribution (SSAEPD). This innovation allows the model to effectively capture the inherent skewness and kurtosis typically observed in financial data. Utilizing Markov Chain Monte Carlo (MCMC) techniques for simulating error terms and Maximum Likelihood Estimation (MLE) for parameter estimation in MATLAB, the model's robustness is significantly enhanced. We employ Impulse-Response Function (IRF) and Variance Decomposition (VD) for structural analysis to examine the relevance of classic Keynesian theory in today's US economic context. Our empirical findings suggest that the SSAEPD-enhanced SVAR model not only outperforms traditional SVAR models in capturing data characteristics but also aligns more closely with Keynesian principles. The results underscore the continued applicability of the IS-LM framework in understanding contemporary economic dynamics, demonstrating the model's superior performance in variance decomposition and structural analysis.

Keywords: Structural Vector Autoregressive Regression (SVAR); Standardized Standard Asymmetric Exponential Power Distribution (SSAEPD); Classic Keynesian Theory, Impulse Response Function (IRF).

1. Introduction

The Vector Autoregression Model (VAR) is a cornerstone linear methodology that effectively captures the complex dynamics present in multiple time series. Its utility and flexibility as a statistical tool are well-documented, yet it is inherently limited as a reduced form model. Specifically, the VAR model does not offer explanations for the instantaneous relationships among variables [1, 2]. To overcome this limitation, the Structural Vector Autoregression model (SVAR) was developed. SVAR enhances our understanding by interpreting the instantaneous correlations among variables through a carefully designed set of restrictions.

One of the distinct advantages of the SVAR model over other simultaneous equation models is its identification strategy. While other models impose extensive restrictions on autoregressive coefficients—often more than necessary—the SVAR model applies just enough constraints on error terms to identify the system. This minimalist approach is preferable since economic theory frequently offers limited guidance on these coefficients, thus making data-driven determination a more practical choice [3].

Historically, since Sims' original proposal, the SVAR model has evolved into a significant tool for macroeconomic analysis, demonstrating its worth across various empirical studies [4]. Many researchers have focused on implementing restrictions within different SVAR systems guided by economic theory. For instance, restrictions based on traditional Keynesian models have been applied to analyze macroeconomic fluctuations in the US [5]. Moreover, the imposition of restrictions on the long-run effects of shocks has allowed for the exploration of dynamic responses to demand and supply disturbances in bivariate SVAR systems [6].

In conventional applications, structural shocks within SVAR models are assumed to be serially uncorrelated and mutually orthogonal white noise disturbances, typically modeled using a normal distribution. However, this assumption does not hold well in practice where financial data often display skewness, fat tails, and asymmetric kurtosis, attributes poorly captured by normal distributions. Addressing this challenge, we introduce the Standardized Standard Asymmetric Exponential Power Distribution (SSAEPD) as a novel approach to model error terms in SVAR, enhancing the model's capability to account for these data characteristics [7].

Different from previous studies, our approach extends the SVAR model by focusing on improving the fit of residuals rather than merely refining the system's identification. In this paper, SSAEPD margins are integrated using a Gaussian Copula in what we term the SVAR-GC SSAEPD model. We employ Maximum Likelihood (ML) for parameter estimation and use Markov Chain Monte Carlo (MCMC) for error term simulation [8]. Additionally, we implement the Kolmogorov-Smirnov (KS) test for model diagnostics and utilize Impulse-Response Function (IRF) and Variance Decomposition (VD) for structural analysis. Our research aims to reassess the validity of the Keynesian theory in the current U.S. economic framework under the assumption of non-Normal error terms [9].

The paper is organized as follows: Section 2 presents the model and methodology, Section 3 describes the simulation analysis, Section 4 discusses empirical results, and Section 5 concludes with future research directions.

Table 1. Empirical Applications about SVAR [10].

Author	Contents	Sample	Data
Bernanke (1986)	Money-income correlation	U.S.	1954: Q1-1984: Q4
Blanchard and Quah (1989)	Effects of demand and supply on GNP and unemployment	U.S.	1950: Q2-1987: Q4
Dalsgaard and Serres(1999)	Source of government deficit ratio of 4 economic variances	EU	1996.01-1997.12
Monticelli et.al. (1999)	Effects of monetary policy on macroeconomy	U.S. and 3 European countries	1978: Q1-1997: Q4
Breitung (2000)	IS-LM model	U.S.	1970: Q1-1997: Q4
Dungey and Pagan (2000)	Effects of 5 types of shocks on macroeconomy	Australia and U.S.	1980: Q1-1998: Q4
Gottschalk and Zandweghe (2001)	Source of output fluctuations	Germany	1962: Q1-1998: Q4
Garc´ia et al. (2002)	Core inflation	EU	1981.01-2000.12
Ho¨ppner (2002)	Effects of fiscal policy on macroeconomy	Germany	1970: Q1-2000: Q4
Mohr (2002)	Effects of fiscal policy on macroeconomy	Germany	1970: Q1-2000: Q2
Bankasi (2003)	Effects of fiscal policy on macroeconomy	Turkish	1987: Q1-2005: Q4
Bruneau and Bandt (2003)	Effects of ficsal and moentary policy on macroeconomy	EU	1979: Q1-2000: Q2
Djivre and Ribon (2003)	Effects of monetary policy on macroeconomy	Israeli	1990: Q1-1999: Q4
Giacinto (2003)	Regional effects of monetary policy on macroeconomy	U.S.	1958: Q2-2000: Q4
Michal (2003)	Source of natural interest rate	U.S.	1960.01-2002.06
Claeys (2004)	Moentary-budgetary policy correlation	OECD	mid 1960s -2001: Q2 mids 1970s-2001: Q2
Peersman and Straub (2004)	GDP-GDP deflator- interest rate- hours worked-real wage correlation	EU	1982: Q1-2002: Q4
Giordano (2005)	Effects of fiscal policy on macroeconomy	Italy	1982: Q2-2003: Q4
Ferna´ndez (2006)	Effects of fiscal policy on macroeconomy	Spain	1980: Q1-2004: Q4
Mitra (2006)	Effects of government investment on private investment	India	1969.01-2005.12
Simorangkir (2006)	Openness-economic performance correlation	Latin America and East Asia	1965.01-1989.12
Sousa and Zaghini (2006)	Effects of global monetary policy on macroeconomy	G5 countries	1980: Q1-2001: Q4
GUAY (2007)	Effects of structural policy settings on macroeconomy	OECD	1975: Q1-2004: Q4
Elbourne (2008)	Housing market-monetary policy correlation	UK	1987.01-2003.05
Mirdala (2008)	Effects of macroeconomic shocks on interest rate and GDP	EU	1995.01-2007.12
Dungey and Pagan (2009)	NK model	Austrilia and U.S.	1980:Q1-2006:Q4
Afonso and Sousa (2009)	Effects of fiscal policy on macroeconomy	Portugal	1979:Q1-2007:Q4
Beetsma et.al. (2009)	Effects of government spending on GDP	EU	1965:Q1-2004:Q4
Bencˇ´ik (2009)	Effects of fiscal policy on business cycle	Slovakia	1997.01-2007.12
Estrada and Montero (2009)	Effects of R&D investment on GDP	Spain and 6 developed countries	1970.01-2006.12
Inoue and Hamori (2009)	Source of exchange rate fluctuations	India	1999.01-2009.02
Mirdala (2009)	Effects of monetary policy on interest rate	EMU	1999:Q1-2008:Q3
Partridge and Rickman (2009)	Source of labor market fluctuations	Canada	1976-2003
Alexandru (2010)	Shadow economy-unemlpoyment rate correlation	U.S.	1980:Q1-2009:Q2
Loria et.al. (2010)	Effects of monetary approach on exchange rate	Mexico	1994.01-2007.12
Ravnik and Zˇilic´ (2010)	Effects of fiscal policy on macroeconomy	Croatia	2001.01-2009.12
Ahmed and Wadud(2011)	Effects of oil price on macroeconomy	Malaysian	1986.01-2009.12
Hausman et.al. (2012)	Effects of acreage supply on crop prices	U.S.	1956.01-2007.12
Zhao (2010)	Source of price fluctuation of real estate	China	1999:Q1-2009:Q2
Huang (2010)	Effects of monetary policy on real estate price	China	2005.07-2009.09
Zhao (2008)	Source of core inflation	China	1986.01-2007.12
Jiang (2009)	Regional effects of monetary policy on macroeconomy	China	1978-2006
Liang (2011)	Effects of exchange rate and export tax rebates on export	China	1984.01-2009.12
Lv (2009)	Source of demand & supply shocks	China	1992:Q1-2008:Q3

Table 2. Extensions and Applications of the Normal Distribution [11].

