

Research on Default Risk of Urban Investment Bonds Based on Multi-factor Model and KMV Model

-- Take Shanghai as an example

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Abstract: With the acceleration of urbanization in China, the urban investment bond market is facing increasing risks and challenges while providing financial support for infrastructure construction. Aiming at the complexity and uncertainty of the market, the study aims to improve the accuracy and applicability of risk measurement through model fusion. This study will comprehensively consider market factors, company financial situation and macroeconomic indicators, and use the comprehensiveness of multi-factor model and the accurate estimation of default probability by KMV model. The results show that the combination of data collection, model construction, factor optimization and model validation can provide investors and regulators with scientific decision-making and effective supervision. Promote the stable development of the market.

Keywords: Default risk of urban investment bonds, Multi-factor model, KMV model, Mixed research method, Risk assessment model.

1. Introduction

With the rapid development of China's urbanization and the increasing investment in infrastructure construction, urban investors and institutions have participated in China's urban investment bond market. The vigorous development of this market has provided financial support for the construction of urban infrastructure, but it is also accompanied by an increase in risk. Therefore, it is particularly important to study how to measure the risk of China's urban investment bonds scientifically and effectively.

KMV model is a structural model based on the probability of corporate default, which can accurately estimate the probability of debt default by considering the structure of the debtor's balance sheet and market factors. The multi-factor model evaluates the risk of bonds comprehensively by considering multiple factors, such as market factors, credit rating, macroeconomic indicators, etc. [1]. This paper empirically studies the default risk of Shanghai municipal investment bonds by using multi-factor model and KMV model.

2. Literature Review

Zhu Jie and Li Qiyun calculated the issuance scale of local government bonds from the perspective of credit risk through KMV model, which provided an empirical analysis for understanding the relationship between the scale and risk of urban investment bonds [2]. Xia Shiyuan used KMV model to conduct early warning research on local government debt risk, demonstrating the application potential of structural model in default risk assessment [3]. Sun Li and Kong Wenxi studied the measurement and early warning of default risk of urban investment bonds based on KMV and BP models, which provided a new perspective for risk management [4]. In her research, yuan Xinyu tried to improve the credit rating model by introducing more factors to improve the accuracy

of credit risk assessment of urban investment bonds [1]. Abdul Bahri assessed the debt risk of green bonds issued by local governments using three principal component factors extracted from the KMV model combined with principal component analysis. This highlights the importance of including multiple factors when assessing the default risk of urban investment bonds [5]. Peykani's research explores the application of structural and machine learning models to predict the non-standard default risk of urban investment and development companies in the urban investment bond market [6].

3. Multi-factor Model Analysis Based on The Issuer's Perspective

3.1. Data Source and Processing

The data used in this paper are all from the authoritative Wind database, excluding the samples that can not match the maturity of treasury bonds, can not calculate credit spreads by matching with treasury bonds, and have serious data deficiencies, including the samples that the issuers have no credit rating and the samples that have a large number of missing financial data.

3.2. Indicator Factors

Usually, the yield of treasury bonds with the same maturity is regarded as a substitute for the risk-free interest rate, and the credit spread of bonds is measured by the difference between the issuance price of bonds and the yield of treasury bonds.

$$CSt=Yt-Rt \quad (1)$$

Where, CSt is the credit spread of the urban investment bond with the remaining maturity of t, Yt is the issuing interest rate of the urban investment bond with the remaining maturity of t, and Rt is the yield to maturity of the treasury bond matching the urban investment bond, that is, consistent

with the remaining maturity of the urban investment bond.

The fund sources for urban investment companies to repay the principal and interest of their debts can be divided into three parts: endogenous funds, exogenous funds and coordinated funds. The stability of such funds is highly dependent on the financial ecological environment of the region where the urban investment companies are located, that is, the comprehensive effect of a series of non-economic factors [7].

In order to accurately quantify this complex risk, it is necessary to build a multi-dimensional and multi-factor evaluation model system.

3.3. Factor Analysis

(1) Feasibility test

It can be seen from Table 1 that the data are relevant and suitable for factor analysis.

Table 1. KMO test

	KMO	0.812
	Approximate x2	21031
Bartley sphere test	Degrees of freedom	615
	Sig.	0.000

(2) extracting principal components;

It is mainly based on the cumulative contribution value as the screening criteria, supplemented by the characteristic value as a reference. After screening, we found that there were eight factors whose characteristic roots were greater than 1, and the cumulative variance contribution rate of these factors was as high as 85.01%. These eight factors were regarded as suitable principal components. In order to clarify the influencing factors of each principal component, the maximum variance orthogonal rotation method is used to process the eight principal component factors extracted before. The rotated factor loading matrix is shown in Table 2.

