

Optimal Risk Control of Complex Supply Chain Networks under Public Emergencies

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Abstract: Public emergency is an event that occurs suddenly, has a wide range of influence, and brings great impact to the society. It has the characteristics of suddenness, publicity, urgency and uncertainty. Frequent public emergencies pose a great threat to supply chain security, causing serious economic losses and social impacts. In order to improve the ability of supply chain network to cope with public emergencies, it is of great significance to study the law of supply chain risk transmission and risk control measures. Through the review of the relevant theories of supply chain sudden risk and infectious disease model, the existing problems and research prospects are pointed out. Then, based on the SEIR model, a complex supply chain network risk control model under government intervention is constructed to explore the transmission mechanism of supply chain risk and the optimal control rate of the government. Finally, the simulation verifies that by controlling the basic reproduction number, the government can effectively curb the further spread and diffusion of risks, and achieve the goal of controlling risks and balancing cost control.

Keywords: Public Emergencies, Supply Chain Risk, Infectious Disease Model, Optimal Control.

1. Introduction

In recent years, international trade cooperation has shown a deepening trend, economic globalization and the rapid development of information technology have made the links between enterprises in the global industrial chain increasingly close. Initially, the supply chain presents a chain structure, in which logistics, capital flow and information flow form a chain structure among manufacturers, distributors, retailers and consumers. With the increasing interdependence of global economy, supply chain becomes more and more complex, which promotes the transformation of supply chain from traditional chain structure to supply chain network structure.

With the network of supply chain system, the risk of supply chain becomes complicated, and the management of supply chain network is more and more challenging. The complexity of supply chain network makes it more vulnerable to public emergencies. Supply chain risk comes from the existence of various uncertain factors. Due to the strong dependence of enterprises on the supply chain network, the problem of any enterprise is likely to affect the normal operation of other enterprises, and even lead to the breakdown of the supply chain. When public emergencies (such as COVID-19 in 2020, the Suez Canal blockage in 2021, the Russia-Ukraine conflict in 2022, etc.) occur, the supply chain network will face various risks, such as disruption risk, financial risk, and logistics delay risk. The transmission of these risks is usually distributed. A risky link can pass the risks to neighboring enterprises through the relationship between enterprises, and then to other neighboring enterprises. Like dominoes, its transmission will have a far-reaching impact on the entire supply chain network, which may lead to the paralysis of the entire supply chain network, causing serious economic losses and social impacts. Therefore, how to carry out emergency management under public emergencies has received great attention. In this case, more and more scholars began to pay attention to the study of sudden risks in the supply chain.

2. Literature Review

2.1. Supply Chain Emergency Risk Management

At present, the research on supply chain emergency risk management is mainly carried out from the perspectives of risk assessment, risk identification, risk avoidance and coordination strategy. Among them, Kleindorfer et al [1] proposed a conceptual framework for risk assessment and risk mitigation by considering the risk of emergencies caused by natural disasters, strikes and economic damage caused by human activities, including terrorists, to lay a foundation for supply chain emergency risk management. Yu et al [2] studied the emergency management measures of supply chain to deal with emergencies under wholesale price contract and repurchase contract. Pang [3] studied the optimal coordination strategy of supply chain system to cope with emergencies when market demand changes due to emergencies. Giri et al [4] propose that the risk of production disruptions in supply chain networks can be circumvented by designing buyback and revenue sharing contracts. Ivanov et al [5] proposed that in the context of the COVID-19 pandemic, the global supply chain has been hit, and the need to study supply chain resilience to mitigate the risks posed by such events. Hosseini et al [6] used Markov chain and dynamic Bayesian network model to simulate the vulnerability and resilience of suppliers, divided suppliers into three states of full operation, semi-interruption and complete interruption, and quantified the disruption risk propagation process of the secondary supply chain network, so as to improve the resilience and risk response ability of the supply chain.

The above studies mostly put forward measures and suggestions to deal with sudden risks from the perspective of enterprises themselves, and rarely focus on the risk control measure of supply chain network from the perspective of external environment such as government.