Authors	Distributions and their Applications
De Moivre (1738)	Normal distribution
Gauss (1809)	Normal applied in astronomy
Subbotin (1923)	EPD
Aitchison J. and Brown J.A.C. (1957)	Lognormal distribution
Azzalini (1986)	SEPD
Zolotarev V.M. (1986)	Stable distribution
Bolleslev (1987)	Student-t distribution
Fernandez et al. (1995)	Modified SEPD
Swamee P.K. (2002)	Near lognormal distribution
Aybeo and Kozubowski (2004)	SEPD in finance
DiCiccio and Monti (2004)	Properties of MLE of the SEPD
Zhu and Zinde-Walsh (2009)	AEPD

Notes: EPD=Exponential Power Distribution; SEPD=Skewed Exponential Power Distribution. AEPD=Asymmetric Exponential Power Distribution.

2. Model and Methodology

2.1. SVAR-MGC SSAEPD Model

Based on the AB-type SVAR model, we propose a new SVAR(p) model with non-Normal margins. The structure form of this new model is (denoted as SVAR(p)-MGC SSAEPD):

$$AY_t = \Gamma_1 Y_{t-1} + \Gamma_2 Y_{t-2} + \cdots + \Gamma_{t-p} Y_{t-p} + B\varepsilon_t, (t = 1,2, \ldots T) \quad (1)$$

where

$$A = \begin{bmatrix} 1 & a_{01} & a_{02} \\ b_{01} & 1 & b_{02} \\ c_{01} & c_{02} & 1 \end{bmatrix}$$

$$\Gamma_i = \begin{bmatrix} a_{i1} & a_{i2} & a_{i3} \\ b_{i1} & b_{i2} & b_{i3} \\ c_{i1} & c_{i2} & c_{i3} \end{bmatrix}, (i = 1,2, \ldots, p)$$

$$B = \begin{bmatrix} a_{03} & 0 & 0 \\ 0 & b_{03} & 0 \\ 0 & 0 & c_{03} \end{bmatrix}$$

$$Y_t = \begin{bmatrix} y_{1t} \\ y_{2t} \\ y_{3t} \end{bmatrix}, \varepsilon_t = \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{bmatrix}$$

$$(\varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{3t}) \sim MGC\_SSAEPD(\alpha_1, p_{11}, p_{12}, \alpha_2, p_{21}, p_{22}, \alpha_3, p_{31}, p_{33}, R) \quad (2)$$

$$\varepsilon_{it} \sim SSAEPD(\alpha_i, p_{i1}, p_{i2}), \alpha_i \in (0,1), p_{ij} > 0, i = 1,2,3, \quad (3)$$

where MGC_SSAEPD is the joint probability density function (PDF) of errors ε_{1t}.

$$c_R^{Ga}(\varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{3t}; R) = \frac{1}{|R|^{1/2}} \exp\left\{-\frac{\varsigma_t'(R^{-1}-I)\varsigma_t}{2}\right\} \quad (4)$$

where

$$\varsigma_t = [\varsigma_{1t}, \varsigma_{2t}, \varsigma_{3t}]', \qquad \varsigma_{it} = \Phi^{-1}\big(F_i(\varepsilon_{it})\big), i = 1,2,3,$$

For equation (4), the $c_R^{Ga}(\varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{3t}; R)$ is the probability density function (PDF) of multivariate Gaussian copula (MGC) with correlation matrix R [12]. Φ (.) is the cumulative density function (CDF) of standard Normal. F_i(.) is cumulative density function (CDF) the of SSAEPD.

By premultiplying with A^{-1}, the reduced form of the new SVAR model is:

$$Y_t = \Pi_1 Y_{t-1} + \Pi_2 Y_{t-2} + \cdots + \Pi_{t-p} Y_{t-p} + u_t \quad (5)$$

where $\Pi_i = A^{-1} F_i$. Moreover, the reduced-form disturbances u_t can be modeled as an interdependent system of linear equations of the residuals ε_t in its structure form (see equation (6)).

$$Au_t = B\varepsilon_t \tag{6}$$

To identify the structure form parameters, we must place restrictions on the parameters of A and B matrices. For this model, a set of *n(n + 1)/2* restritions should be imposed, leaving overall *2n² − n(n + 1)/2* free elements.

2.2. Standardized Standard AEPD (SSAEPD)

The probability density function (PDF) of a random variable ε_{it} with Standardized Standard AEPD (i.e., ε_{it} ~ SSAEPD (α_i, p_{i1}, p_{i2})) is:

$$f(\varepsilon_{it}) = \begin{cases} \delta_i\left(\frac{\alpha_i}{\alpha_i^*}\right)K(p_{i1})\exp\left(-\frac{1}{p_{i1}}\left|\frac{\omega_i+\delta_i\varepsilon_{it}}{2\alpha_i^*}\right|^{p_{i1}}\right), & \text{if } \varepsilon_{it} \leqslant -\frac{\omega_i}{\delta_i} \\ \delta_i\left(\frac{1-\alpha_i}{1-\alpha_i^*}\right)K(p_{i2})\exp\left(-\frac{1}{p_{i2}}\left|\frac{\omega_i+\delta_i\varepsilon_{it}}{2(1-\alpha_i^*)}\right|^{p_{i2}}\right), & \text{if } \varepsilon_{it} > -\frac{\omega_i}{\delta_i} \end{cases} \tag{7}$$

And its cummulative density function (CDF) of SSAEPD is:

$$F(\varepsilon_{it}) = \begin{cases} \alpha_i\left[1 - G\left(\frac{1}{p_{i1}}\left(\frac{|\varepsilon_{it}\delta_\delta+w_i|}{2\alpha_i^*}\right)^{p_{i1}};\frac{1}{p_{i1}}\right)\right], & \text{if } \varepsilon_{it} \leq -\frac{\omega_i}{\delta_i} \\ \alpha_i + (1-\alpha_i)G\left(\frac{1}{p_{i2}}\left(\frac{|\varepsilon_{ii}\delta_i+w_i|}{2(1-\alpha_i^*)}\right)^{p_{i2}};\frac{1}{p_{i2}}\right), & \text{if } \varepsilon_{it} > -\frac{\omega_i}{\delta_i} \end{cases} \tag{8}$$

Where

$$\omega_i = \frac{1}{B_i}\left[(1-\alpha_i)^2\frac{p_{i2}\Gamma(2/p_{i2})}{\Gamma^2(1/p_{i2})} - \alpha_i^2\frac{p_{i1}\Gamma(2/p_{i1})}{\Gamma^2(1/p_{i1})}\right] \tag{9}$$

$$\delta_i^2 = \frac{1}{B_i^2}\left[(1-\alpha_i)^3\frac{p_{i2}^2\Gamma(3/p_{i2})}{\Gamma^3(1/p_{i2})} + \alpha_i^3\frac{p_{i1}^2\Gamma(3/p_{i1})}{\Gamma^3(1/p_{i1})}\right] - \frac{1}{B_i^2}\left[(1-\alpha_i)^2\frac{p_{i2}\Gamma(2/p_{i2})}{\Gamma^2(1/p_{i2})} - \alpha_i^2\frac{p_{i1}\Gamma(2/p_{i1})}{\Gamma^2(1/p_{i1})}\right]^2 \tag{10}$$

$$K(p_{i1}) = \frac{1}{2p_{i1}^{1/p_{i1}}\Gamma(1+1/p_{i1})}, \quad K(p_{i2}) = \frac{1}{2p_{i2}^{1/p_{i2}}\Gamma(1+1/p_{i2})} \tag{11}$$

where p_1 and p_2 are the parameters which control the left tails and right tails respectively. α is the skewness parameter of SSAEPD. The SSAEPD can be reduced to many distributions. If α = 0.5 and $p_1 = p_2$, it becomes the EPD. If $p_1 = p_2$, the AEPD reduces to Skewed EPD [13, 14]. If α = 0.5, the AEPD becomes symmetric EPD [15]. If α = 0.5 and $p_1 = p_2$= 1, it reduces to the Laplace distribution. If α = 0.5 and $p_1 = p_2$= 2, it becomes the Normal distribution and then the new model will be reduced to the so-called AB-model [16].

