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Option Pricing Based on REGARCH Model with High-Frequency Information

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Abstract: As a prevalent tool for hedging risk, the trading volume of options has been growing increasingly in the derivatives market. Precision in the estimation of volatility leads to accurate option pricing. Since volatility is time-varying and has a clustering effect, GARCH class of volatility models is effective in modeling volatility precisely. This paper utilizes the realized EGARCH (REGARCH) model combined with Monte Carlo simulation to investigate the role of high-frequency information in option pricing. The parameter estimates of the REGARCH model are obtained via joint maximum likelihood estimation using observations on returns and realized measure. Applying the model to S&P options market, the empirical results show that the REGARCH model that using high-frequency data is more efficient than the model that only use daily closing prices, including the EGARCH, NGARCH and GJR-GARCH models. This paper demonstrates that incorporating realized measures into volatility models can improve the accuracy of option pricing. The REGARCH model contained more intraday trading information from high-frequency data, can measure the additional risk premiums and specific volatility shocks.

Keywords: Option pricing, High-frequency information, Realized volatility, REGARCH model, S&P 500 index options.

1. Introduction

Options are an important financial instrument in the derivatives trading market, and the trading volume has been growing rapidly in recent decades [1-3]. The necessity of accurate option pricing is self-evident. The Black-Scholes model, proposed by Scholes and Black [4], is a groundbreaking work in the field of option pricing. However, the model assumes that the returns on the underlying asset follow a normal distribution and the volatility is a constant, which does not align with the volatility patterns observed in real financial markets. To better capture the stylized facts of the dynamics of volatility, including clustering, long memory, leverage effect and heavy tails of distribution, researchers have begun to propose stochastic volatility models for option pricing. These models evolve the assumption of constant volatility to time-varying volatility [5-7].

Accurately modeling the dynamics of the underlying asset is crucial for an effective option pricing model. Duan [8] employs the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model for option pricing through a local risk-neutral valuation approach. The model effectively captures empirical characteristics of asset returns, such as leptokurtosis, volatility clustering, and time-varying volatility, and it offers good predictive performance for volatility, making it widely used in option pricing. However, the GARCH model assumes that asset return innovations follow a normal distribution, which does not reflect the non-normal characteristics of asset returns. Moreover, the dynamics of asset returns under the objective and risk-neutral measures are based on consistent parameters, leading to certain errors in option pricing. The Exponential GARCH (EGARCH) model, proposed by Nelson [9], is type of volatility model that is particularly adept at capturing the asymmetric effects of positive and negative shocks on volatility. Engle and Ng [10] introduce the Nonlinear GARCH (NGARCH) model, which is designed to capture the asymmetric effect of shocks on volatility. Glosten et al. [11] propose the GJR-GARCH model, which particularly designed to address the leverage effect observed in financial markets. In comparison to NGARCH model, the GJR-GARCH model is more flexible in capturing the leverage effect.

Enhancing the information content or refining the model architecture can offer a more comprehensive depiction of daily volatility, thereby enhancing the accuracy of option pricing models. With the growing accessibility of high-frequency data, which encompasses a richer set of trading information, it has become an increasingly popular tool for option pricing due to its more accurate reflection of volatility. Studies such as those by Coris et al. [12] and Christoffersen et al. [13] have demonstrated the superiority of models that simultaneously consider the dynamics of returns and realized variances over those that rely solely on return data. To more effectively capture the interplay between returns and volatility, Hansen and Huang [14] introduced the REGARCH model, featuring a more adaptable leverage function. Subsequent research by Hansen et al. [15] confirmed that the REGARCH model outperforms other alternatives in terms of VIX pricing accuracy.

In this paper, we derive the option pricing formula for the REGARCH framework, which may result in better option pricing performance. This is because REGARCH framework has proven to be superior to conventional GARCH models for the modeling of returns and for forecasting volatility. The REGARCH model allows for an additional risk premium that relates to volatility shocks, and the model benefits from having both return and volatility shocks. Additionally, the measurement equation in the model does not require the realized measure to be an unbiased estimator of daily volatility. Unbiased estimators are difficult to obtain because high-frequency data is only available for a fraction of the day. Market microstructure noise that is not properly accounted for can also induce bias in realized measures.