Table 2. Factor loading matrix after orthogonal rotation

Specific Indicators	Ingredient							
	f1	f2	f3	f4	f5	f6	f7	f8
GDP	0.995	-0.005	-0.023	0.002	-0.013	-0.007	0.005	0.031
GDP per capita	0.979	-0.013	-0.023	0.004	-0.010	0.005	-0.023	0.025
Year-on-year GDP growth	-0.511	0.006	-0.053	0.034	-0.001	-0.049	0.709	0.025
Debt ratio	0.978	-0.018	0.029	-0.019	-0.003	0.034	0.043	-0.032
Debt ratio	0.968	0.001	0.031	-0.029	0.008	0.057	0.165	-0.054
Urbanization rate	0.945	-0.047	-0.012	0.028	-0.025	-0.020	-0.082	0.039
Tax Revenue	0.971	-0.022	-0.031	0.019	-0.024	-0.044	0.107	0.041
Financial self-sufficiency rate	0.701	0.023	0.021	-0.029	0.007	0.025	0.736	-0.057
Balance of local government debt	0.987	-0.004	0.010	-0.026	-0.006	0.015	0.022	-0.004
Local public finance revenue	1.001	-0.003	-0.007	-0.009	-0.011	-0.009	0.075	0.013
Year-on-year growth rate of local public revenue	0.081	-0.017	-0.027	0.034	-0.007	-0.015	0.937	0.020
Interest-bearing debt	0.051	-0.055	-0.590	0.051	0.064	-0.095	-0.033	-0.317
Current ratio	-0.094	-0.055	-0.021	-0.044	0.853	-0.056	0.006	0.073
Quick ratio	-0.031	0.052	0.074	0.201	0.901	0.019	-0.008	-0.043
Interest-bearing liabilities	-0.049	1.006	-0.071	-0.014	0.002	-0.034	-0.004	-0.025
Short-term debt	0.033	0.986	-0.092	-0.019	-0.039	-0.031	0.004	-0.051
Cash turnover	0.104	-0.137	-0.079	0.132	0.027	-0.034	-0.021	0.931
return on equity	0.000	-0.319	0.885	-0.055	0.008	-0.038	-0.035	-0.166
Return on total assets	-0.002	0.193	0.868	-0.004	-0.063	0.000	0.001	0.028
Turnover of total assets	0.090	0.510	0.299	0.122	-0.073	-0.131	0.007	0.351
Cash-to-maturity debt ratio	-0.069	-0.011	-0.019	0.091	-0.234	0.794	-0.053	-0.080
Total profit/total assets	-0.036	-0.094	0.933	-0.046	0.126	0.046	-0.023	-0.066
Registered capital (RMB 100 million)	-0.097	0.866	-0.035	0.167	-0.064	-0.059	0.001	-0.053
Total assets (100 million yuan)	-0.021	1.004	-0.062	-0.004	-0.019	-0.023	-0.006	-0.032
Operating income (100 million yuan)	-0.053	1.011	-0.048	-0.057	0.028	-0.032	-0.001	0.018
Total profit (100 million yuan)	0.061	0.931	0.201	0.019	0.036	-0.025	-0.005	-0.097
Total owner's equity (100 million yuan)	0.015	0.100	-0.047	0.006	0.029	0.001	-0.005	-0.051
Net cash flow from operating activities	-0.022	0.799	-0.019	-0.201	0.058	0.391	0.011	0.081
Net cash flow from operating activities/interest-bearing liabilities	0.058	-0.011	0.049	0.089	0.010	0.935	0.015	0.019
Coupon rate	-0.385	-0.198	-0.137	-0.369	-0.006	0.031	0.098	0.403
Term of issue (years)	-0.441	0.018	0.133	0.183	-0.153	-0.131	0.116	-0.082
Issue size (100 million yuan)	-0.145	-0.034	-0.051	0.995	0.053	0.089	0.038	0.068
Balance of bonds (100 million yuan)	0.089	0.000	-0.062	0.972	0.027	0.058	0.019	0.103

(3) nomenclature of principal factor

Through principal component analysis, 8 common factors

are extracted, as shown in Table 3:

Table 3. Index system of multi-factor explanatory variables

Classification of indicators	Specific indicators and abbreviations
Local economic and financial strength (f1)	GDP/GDP GDP per capita Urbanization rate Fiscal revenue of local government Tax Revenue Debt ratio Debt ratio
Capital structure (f2)	Total assets Owner's equity Operating income Total profit Interest-bearing liabilities Net cash flow from operating activities Short-term debt
Profitability (f3)	Total profit/total assets Return on equity ROE Return on total assets (ROA)
Debt factor (f4)	Issue size Bond balance
Liquidity of assets (f5)	Current ratio Quick ratio
Solvency (f6)	Net cash flow from operating activities/interest-bearing liabilities Cash-to-maturity debt ratio
Local economic and financial growth (f7)	GDP growth Growth rate of local public finance revenue
Asset management efficiency (f8)	Cash turnover