2.3. Maximum Likelihood Estimation

Method of Maximum Likelihood Estimation (MLE) is used to estimate this new model. The reduced form residuals u_t corresponding to structure form disturbances ε_t have the form $u_t = A^{-1}B\varepsilon_t$. ε_t is assumed to be uncorrelated white noise, $\varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{3t}$~ MGC_SSAEPD ($\{\alpha_i, p_{i1}, p_{i2}\}_{i=1}^3$, $I_{3\times3}$). The log likelihood function of the new model is shown as follows.

$$\begin{aligned} L(y_{1t}, y_{2t}, y_{3t}; \theta) &= \textstyle\sum_{t=1}^T \ln f(y_{1t}, y_{2t}, y_{3t}) \\ &= \tfrac{T}{2}\ln |A|^2 - \tfrac{T}{2}\ln |B|^2 + \textstyle\sum_{t=1}^T \ln\left(f(\varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{3t})\right), \\ &= \tfrac{T}{2}\ln |A|^2 - \tfrac{T}{2}\ln |B|^2 + \textstyle\sum_{t=1}^T \ln c(\varsigma_t; I_{3\times3}) \\ &+ \textstyle\sum_{t=1}^T \ln \begin{cases} \delta_1\left(\frac{\alpha_1}{\alpha_1^*}\right)K(p_{11})\exp\left(-\frac{1}{p_{11}}\left|\frac{\omega_1+\delta_1\varepsilon_{1t}}{2\alpha_1^*}\right|^{p_{11}}\right), & \text{if } \varepsilon_{1t} \leqslant -\frac{\omega_1}{\delta_1}, \\ \delta_1\left(\frac{1-\alpha_1}{1-\alpha_1^*}\right)K(p_{12})\exp\left(-\frac{1}{p_{12}}\left|\frac{\omega_1+\delta_1\varepsilon_{1t}}{2(1-\alpha_1^*)}\right|^{p_{12}}\right), & \text{if } \varepsilon_{1t} > -\frac{\omega_1}{\delta_1}, \end{cases} \\ &+ \textstyle\sum_{t=1}^T \ln \begin{cases} \delta_2\left(\frac{\alpha_2}{\alpha_2^*}\right)K(p_{21})\exp\left(-\frac{1}{p_{21}}\left|\frac{\omega_2+\delta_2\varepsilon_{2t}}{2\alpha_2^*}\right|^{p_{21}}\right), & \text{if } \varepsilon_{2t} \leqslant -\frac{\omega_2}{\delta_2}, \\ \delta_2\left(\frac{1-\alpha_2}{1-\alpha_2^*}\right)K(p_{22})\exp\left(-\frac{1}{p_{22}}\left|\frac{\omega_2+\delta_2\varepsilon_{2t}}{2(1-\alpha_2^*)}\right|^{p_{22}}\right), & \text{if } \varepsilon_{2t} > -\frac{\omega_2}{\delta_2}, \end{cases} \\ &+ \textstyle\sum_{t=1}^T \ln \begin{cases} \delta_3\left(\frac{\alpha_3}{\alpha_3^*}\right)K(p_{31})\exp\left(-\frac{1}{p_{31}}\left|\frac{\omega_3+\delta_3\varepsilon_{3t}}{2\alpha_2^*}\right|^{p_{31}}\right), \text{if } \varepsilon_{3t} \leqslant -\frac{\omega_3}{\delta_3}, \\ \delta_3\left(\frac{1-\alpha_3}{1-\alpha_3^*}\right)K(p_{32})\exp\left(-\frac{1}{p_{32}}\left|\frac{\omega_3+\delta_3\varepsilon_{3t}}{2(1-\alpha_3^*)}\right|^{p_{32}}\right), \text{if } \varepsilon_{3t} > -\frac{\omega_3}{\delta_3}. \end{cases} \end{aligned} \tag{12}$$

$\theta = \{\{a_{i1}, a_{i2}, a_{i3}\}_{i=0}^p, \{b_{i1}, b_{i2}, b_{i3}\}_{i=0}^p, \{c_{i1}, c_{i2}, c_{i3}\}_{i=0}^p, \{\alpha_i, p_{i1}, p_{i2}\}_{i=1}^3\}$ are the parameters of the model to be estimated. All the numerical computations are run in MatLab.

2.4. Impulse Response Function (IRF)

Once the VAR system is estimated, we apply the Impulse-Response Function (IRF) to trace out the effect of exogenous shocks on realizations of the random variable across time. By Wold (1960)'s scheme, VAR (p) in equation (5) can be written as VMA (∞) evaluate the relative importance of error shocks [17].

$$Y_t = \textstyle\sum_{i=0}^{\infty} \Phi_i u_{t-i} \tag{13}$$

$$\Phi_i = \textstyle\sum_{j=1}^{i} \Phi_{i-j}\Pi_j \tag{14}$$

The s-period ahead forecast value is [17]:

$$Y_{t+s} = \textstyle\sum_{i=0}^{\infty} \Phi_i u_{t+s-i} \tag{15}$$

The *(i, j) th* elements of the matrices Φ_s, regarded as a function of *s*, trace out the expected responses of $y_{i,t+s}$ to a unit change in $y_{j,t}$, holding constant all past values of y_t. Equation (16) is called the Impulse-Response Function (IRF) [18].

$$\{\Phi_s\}_{i,j} = \frac{\partial y_{i,t+s}}{\partial u_{j,t}} \tag{16}$$

For the AB-type SVAR model, the relation of dependent variables to the reduced form residuals is given by $Au_t = B\varepsilon_t$. Therefore, the VMA (∞) form of SVAR model can be shown as

$$Y_{t+s} = \textstyle\sum_{i=0}^{\infty} \Psi_i \varepsilon_{t+s-i} \tag{17}$$

where $\Psi i = A^{-1}B\Phi i$ [17]. The impulse responses in a general SVAR model may be obtained from Equation (18):

$$\{\Psi_s\}_{i,j} = \frac{\partial y_{i,t+s}}{\partial \varepsilon_{j,t}} \tag{18}$$

2.4.1. Variance Decomposition

Variance decomposition is also a popular tool for interpreting SVAR model. By separating the variation of an endogenous variable into error shocks, the variance decomposition provides information about the relative importance of each random shocks in affecting dependent variable. Recall the s-step forecast error of Y_{t+s} from a VAR model is:

$$Y_{t+s} - E(Y_{t+s}) = \sum_{i=0}^{s-1} \Psi_i E_{t+s-i} \tag{19}$$

The variance of the Y_{t+s} is:

$$\text{Var}\,(Y_{t+s}) = \sum_{i=0}^{s-1} \text{Var}\,(\Psi_i E_{t+s-i}) = \sum_{i=0}^{s-1} \Psi_i^2 \sigma_{t+s-i}^2 \tag{20}$$

Given that the ε_t are serially uncorrelated and have unit variance, the s-step forecast error variance of variance *i* is:

$$\sigma_i^2(s) = \sum_{j=1}^{n} \sum_{k=0}^{s-1} (\Psi_k)_{i,j}^2 \tag{21}$$

Dividing the preceding terms by σ_i^2 gives the percentage contribution of variable to the s-step forecast error variance of variable *i*,

$$\omega_{j\to i}(s) = \frac{\sum_{k=0}^{s-1} (\Psi_k)_{i,j}^2}{\sum_{j=1}^{n} \sum_{k=0}^{s-1} (\Psi_k)_{i,j}^2} \tag{22}$$

3. Simulation Results

In this section, we run the simulation for this new model to check the validity of the MatLab program. Simulation results show our program is valid and can be used for empirical analysis.

3.1. Simulation for Model Estimation

The results from both MatLab and EViews (Version 7.2) are compared with the true values. The simulation process is as follows.

1. Select a set of true parameter values for
$\theta = \{\{a_{i1}, a_{i2}, a_{i3}\}_{i=0}^{4}, \{b_{i1}, b_{i2}, b_{i3}\}_{i=0}^{4}, \{c_{i1}, c_{i2}, c_{i3}\}_{i=0}^{4}, \{\alpha_i, p_{i1}, p_{i2}\}_{i=1}^{3}\}$

2. Generate error terms ε_t from the joint density of

$$(\varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{3t}) \sim MGC\_SSAEPD(\{\alpha_i, p_{i1}, p_{i2}\}_{i=1}^{3}\}, R)$$

with the restriction that the error terms u_t are uncorrelated (i.e. the covariance matrix R is $I_{3\times 3}$). SSAEPD $(\alpha_i, p_{i1}, p_{i2})$ is used as the proposal density for ε_{it}. Another method is used to get the temporate draws of SSAEPD (α, p_1, p_2) [11]. MCMC is used to draw the tri-variate errors from $(\varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{3t}) \sim MGC\_SSAEPD(\{\alpha_i, p_{i1}, p_{i2}\}_{i=1}^{3}\}, R)$.