In this study, we develop an option pricing formula within the REGARCH framework, potentially enhancing the accuracy of option pricing. The REGARCH framework has demonstrated its effectiveness over traditional GARCH models in capturing return dynamics and forecasting volatility. This model accommodates an extra risk premium associated with volatility shocks, capitalizing on the inclusion of both return and volatility shocks. Moreover, the REGARCH model does not necessitate that the realized measure serves as an unbiased estimator of daily volatility, which is a challenging requirement due to the limited availability of high-frequency data throughout the day. The model also accounts for market microstructure noise, which, if not properly addressed, can lead to biased realized measures.

In this paper, we conduct an empirical analysis with S&P 500 option prices. The data spans from January 2, 2019 to December 31, 2023. We consider a range of models in our comparisons, including the EGARCH, NGARCH and GJR-GARCH. And the REGARCH model outperforms the competing models both in-sample and out-of-sample. The main conclusion is that the use of high-frequency information greatly reduces option pricing errors. The results highlight the empirical gains on a non-affine model with leverage effect and reinforce the existing literature on the importance of including realized measures into option pricing.

In this research, we perform an empirical study utilizing S&P 500 option pricing data from January 2, 2019, to December 31, 2023. Our analysis encompasses a variety of models, such as the EGARCH, NGARCH, and GJR-GARCH, in our comparative evaluation. The REGARCH model surpasses these alternative models in both in-sample and out-of-sample tests. The main finding is that incorporating high-frequency data significantly diminishes the discrepancies in option pricing. The outcomes underscore the empirical advantages of employing a non-affine model that accounts for the leverage effect, thereby reinforce the existing literature on realized measures in the pricing of options.

The remainder of the paper is organized as follows: Section 2 introduces the REGARCH model, the competing models, and option valuation; Section 3 discusses the estimation method and the empirical results, including in-sample and out-of-sample; and Section 5 concludes.

2. Methodology

2.1. REGARCH Model

To describe the dynamics of the underlying asset returns under physical measure, we consider the following specification of the REGARCH model,

$$\begin{aligned} r_{t+1} &\equiv \log\left(\frac{S_{t+1}}{S_t}\right) \\ &= r + \lambda\sqrt{h_{t+1}} - \frac{1}{2}h_{t+1} + \sqrt{h_{t+1}}z_{t+1}, \end{aligned} \tag{1}$$

$$\begin{aligned} \log h_{t+1} &= \omega + \beta \log h_t + \tau_1 z_t \\ &\quad + \tau_2(z_t^2 - 1) + \gamma\sigma_u u_t, \end{aligned} \tag{2}$$

$$\begin{aligned} \log x_t &= \xi + \phi \log h_t + d_1 z_t \\ &\quad + d_2(z_t^2 - 1) + \sigma_u u_t, \end{aligned} \tag{3}$$

Where r_{t+1} denotes the log-return on day $t+1$, S_t is the price of underlying asset on day t. r is the risk-free interest rate, and λ is the equity risk premium parameter. $h_t = \mathrm{Var}_{t-1}^{\mathrm{P}}(r_t)$ denotes the conditional variance of return on day t, and x_t is the realized measure of volatility. The shocks of return follow standard normal distributions, that is, $z_t \sim i.i.d.N(0,1)$ and $u_t \sim i.i.d.N(0,1)$. We specify u_t to be independent of z_t, $corr(z_t, u_t) = 0$. The leverage functions, $\tau_1 z_t + \tau_2(z_t^2 - 1)$ and $d_1 z_t + d_2(z_t^2 - 1)$, capture the leverage effect.

The model has two characteristics: First, the REGARCH model has a key feature of a stochastic volatility model, that is, it has an innovation u_t directly related to volatility, and the model is easier to estimate due to the observation-driven framework. Second, there is fewer parameter restrictions of REGARCH model than other discrete models that use realized variance for option pricing, which greatly simplifies the estimation process.