3.4. Model Construction and Analysis

This paper constructs the following multiple regression model to examine the impact of variables on the credit of urban investment bonds. Where CSI

It is the credit spread for the issuance of urban investment bonds. Variables F5 and F6 were removed by stepwise regression analysis using Stata software and a screening process of backward elimination, and the remaining factors were statistically significant at the 1% significance level, as shown in Table 4.

$$CSI = \beta_0 + \beta_1 \times f1 + \beta_2 \times f2 + \beta_3 \times f3 + \beta_4 \times f4 + \beta_5 \times f5 + \beta_6 \times f6 + \beta_7 \times f7 + \beta_8 \times f8 \quad (2)$$

Table 4. Regression results

	(1) ctedit_spread	(2) ctedit_spread	(3) ctedit_spread
f1	-0.302*** (-9.173)	-0.321*** (-9.690)	-0.282*** (-9.468)
f2	-0.352*** (-9.631)	-0.324*** (-9.867)	-0.301*** (-9.773)
f3	-0.224*** (-7.577)	-0.234*** (-7.533)	-0.232*** (-7.403)
f4	-0.542*** (-18.123)	-0.522*** (-17.315)	-0.542*** (-17.603)
f5	0.032 (0.773)		
f6	0.062** (2.176)	0.062** (2.131)	
f7	0.162*** (4.893)	0.139*** (4.973)	0.139*** (4.883)
f8	0.623*** (19.579)	0.612*** (19.569)	0.619*** (19.836)
Constant term	2.682*** (95.237)	2.702*** (95.237)	2.702*** (95.173)
N	204	204	204
R2	0.601	0.578	0.561
F	163.793	198.364	228.702

Note: (1) The values in brackets are t values; (2) After the data, "*" means that the coefficient is significant at the 10% confidence level, "* *" means that the coefficient is significant at the 5% confidence level, and "* * *" means that the coefficient is significant at the 1% confidence level.

3.5. Robustness Test

The explained variable is changed from the issuance spread to the issuance interest rate, and the other explanatory variables in the model are kept unchanged for regression. The results are shown in Table 5:

Table 5. Robustness test results

	(1) ctedit_spread	(2) ctedit_spread	(3) ctedit_spread
f1	-0.576*** (-19.013)	-0.582*** (-19.090)	-0.580*** (-19.018)
f2	-0.302*** (-9.521)	-0.294*** (-9.486)	-0.291*** (-9.413)
f3	-0.212*** (-6.623)	-0.213*** (-6.613)	-0.199*** (-6.495)
f4	-0.556*** (-17.823)	-0.557*** (-17.695)	-0.552*** (-17.831)
f5	-0.022 (-0.373)		
f6	0.052 (1.321)	0.042 (1.431)	
f7	0.146*** (4.993)	0.143*** (4.997)	0.143*** (5.131)
f8	0.621*** (19.524)	0.611*** (19.561)	0.602*** (19.703)
Constant term	5.482*** (193.532)	5.472*** (193.772)	5.501*** (193.571)
N	204	204	204
R2	0.606	0.607	0.601
F	212.325	242.964	282.782

The original model successfully passed the robustness test,

showing high stability and reliability.

4. Analysis of KMV Model Based on Bond

4.1. Sample Selection

This paper takes Shanghai as the research object of urban investment bonds, derives the financial data of Shanghai from Wind database from 2014 to 2023, calculates the default distance and default probability of Shanghai's urban investment bonds as a whole in 2024, and on this basis, calculates the maturity repayment scale of Shanghai's urban investment bonds when the default probability is 0, and compares it with the actual maturity repayment scale of urban investment bonds. The urban investment bonds with high default risk are obtained.

4.2. Calculation Method of KMV Model

According to the BS formula:

$$S_t = V_0 N(d_1) - De^{-rt} N(d_2) \quad (3)$$

$$d_1 = \frac{\ln(\frac{V_0}{D}) + (r + 1/2\sigma^2)t}{\sigma\sqrt{t}} \quad (4)$$

According to the KMV model, the financial guarantee income obeys the lognormal distribution and meets the following conditions:

$$\ln R_t = \ln A + (\mu - \frac{1}{2\sigma^2})t + \sigma\sqrt{t}Z_t \quad (5)$$

$$R_t = \text{REXP} \left\{ (\mu - \frac{1}{2\sigma^2})t + \sigma\sqrt{t}Z_t \right\} \quad (6)$$

Where Z_t is the standard normal distribution;

R_t represents the local government financial guarantee revenue at time T ;

μ represents the growth rate of local government financial guarantee revenue;

σ represents the fluctuation rate of local government financial guarantee income;

B_t represents the debt repayment scale of the urban investment enterprise at time T .