3. Generate data Y_t by equation (23) with the initial values $Y_i = [0.1, \quad 0.1, \quad 0.1]^T, (i = -3, -2, -1, 0)$.

$$AY_t = \Gamma_1 Y_{t-1} + \Gamma_2 Y_{t-2} + \Gamma_3 Y_{t-3} + \Gamma_4 Y_{t-4} + B\varepsilon_t, (t = 1, 2, \dots T) \tag{23}$$

The simulation results are listed in Table 3. We find out that all estimates from MatLab and Eviews are close to the true values. Hence, we may conclude that our program in MatLab is valid and can be used to analyze empirical data.

Table 3. Simulation results.

Par.	T	M	E	T	M	E	T	M	E	T	M	E
α1	0.5	0.4860	-	0.5	0.4956	-	0.5	0.5105	-	0.5	0.5437	-
p11	2	1.9291	-	2	1.9380	-	1.5	1.5584	-	1.5	1.5652	-
p12	2	2.0351	-	2	1.9203	-	2	1.9373	-	2	1.8305	-
a01	0.2	-	-	0.2	-	-	0.2	-	-	0.2	-	-
a02	0.1	0.0793	0.0441	0.1	0.0910	0.1013	0.1	0.0876	0.0783	0.1	0.1142	0.0982
a03	1	-	-	0.8	-	-	1	-	-	0.7	-	-
a11	-0.2	-0.1949	-0.2555	-0.2	-0.1672	-0.3567	-0.2	-0.1967	-0.1892	-0.2	-0.1652	-0.1872
a12	0.2	0.2300	0.2750	0.2	0.1950	0.2587	0.2	0.2000	0.1732	0.2	0.2210	0.1979
a13	-0.3	-0.2971	-0.3478	-0.3	-0.3005	-0.3537	-0.3	-0.3139	-0.2793	-0.3	-0.3003	-0.2987
a21	0.2	0.2181	0.2673	0.2	0.2272	0.3023	0.2	0.2089	0.1394	0.2	0.1931	0.1872
a22	-0.2	-0.1990	-0.2541	-0.2	-0.2063	-0.2871	-0.2	-0.1999	-0.1674	-0.2	-0.1908	-0.1689
a23	0.1	0.0873	0.1293	0.1	0.1195	0.1589	0.1	0.0928	0.0784	0.1	0.1000	0.0987
a31	0.1	0.1184	0.1207	0.1	0.0849	0.1490	0.1	0.0713	0.0821	0.1	0.0873	0.0682

a32	-0.2	-0.2044	-0.1931	-0.2	-0.1917	-0.1961	-0.2	-0.2279	-0.1782	-0.2	-0.2087	-0.1982
a33	0.2	0.1989	0.1476	0.2	0.1955	0.1676	0.2	0.1952	0.1872	0.2	0.1868	0.1680
a41	-0.1	-0.1072	-0.1201	-0.1	-0.1055	-0.0627	-0.1	-0.1037	-0.0982	-0.1	-0.0961	-0.0892
a42	0.2	0.2096	0.2091	0.2	0.2248	0.2173	0.2	0.1779	0.1572	0.2	0.1974	0.1808
a43	-0.2	-0.2210	-0.1918	-0.2	-0.2116	-0.1853	-0.2	-0.2310	-0.1782	-0.2	-0.2019	-0.1972
α2	0.5	0.4907	-	0.5	0.5374	-	0.6	0.5268	-	0.6	0.5540	-
p21	2	1.9873	-	2	2.2210	-	2	1.7920	-	2	1.9074	-
p22	2	2.0491	-	2	1.8782	-	1.5	1.7911	-	1.5	1.6124	-
b01	0.2	0.2084	0.2154	0.2	0.2070	0.2781	0.2	0.1908	0.2012	0.2	0.1956	0.1782
b02	0.1	0.1023	0.0766	0.1	0.1014	0.1331	0.1	0.1260	0.0982	0.1	0.0994	0.0987
b03	1	1.0034	1.0036	0.8	0.7844	0.7922	1	1.0006	0.9989	0.8	0.8019	0.7972
b11	-0.2	0.2307	0.2165	0.2	0.2128	0.2160	0.2	0.2441	0.1982	0.2	0.1911	0.1672
b12	0.1	-0.3020	-0.3351	-0.3	-0.2578	-0.3699	-0.3	-0.2818	-0.2541	-0.3	-0.3246	-0.2981
b13	-0.1	0.2298	0.2643	0.2	0.2015	0.2752	0.2	0.1993	0.1362	0.2	0.2041	0.2492
b21	-0.2	-0.1788	-0.2202	-0.2	-0.1941	-0.2280	-0.2	-0.2348	-0.1872	-0.2	-0.2062	-0.2301
b22	0.1	0.0873	0.1341	0.1	0.1044	0.1059	0.1	0.1242	0.0792	0.1	0.0945	0.0876
b23	-0.1	-0.0798	-0.0853	-0.1	-0.1097	-0.1000	-0.1	-0.1086	-0.0982	-0.1	-0.0900	-0.0697
b31	-0.1	-0.1093	-0.1101	-0.1	-0.0896	-0.1188	-0.1	-0.0990	-0.1320	-0.1	-0.1145	-0.0980
b32	-0.1	-0.1360	-0.0986	-0.1	-0.1098	-0.0687	-0.1	-0.0783	-0.0324	-0.1	-0.1179	-0.1392
b33	0.2	0.1821	0.1672	0.2	0.2095	0.1556	0.2	0.1787	0.1592	0.2	0.2032	0.1987
b41	-0.1	-0.0698	-0.0948	-0.1	-0.3205	-0.3147	-0.1	-0.3003	-0.3421	-0.1	-0.3025	-0.3092
b42	0.1	0.0873	0.0809	0.1	0.1021	0.0778	0.1	0.1086	0.0932	0.1	0.0850	0.0897
b43	-0.2	-0.183	0.1672	-0.2	-0.2094	-0.1769	-0.2	-0.2042	-0.1872	-0.2	-0.1942	-0.1797
α3	0.5	0.4911	-	0.5	0.4826	-	0.5	0.4785	-	0.5	0.4801	-
p31	2	2.0484	-	2	2.0247	-	2.5	2.5050	-	2.5	2.2595	-
p32	2	1.9928	-	2	2.0644	-	2	2.0219	-	2	2.0044	-
c01	0.1	0.1237	0.1538	0.1	0.1348	0.1504	0.1	0.1178	0.0672	0.2	0.1415	0.1982
c02	-0.2	-	-	-0.2	-	-	-0.2	-	-	-0.2	-	-
c03	1	0.9889	0.9921	1.5	1.5013	1.5044	1	1.0103	1.0023	1.5	1.5154	1.4982
c11	0.2	0.2111	0.2660	0.2	0.1732	0.2487	0.2	0.1968	0.1409	0.2	0.1703	0.1672
c12	0.1	0.1347	0.0966	0.1	0.1303	-0.0189	0.1	0.1048	0.1092	0.1	0.0873	0.0786
c13	-0.2	-0.2171	-0.0987	-0.2	0.2164	-0.2434	-0.2	-0.2077	-0.1778	-0.2	-0.1979	-0.1798
c21	-0.2	-0.1930	-0.2723	-0.2	-0.1619	-0.3334	-0.2	-0.1950	-0.1872	-0.2	-0.2633	0.2381

c22	0.1	0.0897	0.1484	0.1	0.1178	0.1878	0.1	0.0959	0.0787	0.3	0.3036	0.2987
c23	-0.2	-0.1905	-0.2172	-0.2	-0.2108	-0.2781	-0.2	-0.1730	-0.1492	-0.2	-0.2277	-0.1978
c31	-0.1	-0.0838	-0.1311	-0.1	-0.0792	-0.1503	-0.1	-0.1147	-0.0897	-0.1	-0.1209	-0.0978
c32	0.2	0.1821	0.1808	0.2	0.1785	0.1648	0.2	0.1631	0.1583	0.2	0.2026	0.1682
c33	0.1	0.1412	0.1394	0.1	0.1078	0.1265	-0.1	-0.1037	-0.0872	-0.1	-0.0747	-0.0971
c41	0.1	0.0844	0.0964	0.1	-0.1430	-0.1036	0.1	0.1173	0.1492	0.1	0.1053	0.0989
c42	-0.2	-0.1766	-0.2162	-0.2	-0.1948	-0.1963	-0.2	-0.2122	-0.1872	-0.2	-0.2261	-0.2871
c43	0.1	0.0913	0.1014	0.1	0.1179	0.1226	0.1	0.1243	0.0938	0.1	0.1285	0.1682

Notes: Par. = Parameter; T = True values; M = Estimates by MatLab; E = Estimates by EViews.