2.2. Risk Neutralization

For the purpose of option pricing, we need to derive the risk-neutral dynamics of the REGARCH model. Following Christoffersen et al. [16] and Huang et al. [17], we apply the exponential affine pricing kernel:

$$\begin{aligned} \zeta_{t+1} &= \frac{\exp(\nu_{1,t} z_{t+1} + \nu_{2,t} u_{t+1})}{\mathrm{E}_t^{\mathrm{P}}[\exp(\nu_{1,t} z_{t+1} + \nu_{2,t} u_{t+1})]} \\ &= \exp\left(\nu_{1,t} z_{t+1} + \nu_{2,t} u_{t+1} - \frac{\nu_{1,t}^2}{2} - \frac{\nu_{2,t}^2}{2}\right), \end{aligned} \tag{4}$$

Where $\nu_{1,t}$ and $\nu_{2,t}$ are associated with equity and volatility risk premia, respectively, capturing the discrepancy between the physical measure and risk-neutral measure. Imposing the non-arbitrage condition,

$$\mathrm{E}_t^{\mathbb{Q}}(\exp(r_{t+1})) = \exp(r), \tag{5}$$

Yields the condition

$$\begin{aligned} &\mathrm{E}_t^{\mathbb{Q}}(\exp(r_{t+1})) = \mathrm{E}_t(\zeta_{t+1}\exp(r_{t+1})) \\ &= \exp(r + \lambda\sqrt{h_{t+1}} + \nu_{1,t}\sqrt{h_{t+1}}) = \exp(r), \end{aligned} \tag{6}$$

With

$$\nu_{1,t} = -\lambda. \tag{7}$$

The volatility risk premium $\nu_{2,t}$ cannot be determined uniquely and needs to be estimated using option data. To simplify the analysis, it is assumed to be a constant, $\nu_{2,t} = \chi$.

Then we consider the risk-neutral moment-generating function (MGF) and the sufficient condition to derive the model under risk-neutral measure. The MGF of the two random shocks can be written as

$$\begin{aligned} &\mathrm{E}_t^{\mathbb{Q}}(\exp(s_1 z_{t+1} + s_2 u_{t+1})) \\ &= \mathrm{E}_t(\zeta_{t+1}\exp(s_1 z_{t+1} + s_2 u_{t+1})) \\ &= \exp(-s_1\lambda - s_2\chi + \frac{s_1^2}{2} + \frac{s_2^2}{2}). \end{aligned} \tag{8}$$

We maintain the mapping relationship between the physical and risk-neutral innovation:

$$z_{t+1}^* = z_{t+1} + \lambda, \tag{9}$$

$$u_{t+1}^* = u_{t+1} + \chi. \tag{10}$$

Then, the corresponding risk-neutral dynamics is

$$r_{t+1} = r - \frac{1}{2}h_{t+1} + \sqrt{h_{t+1}}z_{t+1}^*, \tag{11}$$

$$\begin{aligned} \log h_{t+1} &= \omega + \beta\log h_t + \tau_1(z_t^* - \lambda) \\ &\quad + \tau_2((z_t^* - \lambda)^2 - 1) + \gamma\sigma_u(u_t^* + \chi), \end{aligned} \tag{12}$$

$$\begin{aligned} \log h_{t+1} &= \xi + \phi\log h_t + d_1(z_t^* - \lambda) \\ &\quad + d_2((z_t^* - \lambda)^2 - 1) + \sigma_u(u_t^* + \chi), \end{aligned} \tag{13}$$

Where z_t^* and u_t^* are independently normally distributed. When estimating the dynamic equation under the risk-neutral measure, the effective model is

$$r_{t+1} = r - \frac{1}{2}h_{t+1} + \sqrt{h_{t+1}}z_{t+1}^*, \tag{14}$$

$$\begin{aligned} \log h_{t+1} &= \omega^* + \beta\log h_t + \tau_1^* z_t^* \\ &\quad + \tau_2(z_t^{*2} - 1) + \gamma\sigma_u u_t^*, \end{aligned} \tag{15}$$

$$\begin{aligned} \log h_{t+1} &= \xi^* + \phi\log h_t + d_1 z_t^* \\ &\quad + d_2(z_t^{*2} - 1) + \sigma_u u_t^*, \end{aligned} \tag{16}$$