According to the definition of mean and standard deviation, we can get:

$$\mu = \frac{1}{n-1} \sum_{i=1}^n \ln \frac{R_{i+1}}{R_i} + \ln R \quad (7)$$

$$\sigma = \frac{1}{n-2} \sum_{i=1}^{n-1} \left(\ln R * \frac{R_{i+1}}{R_i} - \frac{1}{n-1} \sum_{i=1}^{n-1} \ln R * \frac{R_{i+1}}{R_i} \right) \quad (8)$$

Therefore, the default distance DD and the default probability p are:

$$DD = \frac{\ln(\frac{R}{B}) + (\mu - 1/2\sigma^2)t}{\sigma\sqrt{t}} \quad (9)$$

$$P = N \left\{ -\frac{\ln(\frac{R}{B}) + (\mu - 1/2\sigma^2)t}{\sigma\sqrt{t}} \right\} \quad (10)$$

4.3. Financial Guarantee Revenue Forecast

In Eviews, ADF is used to test whether the time series is stable or not, and the time series Y is the financial data of Shanghai from 2014 to 2023. Taking the logarithm of the original data, the series Y is obtained, and the result of ADF test shows that the time series is not stable.

Table 6. Statistical Table of Test Results

	Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
Z(t)	-0.583	-3.823	-3.185	-2.931

Take the first order difference of the time series to get a new sequence dY , and then carry out the ADF test again, the test results show that the time series is stable.

Table 7. Statistical Table of Test Results

	Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
Z(t)	-6.101	-3.823	-3.185	-2.931

The values of p and Q in different ARIMA models are substituted and the values of AIC are observed. After testing, the fitting effect of ARIMA (0,2,0) is the best, the value of AIC is 483.731, and the predicted value of Shanghai's fiscal revenue in 2024 is 897.456 billion yuan. After multiplying by the guarantee coefficient of 0.5, the financial guarantee income of Shanghai is calculated. According to the model, Shanghai's fiscal revenue in 2023 is 897. The financial guarantee revenue was 448.728 billion yuan. According to the budget implementation report or Report on the Work of the Government published by the Shanghai Municipal Government, the actual fiscal revenue is 872.8 billion yuan, and the deviation between the predicted fiscal revenue and the actual fiscal revenue is 24.656 billion yuan and the deviation is -2.75%. It shows that the prediction results of the model are effective and the analysis results are accurate.

4.4. Growth Rate and Volatility

Substituting Shanghai's fiscal guarantee income into formulas 6 and 7, $\mu = 0.143565$, $\sigma = 0.156867$, $\sigma^2 = 0.024601$, we can see that Shanghai's fiscal revenue is growing and the volatility is low, indicating that the economic level of Shanghai has improved significantly in the past ten years. It is closely related to attracting investment, technological research and development and innovation, as well as supporting the development of all kinds of enterprises.

4.5. Default Distance and Probability of Default

The default distance is calculated according to the formula of KMV model:

$$DTD = \frac{\text{Asset Value} - \text{Liability Value}}{\text{Asset Volatility} \times \sqrt{\text{Time to Maturity}}} \quad (11)$$

Among them, the asset value is based on market valuation, the liability value is the current total debt, the asset volatility is derived from historical price data, and the maturity time is determined according to the maturity date of the bond.

Taking Shanghai's financial guarantee income, the volatility and growth rate of financial guarantee income and the actual maturity scale of urban investment bonds in 2024 into Formula 8 and 9, we get $DD = 3.4525$, $p = 0.105$.

Taking the default distance and default probability as the bridge, the scale of Shanghai's security debt is 660.034 billion yuan, and the actual repayment scale is 129.243 billion yuan.

When the actual debt repayment scale is similar to or even larger than safe debt scale, the default risk of the province is higher, and the credit risk of Shanghai is lower combined with the default probability.

5. Conclusion

Local economic and financial strength, corporate capital structure, profitability and debt factor are the main factors affecting the credit spreads of urban investment bonds. The default risk of Shanghai's urban investment bonds is low, which is closely related to Shanghai's strong economic strength and sound financial situation. It is suggested that local governments should continue to optimize the structure of financial expenditure, improve the efficiency of the use of financial funds, and strengthen the supervision of urban investment companies to ensure their financial health and solvency. For investors, Shanghai urban investment bonds are a relatively safe investment choice, but attention should be paid to diversification of investment to reduce risk. This study is mainly based on historical data, and future research can consider more macroeconomic factors and market dynamics to improve the prediction accuracy and applicability of the model.

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