3.2. Simulation for Structure Analysis

To check the validity of Impulse Response Function (IRF) and Variance Decomposition (VD), the results from MatLab are compared with those from EViews (Version 7.2). The results of Table 4 and 5 show the estimates from our MatLab program are very close to those from EViews. That means our MatLab program can be used for empirical data. The simulation process for the Impulse Response Function (IRF) is as follows:

1. Set true parameter values $\theta = \{\{a_{i1}, a_{i2}, a_{i3}\}_{i=0}^{4}, \{b_{i1}, b_{i2}, b_{i3}\}_{i=0}^{4}, \{c_{i1}, c_{i2}, c_{i3}\}_{i=0}^{4}, \{\alpha_i, p_{i1}, p_{i2}\}_{i=1}^{3}\}$.

2. Generate a group of simulation data.

3. Calculate values for Impulse Response Functions (IRFs) and Variance Decomposition (VD) by MatLab and EViews.

Table 4. Simulation of Impulse Response Function (IRF).

Lag	**Response of y1**					
	Ψ11		**Ψ12**		**Ψ13**	
	E	M	E	M	E	M
1	0.6491	0.6491	-0.2095	-0.2095	-0.3673	-0.3661
2	-0.2856	-0.2855	0.2416	0.2416	-0.4362	-0.4350
3	0.3464	0.3463	-0.4103	-0.4102	0.3816	0.3805
4	0.0036	0.0037	-0.0005	-0.0005	-0.0952	-0.0949
5	-0.0835	-0.0842	0.1359	0.1362	-0.3445	-0.3431
6	0.0079	0.0084	0.0036	0.0033	0.1058	0.1057
7	0.1670	0.1663	-0.1560	-0.1554	0.0845	0.0840
8	-0.1012	-0.1010	0.0952	0.0950	-0.1963	-0.1953
9	-0.0016	-0.0016	0.0102	0.0102	0.0440	0.0440
10	0.0529	0.0529	-0.0599	-0.0599	0.0678	0.0675
Lag	**Response of y2**					
	Ψ21		**Ψ22**		**Ψ23**	
	E	M	E	M	E	M

1	-0.0767	-0.0767	0.8202	0.8202	-0.0039	-0.0040
2	0.2457	0.2457	-0.3298	-0.3298	0.3340	0.3331
3	-0.2952	-0.2951	0.2826	0.2824	-0.3273	-0.3263
4	0.1228	0.1227	-0.1756	-0.1754	0.5346	0.5329
5	-0.2817	-0.2815	0.2058	0.2057	-0.2588	-0.2579
6	0.0898	0.0896	-0.0959	-0.0957	0.1512	0.1508
7	-0.1524	-0.1521	0.1851	0.1849	-0.0664	-0.0662
8	0.1406	0.1403	-0.1960	-0.1958	0.1667	0.1659
9	-0.1529	-0.1525	0.1734	0.1730	-0.1421	-0.1417
10	0.1010	0.1007	-0.1168	-0.1166	0.1557	0.1551

Lag	**Response of y3**					
	Ψ31		**Ψ32**		**Ψ33**	
	E	M	E	M	E	M
1	0.2802	0.2802	0.0686	0.0686	1.4969	1.4925
2	0.1507	0.1507	-0.0706	-0.0706	-0.1563	-0.1557
3	-0.4063	-0.4063	0.4383	0.4383	-0.4034	-0.4023
4	0.1832	0.1831	-0.1399	-0.1398	0.6811	0.6790
5	0.0567	0.0567	-0.2081	-0.2081	0.0668	0.0669
6	-0.2392	-0.2394	0.2714	0.2715	-0.3303	-0.3292
7	0.0066	0.0069	-0.0081	-0.0082	0.2836	0.2826
8	0.1024	0.1021	-0.1338	-0.1336	0.0644	0.0641
9	-0.0782	-0.0780	0.0957	0.0954	-0.1479	-0.1472
10	-0.0394	-0.0393	0.0393	0.0393	0.0630	0.0627

Notes: E=EViews (Version 7.2); M=MatLab; Ψij = a response of yi to a shock of εj. True parameters are:

a01 = 0.2; a02 = 0; a03 = 0.7
b01 = 0.2; b02 = 0.1; b03 = 0.8
c01 = 0.2; c01 = −0.2; c03 = 1.5
a11 = −0.2; a12 = 0.2; a13 = −0.3
b11 = 0.2; b12 = −0.3; b13 = 0.2
c11 = 0.2; c12 = 0.1; c13 = −0.2.
a21 = 0.2; a22 = −0.2; a23 = 0.1.
b21 = −0.2; b22 = 0.1; b23 = −0.1.
c21 = −0.2; c22 = 0.3; c23 = −0.2.
a31 = 0.1; a32 = −0.2; a33 = 0.2.
b31 = −0.1; b32 = −0.1; b33 = 0.2.
c31 = −0.1; c32 = 0.2; c33 = 0.1.
a41 = −0.1;a42 = 0.2; a43 = −0.2.
b41 = −0.3; b42 = 0.1; b43 = −0.2.
c41 = 0.1; c42 = −0.2; c43 = 0.1.

Since EViews (Version 7.2) can only generate IRF of VAR with Normal error terms, we set α1 = 0.5; p11 = p12 = 2

Table 5. Simulation of Variance Decomposition (VD).

Lag	Variance Decomposition of y1					
	Ψ11		Ψ12		Ψ13	
	E	M	E	M	E	M
1	0.7021	0.7031	0.0073	0.0733	0.2247	0.2236
2	0.5406	0.5417	0.1099	0.1102	0.3495	0.3482
3	0.4565	0.4575	0.1984	0.1987	0.3451	0.3438
4	0.4535	0.4545	0.1970	0.1974	0.3494	0.3481
5	0.4151	0.4161	0.1905	0.1910	0.3944	0.3929
6	0.4120	0.4131	0.1891	0.1896	0.3988	0.3973
7	0.4142	0.4152	0.1973	0.1978	0.3884	0.3870
8	0.4059	0.4069	0.1959	0.1964	0.3982	0.3967
9	0.4054	0.4064	0.1958	0.1962	0.3989	0.3974
10	0.4044	0.4054	0.1966	0.1971	0.3990	0.3975

Lag	Variance Decomposition of y2					
	Ψ21		Ψ22		Ψ23	
	E	M	E	M	E	M
1	0.0087	0.0087	0.9913	0.9913	0.0022	0.0000
2	0.0691	0.0691	0.8146	0.8151	0.1163	0.1158
3	0.1244	0.1245	0.6983	0.6990	0.1773	0.1765
4	0.1077	0.1078	0.5700	0.5711	0.3223	0.3211
5	0.1413	0.1415	0.5329	0.5339	0.3258	0.3246
6	0.1427	0.1428	0.5261	0.5271	0.3313	0.3300
7	0.1504	0.1506	0.5270	0.5280	0.3226	0.3214
8	0.1539	0.1541	0.5234	0.5245	0.3226	0.3214
9	0.1599	0.1600	0.5192	0.5203	0.3209	0.3197
10	0.1612	0.1612	0.5137	0.5148	0.3251	0.3239

Lag	Variance Decomposition of y3					
	Ψ31		Ψ32		Ψ33	
	E	M	E	M	E	M
1	0.0340	0.0340	0.0020	0.0020	0.9642	0.9640

2	0.0426	0.0428	0.0041	0.0041	0.9533	0.9531
3	0.0919	0.0924	0.0697	0.0700	0.8384	0.8376
4	0.0879	0.0883	0.0649	0.0652	0.8473	0.8465
5	0.0875	0.0879	0.0764	0.0768	0.8361	0.8363
6	0.0973	0.0978	0.0913	0.0918	0.8114	0.8105
7	0.0952	0.0957	0.0894	0.0899	0.8154	0.8145
8	0.0971	0.0976	0.0933	0.0938	0.8095	0.8086
9	0.0978	0.0983	0.0948	0.0953	0.8074	0.8065
10	0.0980	0.0985	0.0950	0.0955	0.8069	0.8060

4. Empirical Analysis

4.1. Data

Following the work from Breitung et. al., we analyze the traditional Keynesian theory, i.e., IS-LM theory (or IS-LM curves) using quarterly U.S. data from 1959 (i) to 2023 (ii) [19]. The output q_t is measured by real GDP growth rate, m_t is the real monetary base, and i_t is the three-month interbank interest rate. The vector of dependent variables is $Y_t = [q_t, i_t, m_t]$. All data are downloaded from the database of Federal Reserve Economic Data (FRED) maintained at the Federal Reserve Bank of St. Louis.