With

$$w^* = \omega - \tau_1\lambda + \tau_2\lambda^2 - \gamma\sigma_u\chi, \tag{17}$$

$$\xi^* = \xi - d_1\lambda + d_2\lambda^2 - \sigma_u\chi, \tag{18}$$

$$\tau_1^* = \tau_1 - 2\tau_2\lambda, \tag{19}$$

$$d_1^* = d_1 - 2d_2\lambda. \tag{20}$$

2.3. Competing Models

Competing models selected in this paper are the EGRACH, NGARCH and GJR-GARCH models.

2.3.1. EGARCH model

Following Nelson [9], the EGARCH model under physical measure can be written as:

$$r_{t+1} = r + \lambda\sqrt{h_{t+1}} - \frac{1}{2}h_{t+1} + \sqrt{h_{t+1}}z_{t+1}, \tag{21}$$

$$\begin{aligned} \log h_{t+1} &= \omega + \beta\log h_t + \tau_1 z_t \\ &\quad + \tau_2\left(|z_t| - \sqrt{\frac{2}{\pi}}\right), \end{aligned} \tag{22}$$

Where $z_t \sim i.i.d.N(0,1)$, and $\tau_1 z_t + \tau_2(|z_t| - \sqrt{2/\pi})$ is the leverage function, which capture and the corresponding model under risk-neutral measure is:

$$r_{t+1} = r - \frac{1}{2}h_{t+1} + \sqrt{h_{t+1}}z_{t+1}^*, \tag{23}$$

$$\begin{aligned} \log h_{t+1} &= \omega + \beta\log h_t + \tau_1(z_t - \lambda) \\ &\quad + \tau_2\left(|z_t - \lambda| - \sqrt{\frac{2}{\pi}}\right), \end{aligned} \tag{24}$$

Where the persistence parameters under two measures are the same, that is, $\pi^{\mathrm{P}} = \pi^{\mathbb{Q}} = \beta$.

2.3.2. NGARCH model

Under the NGARCH model by Engle and Ng [10], the return dynamics are given by

$$r_{t+1} = r + \lambda\sqrt{h_{t+1}} - \frac{1}{2}h_{t+1} + \sqrt{h_{t+1}}z_{t+1}, \tag{25}$$

$$h_{t+1} = \omega + \beta h_t + \tau_1 h_t(z_t - \tau_2)^2, \tag{26}$$

Where $z_t \sim i.i.d.N(0,1)$. The parameter τ_2 capture the leverage effect. The dynamic equation under risk-neutral measure can be written as:

$$r_{t+1} = r - \frac{1}{2}h_{t+1} + \sqrt{h_{t+1}}z_{t+1}^*, \tag{27}$$

$$h_{t+1} = \omega + \beta h_t + \tau_1 h_t(z_t^* - (\tau_2 + \lambda))^2, \tag{28}$$

Where $z_t^* = z_t + \lambda \sim i.i.d.N(0,1)$.

2.3.3. GJR-GARCH model

The GJR-GARCH model proposed by Glosten et al. [11] is given by

$$r_{t+1} = r + \lambda\sqrt{h_{t+1}} - \frac{1}{2}h_{t+1} + \sqrt{h_{t+1}}z_{t+1}, \tag{29}$$

$$h_{t+1} = \omega + h_t[\beta + \tau_1 z_t^2 + \tau_2\max(0, -z_t)^2], \tag{30}$$

Where $z_t \sim i.i.d.N(0,1)$. The leverage effect is captured by parameter τ_2. The risk-neutral GJR-model can be derived:

$$r_{t+1} = r - \frac{1}{2}h_{t+1} + \sqrt{h_{t+1}}z_{t+1}^*, \tag{31}$$

$$\begin{aligned} h_{t+1} &= \omega + h_t[\beta + \tau_1(z_t - \lambda)^2 \\ &\quad + \tau_2\max(0, -(z_t - \lambda))^2], \end{aligned} \tag{32}$$

Where $z_t^* = z_t + \lambda \sim i.i.d.N(0,1)$.