The descriptive statistics are presented in Table 6. Both interest rate (i_t) and monetary base (m_t) are slightly skewed to the right while GDP (q_t) is slightly skewed to the left. All three-time series are a little leptokurtic. The P-values of JB normality test is 0, which means under 5% significant level, all the data are not distributed as Normal. Hence, non-Normal error distribution may be better to describe the data.

We perform unit root test using Augmented Dicky-Fuller (ADF) test and Phillips-Perron (PP) test for all variables (see Table 7). A unit root is rejected at 5% significance level for all variables, with and without trends. We also perform a Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test for each variable. Results indicate that trend stationarity cannot be rejected for all variables.

Table 6. Descriptive Statistics.

	Mean	**St. Dev.**	**Skewness**	**Kurtosis**	**P-value (JB)**
q_t	0.01	0.01	-0.29	4.41	0
i_t	0.05	0.03	0.70	4.01	0
m_t	0.02	0.01	0.58	4.55	0

Notes: q_t = GDP Growth Rate; i_t = Interest rate; m_t = Monetary Base Growth Rate; St. Dev. = Standard Deviation. H_0 of the JB test: Data is distributed as Normal. CV (5%) = Critical Value under 5% Significance Level.

Table 7. Unit Root Test.

Test	**Determ. Terms**	**GDP (q_t)**		**Interest Rate (i_t)**		**Monetary Base (m_t)**	
		Test Statistic	P-value	Test Statistic	P-value	Test Statistic	P-value
ADF	Intercept	-7.18	0	-6.47	0	-6.89	0
	Trend and intercept	-10.88	0	-6.60	0	-7.11	0
	None	-4.52	0	-6.48	0	-4.13	0
PP	Intercept	-11.04	0	-11.50	0	-6.89	0
	Trend and intercept	-11.19	0	-11.51	0	-7.11	0

	None	-7.98	0	-11.52	0	-2.43	0.02
KPSS	Intercept	-	0.40	-	0.22	-	0.48
	Trend and intercept	-	0.05	-	0.12	-	0.15

Notes: H_0 of the ADF test: Data has a unit root. Or data is not stationary. H_0 of the PP test: Data has a unit root. H_0 of the KPSS test: Data is stationary.

4.2. Model Specification: Lag Length Selection

The lag length for a SVAR (p) model may be determined using model selection criteria. The general approach is to fit a SVAR (p) model with orders $p = 1, 2, ..., p_{max}$ and choose the value of p which minimizes some model selection criteria. The three most common criteria are Akaike Information Criterion (AIC), Schwarz-Bayesian Information Criterion (BIC) and Hannam-Quinn Information Criterion (HQ) [20]. Table 8 shows that all three criteria are minimized when the lag length is determinded as 4. That means, the SVAR (4) model can fit our data better.

Table 8. Results of Model Selection Criteria.

Model	**AIC**	**BIC**	**HQ**	**Rank**
SVAR (1)-MGC_SSAEPD	-21.46(2)	-21.04(1)	-21.50(2)	2.67
SVAR (2)-MGC_SSAEPD	-19.95(4)	-19.39(4)	-20.00(4)	4
SVAR (3)-MGC_SSAEPD	-20.59(3)	-19.89(3)	-20.65(3)	3
SVAR (4)-MGC_SSAEPD	-21.59(1)	-20.75(2)	-21.67(1)	1.33

4.3. Model Estimation

In this section, we estimate our new model for empirical data. To identify the structural form parameters, some restrictions must be placed on the parameter matrices A and B. Based on related studies [21], we use a small macro system from Keynesian arguments to specify the following relations between the reduced form residuals and the structural innovations:

$$u_t^q = -a_{12}u_t^i + b_{01}\varepsilon_t^{IS} \qquad \text{(IS curve)}$$

$$u_t^i = -a_{21}u_t^q - a_{23}u_t^m + b_{02}\varepsilon_t^{LM} \quad \text{(inverse LM curve)}$$

$$u_t^m = b_{03}\varepsilon_t^m \qquad \text{(money supply rule)}$$

4.3.1. Empirical Results

The estimates results are reported in Table 9. For the output (qt), the fat-tail phenomenon is strong since both tail parameters (pˆ11 = 1.4912, pˆ12 = 1.5230) are much lower than 2. However, the skewness is not obvious since the estimate of the skewness parameter α1 is around 0.5. Same is true for both interest rate (it) (αˆ2 = 0.5082, pˆ21 = 0.9866, pˆ22 = 0.8218) and monetary base (mt) (αˆ3 = 0.5448, pˆ31 = 1.4211, pˆ32 = 1.0774). Compared to Normal, SSAEPD can capture the skewness and asymmetric kurtosis of the sample data.

For comparison, we also estimate SVAR (4)-MGC Normal model by setting pi1 = pi2 = 2 and αi = 0.5 (i = 1, 2, 3), which is the traditional SVAR model advocated by some studies from Sims [3] and Bernanke [1]. We can find out that the estimates in SVAR are quite different from those in new model, especially for those in structural model. This discovery implies that the specification of margins is an important determinant for SVAR estimation.

Table 9. Empirical Estimates.

SVAR (4)-MGC_SSAEPD						**SVAR(4)-MGC_Normal**					
q_t		i_t		m_t		q_t		i_t		m_t	
α_1	0.5243	α_2	0.5082	α_3	0.5448	α_1		α_2		α_3	
$p11$	1.4912	$p21$	0.9866	$p31$	1.4211	$p11$		$p21$		$p31$	
$p12$	1.5230	$p22$	0.8218	$p32$	1.0774	$p12$		$p22$		$p32$	
$a01$	-0.0302	$b01$	-0.0937	$c01$		$a01$	0.2653	$b01$	-0.3709	$c01$	
$a02$	-	$b02$	0.2357	$c02$		$a02$		$b02$	0.2490	$c02$	
$a03$	0.0075	$b03$	0.0058	$c03$	0.0060	$a03$	0.0081	$b03$	0.0068	$c03$	0.0063
$a11$	0.2320	$b11$	0.0716	$c11$	-0.0054	$a11$	0.2842	$b11$	-0.0254	$c11$	-0.0193
$a12$	0.0329	$b12$	1.1917	$c12$	-0.1358	$a12$	0.3391	$b12$	0.9972	$c12$	-0.1442
$a13$	0.1654	$b13$	0.1680	$c13$	0.8157	$a13$	0.1496	$b13$	0.0907	$c13$	0.7633
$a21$	0.2455	$b21$	0.0305	$c21$	0.0449	$a21$	0.2967	$b21$	0.1096	$c21$	0.0306
$a22$	-0.1839	$b22$	-0.1320	$c22$	0.2613	$a22$	-0.2702	$b22$	-0.1341	$c22$	0.2438
$a23$	0.1569	$b23$	0.0301	$c23$	-0.1328	$a23$	0.2692	$b23$	0.0345	$c23$	-0.1094

$a31$	-0.0609	$b31$	0.0685	$c31$	0.0668	$a31$	-0.0578	$b31$	-0.0115	$c31$	0.0617
$a32$	0.1749	$b32$	-0.0568	$c32$	-0.0604	$a32$	0.2890	$b32$	0.2022	$c32$	-0.0642
$a33$	-0.0716	$b33$	-0.0184	$c33$	0.1428	$a33$	-0.1512	$b33$	-0.0252	$c33$	0.1675
$a41$	-0.0127	$b41$	0.0186	$c41$	-0.0765	$a41$	0.0304	$b41$	0.0442	$c41$	-0.0959
$a42$	-0.0583	$b42$	-0.0313	$c42$	-0.0222	$a42$	-0.1239	$b42$	-0.0941	$c42$	0.0159
$a43$	0.0077	$b43$	0.0265	$c43$	0.0007	$a43$	0.0121	$b43$	0.0231	$c43$	-0.0002

Notes: q_t= GDP Growth Rate; i_t= Interest rate; m_t= Real Monetary Base.

4.3.2. Model Diagnostics

In this subsection, we first apply Kolmogorov-Smirnov (KS) test to check the residuals in both models. For the new model, all Kolmogorov-Smirnov (KS) test accept the null hypothesis under 5% significance level while for the traditional SVAR model, two null hypotheses are rejected (see Table 10). As expected, SSAEPD can give better fitness compared to Normal.

Second, we apply Likelihood Ratio test (LR) to run Normality tests. The rejection of the null hypothesis in column T22 means that the joint distribution of errors does not follow multi-variate Normal under 5% significance level. For output (qt), the normality assumption is accepted, which is same as the result of KS test. The null for skewness parameter is not rejected in monetary base (mt) while for interest rate (it), both nulls for tail parameters are accepted. This result confirms that the interest rate has fat tailness and the monetary base has skewness. Hence, we conclude that there exists non-Normality in the sample data.

Last, from the graphes by method of "eye-rolling", we may also conclude the residuals of new model fit data better. That is, we compare the probability density functions (PDFs) of the estimated residuals ε^it (i = 1, 2, 3) with those of SSAEPD (α^i, p^i1, p^i2) (i = 1, 2, 3). We find out these curves are very close to each other (see Figure 1). Then, we compare the PDFs of the residuals with that of Normal (0, 1). We find out that there are big differences between these curves. Hence, graph results show the residuals are more likely distributed as SSAEPD.

Table 10. Test for the Normality of Residuals.

q_t	T1	T4	T7	T10	T13	T16	T19	-
	0.4899	0.4066	-5	-1.2	6*	7.6*	4	-
i_t	T2	T5	T8	T11	T14	T17	T20	-
	0.3488	0.0026	-2.6	44.4*	44.4*	55.8*	56.8*	-
m_t	T3	T6	T9	T12	T15	T18	T21	T22
	0.3086	0.0272	35.2*	-3	-12	25.4*	35.2*	155.6*
$\chi^2_{0.05}$	-	-	3.84	3.84	3.84	5.99	7.82	12.59

Note: The null hypothesis of KS test for Tl-T3 is H0: Residuals follow SSAEPD (α^, p^1, p^2)
The null hypothesis of KS test for T4-T6 is H0: Residuals follow Standard Normal T7 means H0:
α1 = 0.5. T8 means H0: α2 = 0.5. T9 means H0: α3 = 0.5. Tl0 means H0: p11 = 2. Tll
means H0: p12 = 2. Tl2 means H0: p21 = 2. Tl3 means H0: p22 = 2. Tl4 means H0: p21 = 2. Tl5
means H0: p22 = 2. Tl6 means H0: p11 = p12 = 2 Tl7 means H0: p21 = p22 = 2Tl8
means H0: p31 = p32 = 2
Tl9 means H0: α1 = 0.5, p11 = p12 = 2. T2O means H0: α2 = 0.5, p21 = p22 = 2. T2l means H0: α3 = 0.5, p31 = p32 = 2.
T22 means H0: α1 = α2 = α3 = 0.5, p11 = p12 = p21 = p22 = p31 = p32 = 2

* Means the parameter restriction is statistically significant. CV (5%) means the Critical Value under 5% Significance Level.

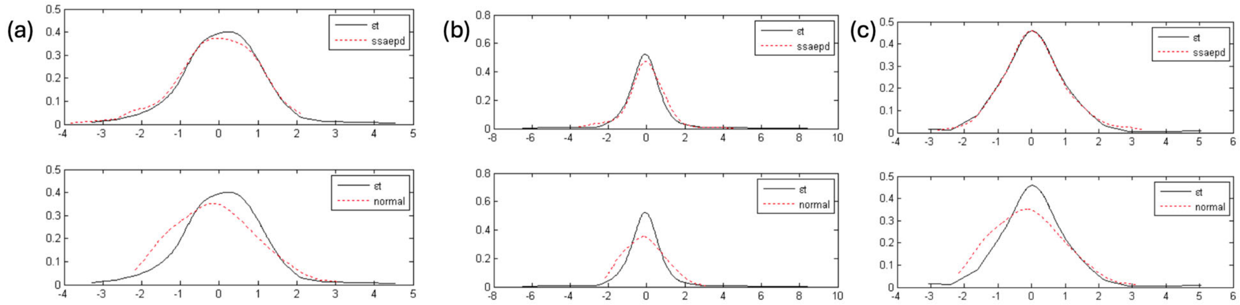

Figure 1. Comparisions of PDFs for Residual εˆt. (a) PDFs of εˆ1t & SSAEPD (αˆ1,pˆ11,pˆ12) (up) and PDFs of εˆ1t & Normal (1,0) (down); (b) PDFs of εˆ2t & SSAEPD (αˆ2,pˆ21,pˆ22) (up) and PDFs of εˆ2t & Normal (1,0) (down); (c) PDFs of εˆ1t & SSAEPD (αˆ3,pˆ31,pˆ32) (up) and PDFs of εˆ3t & Normal (1,0) (down).

4.3.3. Structural Analysis

Impulse-Response Function: We use the Impulse-Response Function (IRF) to evaluate the relative importance of error shocks. For comparison, the IRFs of the traditional SVAR model are also calculated. The graphical results are plotted in Figure 2.

According to the new model, the spending shock (ε^{IS}) increase output immediately, but the increase dies out within three years (see Figure (2a)). In response to a positive money demand shock (ε^{LM}), both output and interest rate experience a positive effect. However, the shock decreases the output after three years while the interest rate still increases during all periods (see Figure (2b) and (2e)). The relationship between output and interest rate is strong and negative at longer horizons, which conforms to the Keynesian theory. Different from the results from Breitung, the positive response of output follows a positive money supply shock (ε^{m}) in this system (see Figure (c)) [22]. The positive correlation between output and money supply proves the view of IS-LM theory.

A positive shift in the IS curve (ε^{IS}) increases interest rates with a raising response whereas real money decreases gradually after two quarters (see Figure (2d) and (2g)). Similarly, a LM shock (ε^{LM}) leads to an increase in interest rates and a decrease in real money base (see Figure(2e) and (2h)). These effects are predicted by the IS-LM theory.

Finally, a positive money supply shock (ε^{m}) leads to an immediate raise in both interest rate and real money (see Figure (2f) and (2i)). This implies the "liquidity trap" which is also an important consequence of the standard IS-LM model.

For the traditional SVAR model, the IRF plots are different from those in the new model. The IRFs of the new model are more consistent with the IS-LM theory than those of the SVAR model. As expected from Keynesian theory, real money (mt) reacts to a money demand shock (ε^{LM}) negatively at longer horizons in the new model while the responses are positive in SVAR model. Moreover, the new model can capture the positive relation between output and interest rate, while the SVAR model cannot.

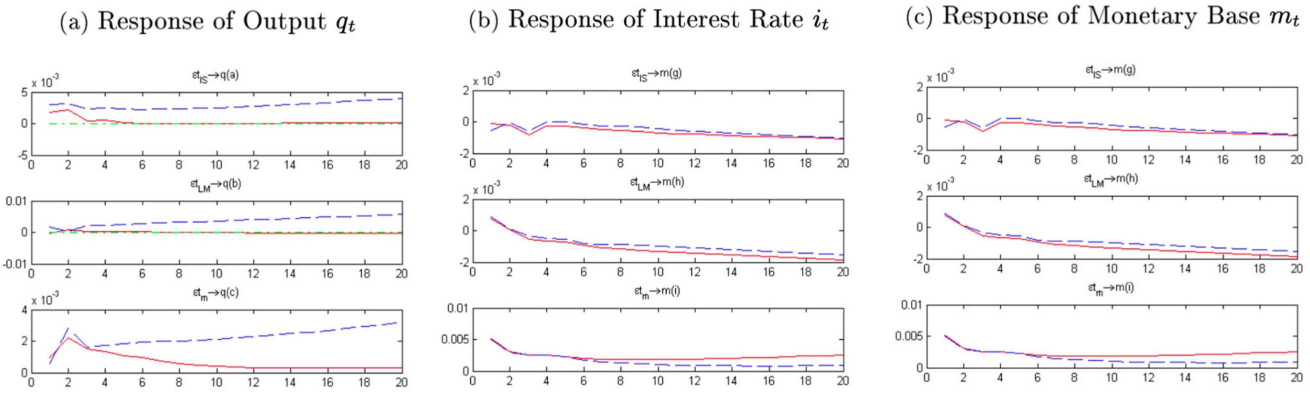
(a) Response of Output q_t (b) Response of Interest Rate i_t (c) Response of Monetary Base m_t

Figure 2. Impulse-Response Functions. Responses of GDP, Interest Rate and Monetary Base are shown from left to right. Solid line is from the SVAR-MGC_SSAEPD model. Dash line is from the SVAR-MGC_Normal model.