2.4. Option Valuation

According to the risk-neutral principle, the European call option price can be written as

$$C_t = \exp(-r(T-t))E_t^{\mathbb{Q}}[\max(S_T - K, 0)], \quad (33)$$

Where T denotes the maturity of the option, T-t is the time to maturity, and K is the trike price. S_T denotes the price of the underlying asset at time T. Since the REGARCH is a non-affine model, whose closed-form solution is not exist, that we use the Monte Carlo simulation method to calculate the option price:

$$C_t \approx \exp(-r(T-t))\frac{1}{MC}\times \sum_{j=1}^{MC}\left[\max\left(S_t \exp\left(\sum_{k=1}^{T-t} r_{t+k}^j\right) - K, 0\right)\right], \quad (34)$$

Where r_{t+k}^j is the return of the underlying asset at time t+k along the j-th simulation path under the risk-neutral measure of REGARCH model. To reduce the simulation error, the empirical simulation method is adopted [18]. The price of a European put option can be derived through the call-put parity relationship.

3. Empirical Analysis

3.1. Data

The data set in this paper contains the S&P 500 return series, the corresponding RV, and the panel data of option prices collected from the Oxford-man Institute Realized Library and Wind. The sample period is from January 2, 2019 to December 31, 2023. We select the call option data on Wednesday as in-sample data for calibrating the pricing model parameters, and choose the call option data on Thursdays as out-of-sample data to examine the pricing ability of the models. Following Christoffersen et al. [13], we use put and call options with positive trading volumes and maturities of 15-180 days, and convert out-of-money put option prices to in-the-money call option prices using the put-call parity. For each maturity on Wednesday and Thursday, we only retain the first six options ranking by trading volume, which ultimately yields a sample of 22,188 option contracts.

3.2. Estimation Method

For option pricing, there are two prevalent methods are employed to determine the model parameters. The first method focuses on minimizing pricing errors only based on option data, while the second approach involves a combined estimation that incorporates both the returns of the underlying assets and the option data. The latter method leverages both physical and risk-neutral information to calibrate the parameters of the models. The parameters to be estimated for option pricing using the REGARCH model can be expressed:

$$\Theta = \{\lambda, \omega, \beta, \tau_1, \tau_2, \gamma, \phi, d_1, d_2, \sigma_u, \log h_1, \chi, \sigma_e\}$$

Where σ_e denotes the vega-weighted pricing standard error for $\{r_t, RV_t \mid t = 1, 2, \ldots, T\}$ and $\{option_i \mid i = 1, 2, \ldots, N\}$.

$$\begin{aligned} L_{r,x,o}(r_t, RV_t, option_i; \Theta) &= L_r(r_t; \Theta) \\ &+ L_x(RV_t; \Theta, r_t) + L_o(option_i; \Theta, r_t, RV_t), \end{aligned} \quad (35)$$

With

$$L_r = -\frac{T}{2}\log(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\log(h_t) - \sum_{t=1}^{T}\frac{(r_t - r - \lambda\sqrt{h_t} + \frac{1}{2}h_t)^2}{2h_t},$$

$$L_x = -\frac{T}{2}\log(2\pi) - \frac{T}{2}\sum_{t=1}^{T}\log(\sigma_u^2) - \sum_{t=1}^{T}\frac{(\log RV_t - \xi - \phi\log h_t - d_1 z_t - d_2(z_t^2 - 1))^2}{2\sigma_u^2},$$

$$L_o = -\frac{N}{2}\log(2\pi) - \frac{N}{2}\sum_{i=1}^{N}\log(\sigma_e^2) - \sum_{i=1}^{N}\frac{((P_i^{Mod} - P_i^{Mkt}) / Vega_i)^2}{2\sigma_e^2},$$

Where P_i^{Mod} is the price obtained from the pricing formula, P_i^{Mkt} is the market price of option i, and the $Vega_i$ denotes the Black-Scholes vega calculated by the P_i^{Mkt}, which measures the sensitivity of the option to changes in implied volatility. The fact that $Vega_i$ is independent of model parameters significantly accelerates the estimation procedure. Thus, $(P_i^{Mod} - P_i^{Mkt}) / Vega_i$ serves as an approximation of the difference in implied volatilities.