4.3.4. Variance Decomposition (VD)

To assess the importance of the three different shocks for the system variables, the forecast error variances of the variables are decomposed with respect to the shocks. The results for different forecast horizons are presented in Table 11. It turns out that the money demand shock contributes a small fraction to the forecast error variance of output. This result proves that the innovation in money does not seem to be an accurate indicator for monetary policy. IS shocks clearly dominate the behavior of the output series but with respect to a longer forecast horizon, IS shocks become less important. Finally, money supply shocks play a minor role in the short run. However, with an increasing forecast horizon, the money supply shocks become more and more important. The relative contribution of the shocks to the forecast error variance of interest rates and real money can be interpreted in a similar manner.

Compared to the new model, the traditional SVAR model shows that the money demand shock contributes a higher fraction to the forecast error variance of output while the money supply becomes less important to interest rate. For the new model, the percentage of the effect from own shocks decreases over time, as the lagged variable's effect starts kicking in. However, for the SVAR model, the rising effect of other shocks can be observed only for the variance of real money. This discovery implies that the use of SSAEPD can improve the performance of variance decomposition.

Table 11. Simulation of Variance Decomposition (VD).

Lag	Variance Decomposition of y1					
	Ψ11		**Ψ12**		**Ψ13**	
	E	M	E	M	E	M
1	99.94	95.03	0	4.71	0	0.25
4	87.65	78.54	1.28	10.47	11.07	10.99
8	82.48	60.89	1.49	23.85	16.03	15.26
12	81.62	41.28	1.51	34.57	16.87	16.67
16	81.16	45.78	1.66	41.66	17.18	17.05
20	80.63	36.59	1.89	46.30	17.48	17.12

Lag	Variance Decomposition of y2					
	Ψ21		**Ψ22**		**Ψ23**	
	E	M	E	M	E	M
1	1.37	15.64	93.10	80.10	5.53	4.26
4	6.44	19.10	90.93	79.25	2.63	1.65
8	13.10	26.17	68.13	70.48	18.77	3.35
12	13.44	28.20	52.06	64.57	34.50	7.23
16	12.65	28.37	43.94	59.20	43.41	10.46
20	12.05	28.12	39.63	61.18	48.32	12.67

Lag	Variance Decomposition of y3					
	Ψ31		**Ψ32**		**Ψ33**	
	E	M	E	M	E	M
1	0	0	0	0	1	1
4	0.94	0.84	1.16	1.06	97.90	98.10
8	1.25	0.77	3.87	2.77	94.89	96.45
12	2.41	1.33	8.40	6.94	89.19	92.67
16	3.90	2.78	13.70	10.02	83.38	87.20

20	5.25	5.00	16.39	14.71	78.36	80.49

4.3.5. IS-LM Theory Still Alive

According to the estimates, the structural model of this new model (i.e. SVAR (4)-MGC SSAEPD) is:

$$\begin{aligned} u_{1t}^{q} &= 0.0075\varepsilon_{1t}^{IS} + 0.0302u_{2t}^{i} \text{ (IS curve)} \\ u_{2t}^{i} &= 0.0058\varepsilon_{2t}^{LM} + 0.0937u_{1t}^{q} - 0.2357u_{3t}^{m} \text{ (inverse LM curve)} \\ u_{3t}^{m} &= 0.0060\varepsilon_{3t}^{MS} \text{ (money supply curve)} \end{aligned} \tag{24}$$

The first equation represents a traditional IS curve with a negative parameter for the interest rate innovation u_{2t}^{i}. However, it turns out that the estimated coefficient a01 = −0.0302 is statistically insignificant. According to our results, an IS shock (ε_{1t}^{IS}) increases output, which is the same as that predicted by IS-LM theory. The second equation reflects a Keynesian LM curve with a relationship between money demand innovation u_{2t}^{i} and interest rate innovation u_{3t}^{m}. The parameters of the inverse LM curve have expected sign, and all parameters are significant at conventional level. An LM shock (ε_{2t}^{LM}) leads to an increase in the output and a decrease in the monetary base. The third equation postulates money supply rule. The parameter of this equation is positive and significant, which means innovations of the monetary base u_{3t}^{m} are driven by exogenous money supply shocks ε_{3t}^{MS}. Hence, we conclude the classic IS-LM theory is still alive in the U.S.

4.4. Model Comparisons

We apply AIC (Akaike Information Criterion), BIC (Bayesian Information Criterion) and HQ (Hannan-Quinn) to compare the new model with other 9 models. Similar to other previous studies, we rank the models according to these criterions and select the best model based on the mean ranks [23]. The results are listed in Table 12. For our sample, the fat-tailness is a strong phenomenon while the skewness is not an obvious character. Hence, a SVAR model with a SEPD margin is the best compromise for precise in-sample fit. Although our new model does not have the best in-sample fit, it still performs better than the classic SVAR model (i.e., M2).

Table 12. Model Comparisons.

Models	M1	M2	M3	M4	M5
AIC	-21.6524(4)	-21.0213(10)	-21.7483(2)	-21.7732(1)	-21.6871(3)
BIC	-20.8140(6)	-20.3226(10)	-20.9565(2)	-21.0279(1)	-20.8953(4)
HQ	-21.7308(4)	-21.0866(10)	-21.8223(2)	-21.8428(1)	-21.7612(3)
Rank	4.67	10	2	1	3.33
Models	M6	M7	M8	M9	M10
AIC	-21.6132(6)	-21.6358(5)	-21.4144(8)	-21.5268(7)	-21.0680(9)
BIC	-20.9146(3)	-20.8285(5)	-20.6071(8)	-20.7195(7)	-20.3228(9)
HQ	-21.6786(6)	-21.7113(5)	-21.4899(8)	-21.6023(7)	-21.1376(9)
Rank	5	5	8	7	9

Notes: Ml means SVAR(4)-GC-SSAEPD. M2 means SVAR(4)-GC-Normal. M3 means SVAR (4)-GC-Skewed EPD
M4 means SVAR(4)-GC-SEPD. M5 means SVAR(4)-GC-EPD M6 means SVAR(4)-GC-Laplace.
M7 means SVAR(4)-GC-SSAEPD with p11 = p21 = 2 M8 means SVAR(4)-GC-SSAEPD with p12 = p22 = 2
M9 means SVAR(4)-GC-SSAEPD with p13 = p23 = 2 MlO means SVAR(4)-GC-SSAEPD with p11 = p12 = p21 = p22 = 2.

5. Conclusions and Future Extensions

In this paper, we have extended the SVAR model by incorporating error terms that follow the Standardized Standard Asymmetric Exponential Power Distribution (SSAEPD). We utilized Markov Chain Monte Carlo (MCMC) techniques for generating error terms and applied the Method of Maximum Likelihood Estimation (MLE) for model estimation. Essential tools like the Impulse-Response Function (IRF) and Variance Decomposition (VD) were employed for structural analysis.

Using the same time series data as previously studied, our findings demonstrate that the new model with SSAEPD residuals can effectively capture the skewness, fat tails, and asymmetric kurtosis of sample data. Compared to the traditional SVAR model, our enhanced model shows better fitness. Moreover, the IRFs derived from our model align more closely with IS-LM theory, indicating that the SSAEPD significantly improves the performance of variance decomposition. Our results support the continued relevance of Keynesian theory in the current economic context.

Looking ahead, several potential extensions could further refine and expand the scope of our work: 1) Optimization Techniques: Future research might explore the use of a Genetic algorithm as an alternative to the current optimization methods to achieve global optimization, potentially enhancing the robustness and efficiency of parameter estimation. 2) Geographical Adaptation: There is scope to re-examine the applicability of economic theories across different countries. Such comparative studies could illuminate distinct economic dynamics and validate the model's utility in various international contexts. 3) Incorporation of Volatility Models: Integrating volatility models like GARCH into our SVAR framework could provide deeper insights into the impact of volatility on macroeconomic variables, thereby enriching the model's analytical capabilities.

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