3.3. Full-sample Parameter Estimates

The parameters under physical measures are ascertained by using the joint maximum likelihood estimation method. Table 1 reports the estimation results of the REGARCH model and the competing models. The coefficients are shown for each model and the corresponding standard errors are presented in parentheses. Allowing for compare the efficiency of the models directly, Table 1 also reports the log-likelihoods, L, and the volatility persistence under physical and risk-neutral measures, π^{P} and $\pi^{\mathbb{Q}}$.

Table 1. Parameter estimates

	EGARCH	NGARCH	GJR-GRACH	REGARCH
λ	0.0489 (0.0069)	-0.0132 (0.0056)	0.6739 (0.0023)	0.0059 (0.0015)
ω	-0.0375 (0.0018)	-0.0036 (0.0009)	-0.4632 (0.0054)	-0.2576 (0.0022)
β	0.9746 (0.0013)	0.7856 (0.0019)	0.9532 (0.0003)	0.9816 (0.0037)
τ_1	−0.1218 (0.0012)	0.0578 (0.0023)	0.0002 (0.0000)	−0.1028 (0.0016)
τ_2	0.1156 (0.8913)	0.0437 (0.0056)	0.0479 (0.0046)	0.0075 (0.0032)
γ				0.0573 (0.0019)
ξ				−0.7318 (0.1252)
ϕ				0.9752 (0.0235)
d_1				−0.2473 (0.0128)
d_2				0.3692 (0.0257)
$\log h_1$	−8.2893 (0.0549)	−8.1465 (0.0257)	−9.2304 (0.0295)	−8.2742 (0.0232)
χ				0.1538 (0.0251)
π^{P}	0.9737	0.9934	0.9248	0.9842
$\pi^{\mathbb{Q}}$	0.9741	0.9934	0.9252	0. 9842
L	39273.6	3374.7	42738.9	44739.2
$\sigma_e \times 100$	4.2383	3.1047	3.3485	2.2983

Note: The number in parenthesis is the standard error. The variables π^{P} and $\pi^{\mathbb{Q}}$ are the persistence parameters under physical and risk-neutral measures and L is the value of log-likelihood function.

For each model, the estimates of λ are positive and statistically significant, suggesting positive volatility risk compensation. The β of most models is close to one, which indicates the strong persistence of volatility dynamic. The parameter estimates, τ_1, τ_2, d_1 and d_2, associated with the leverage function reflect that the models exhibit a significant leverage effect, which is in line with previous studies (see Hansen et al. [14]). The parameter, γ, measures the significant contribution of realized variance, illustrating the importance of realized measures. The volatility risk premium of REGARCH model is positive, which can be seen from the positive value of χ. The smallest σ_e suggests the minimum Vega-weighted option pricing error for the REGARCH model. The competing models without realized measures (the EGARCH, NGARCH, and GJR-GARCH models) all display significantly larger option pricing errors. Additionally, we find that $\pi^{\mathbb{Q}}$ is generally larger than π^{P}, which align with findings in the option pricing literature that under the risk-neutral measure, volatility exhibits more persistence.

3.4. In-sample Pricing Performance

In this paper, we assess the pricing performance of the models using the root mean square of implied volatility (IVRMSE), which is given by

$$IVRMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left[IV_i^{Mod} - IV_i^{Mkt}\right]^2} \times 100, \quad (36)$$

Where IV_i^{Mod} and IV_i^{Mkt} represent the model-based implied volatility and market-based implied volatility, respectively. The pricing results are reported in Table 2. We also compute the IVRMSE for option partitioned by the characteristics: moneyness and time to maturity. The resulting IVRMSEs can be used to identify the inabilities of models to generate sufficient leverage effect and to capture the dynamic properties.

The total IVREMSE indicates that the REGARCH model has the smallest pricing error, while the NGARCH and GJR-GARCH models have comparable pricing abilities, and the EGARCH model performs the worst. We note that the REGARCH model outperforms the competing models in most of cases, especially for at-the-money options and long maturity options. The IVRMSE of all models that do not include realized variance is significantly higher, further supporting the advantage of allowing for high-frequency information.

Table 2. In-sample pricing performance

	EGARCH	NGARCH	GJR-GRACH	REGARCH
Total IVRMSE	4.2738	3.3204	3.4389	2.7535
By moneyness				
Delta<0.3	4.3984	3.3479	4.1398	3.3948
0.3<Delta<0.4	3.8923	3.2304	3.3894	2.5489
0.4<Delta<0.5	3.0328	2.8437	3.2093	2.2397
0.5<Delta<0.6	3.0542	3.2704	3.4998	2.5487
0.6<Delta<0.7	3.4289	3.1928	3.2983	2.2873
0.7<Delta	4.2976	3.6472	3.4837	2.9542
By DTM				
DTM<30	3.4854	3.4583	3.4857	2.4874
30<DTM<60	3.3984	3.3984	3.6740	2.5983
60<DTM<90	4.3947	3.1856	3.3874	2.4458
90<DTM<120	4.3631	3.2394	3.2198	2.7875
120<DTM<150	4.4876	3.3274	3.2391	2.4950
150<DTM	4.9832	3.3457	3.3470	3.2934

3.5. Out-of-sample Pricing Performance

Since the REGARCH model has more parameters than the competing models, there will be a natural concern whether the superior performance is merely due to the in-sample overfitting. Therefore, studying the out-of-sample analysis is a more convincing way to compare the option pricing abilities of the models. We conduct the out-of-sample evaluations from January 8, 2021 to December 31, 2023. The corresponding results is reported in Table 3.

Results reported in Table 3 are similar to those in Table 2, but the REGARCH achieve a larger improvement. To be more specific, the REGARCH model has a better performance according to different moneyness and maturity levels. Overall, the out-of-sample results are consistent with the in-sample results.

Table 3. Out-of-sample pricing performance

	EGARCH	NGARCH	GJR-GRACH	REGARCH
Total IVRMSE	2.9384	2.2873	2.9675	1.8725
By moneyness				
Delta<0.3	5.2653	3.3468	4.8457	1.6534
0.3<Delta<0.4	3.7384	2.8234	3.2365	1.5692
0.4<Delta<0.5	2.9346	2.8902	3.3837	1.2734
0.5<Delta<0.6	2.6523	1.7348	2.9374	1.4837
0.6<Delta<0.7	1.8957	1.7893	2.8954	1.3468
0.7<Delta	1.6732	1.8546	2.2445	1.9755
By DTM				
DTM<30	2.7643	2.2215	2.6287	1.5434
30<DTM<60	2.8374	2.3984	2.3897	1.8279
60<DTM<90	3.2938	2.3490	3.4895	1.4879
90<DTM<120	3.2387	2.3879	3.3748	1.3480
120<DTM<150	3.4562	2.2092	3.3870	1.2057
150<DTM	4.3490	2.2203	3.5434	1.9823

4. Conclusion

This study has introduced the high-frequence information to option pricing by using REGARCH model. An importance feature of the model is that the realized variance is incorporated and linked to the conditional volatility through a measurement equation. This model not only improves the precision of the volatility predictions, but also allows for an additional risk premium that relates to specific volatility shocks. Besides, the parameters of the REGARCH model can be directly estimated by the maximum likelihood method. We compare the REGARCH model and the competing (EGARCH, NGARCH, GJR-GARCH) models in terms of option pricing performance. A consistent finding from both in-sample and out-of-sample analyses is that incorporating realized measures significantly benefits option pricing. The REGARCH model demonstrates superior performance on average. The theoretical and empirical findings point to potential avenues for future research, which could initially focus on enhancing the option pricing capabilities of the REGARCH model. A promising area for subsequent investigation is to explore the volatility spillover effects across derivative markets utilizing the REGARCH model framework.

Declaration of Competing Interest

The author declares no conflict of interest.

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