2.2. Infectious Disease Model

2.2.1. General Application of Infectious Disease Model

In the field of transmission dynamics, many scholars have conducted research on infectious disease models. The SIR Infectious disease model constructed by Kermack et al [7] laid a foundation for the study of infectious disease dynamics. With the gradual deepening of research, many scholars have found that the process of information transmission is similar to the process of virus transmission, and the infectious disease model has been applied to the dissemination of public opinion, knowledge and rumors on complex networks. For example, Wang et al [8], Zhu et al [9], Ding et al [10]. Similarly, May et al [11] proposed that the transmission process of financial risks is similar to the transmission mechanism of infectious diseases, so the infectious disease model was applied to depict the transmission of financial risks. Garas et al [12] introduced the SIR Infectious disease model into the crisis contagion process in the global economic network crisis and conducted simulation.

In the above research, the infectious disease model has played an important role in many fields. The risk transmission process in the supply chain network is similar to the spread of infectious disease in the population, so the analysis idea of the infectious disease model can be used to analyze the risk transmission process in the supply chain network.

2.2.2. Infectious Disease Model in Supply Chains

Due to the similarities between the transmission of viruses and the transmission of supply chain risks in terms of transmission subjects, media, transmission process and transmission mode, more and more scholars have begun to pay attention to the application of infectious disease models in supply chain. For example, Yang et al [13] introduced the SIS model of communication dynamics theory into the study of supply chain risk communication for the first time and established the dynamic model of risk communication. Liang et al [14] applied the SIR Model to dynamically identify and predict the risk states of supply chain risks at different moments. Zhang et al [15] introduced the SEIRS model into the credit risk propagation problem of supply chain finance. Wang et al [16] built a SIRS supply chain risk transfer model based on the actual situation that node-type enterprises were eliminated in the supply chain network. Berger et al [17] demonstrated how the supply chain network structure affects the transmission of enterprise cargo quality risks by establishing an SIS model. The application of the above infectious disease model in supply chain network has obtained a series of high-quality research results and demonstrated the applicability of the infectious disease model in this research field.

2.3. Current Problems and Further Work

According to the literature published in recent years, there are the following problems in the study of sudden risk in supply chain: (1) Most of the existing researches on supply chain emergency risk management are based on the microscopic level, mostly focusing on a single specific linear supply chain or a simple supply chain network, while rarely involving the complex supply chain network of multi-projects, multi-partners, multi-levels, multi-suppliers, multi-manufacturers and multi-sellers. There are even studies of complex supply chain networks where multiple industrial chains are intertwined; (2) Most studies on supply chain emergency risk management put more emphasis on

enterprises within the supply chain, and propose strategies to deal with emergency risks from the perspective of enterprises themselves. However, in reality, due to the public nature of public emergencies, the government will often participate in the supply chain assistance after the outbreak of public emergencies. Therefore, the macro-control role of the government cannot be ignored.

Based on the previous analysis, this paper believes that the following aspects can be discussed in the future study of supply chain network risk: (1) From the perspective of research objects, the rise of economic globalization has made the supply chain network show the characteristics of a complex network. In the future study, the complex supply chain network interwoven by multiple suppliers, manufacturers and sellers can be studied; (2) From the perspective of research methods, the SEIR infectious disease model is applied to the research of supply chain network, and the analysis ideas of SEIR model are used for reference, the government's intervention in supply chain network is introduced into the model for improvement, and the risk transmission in complex supply chain network is simulated and analyzed.

3. Model

3.1. Model Assumption

For the construction model constructed in this paper, there are the following assumptions:

H1: In the model, each chamber is regarded as a node in the network, the communication relationship between each chamber is represented by the edge between the nodes, and the risk is propagated through the transaction between enterprises.

H2: According to the process status of risk transmission, all node enterprises in the supply chain network are divided into four categories: risk susceptible enterprises (S), risk exposed enterprises (E), risk infected enterprises (I) and risk recovery enterprises (R), and the state transformation is carried out according to a certain probability.

H3: In order to control risk transmission, the government intervenes in the management of risk latent enterprises, that is, timely assistance is provided to risk exposed enterprises, so as to block risk transmission and restore the supply chain network.

3.2. Model Building

Based on the above assumptions, the process of government intervention in complex supply chain network risk is shown in Figure 1.

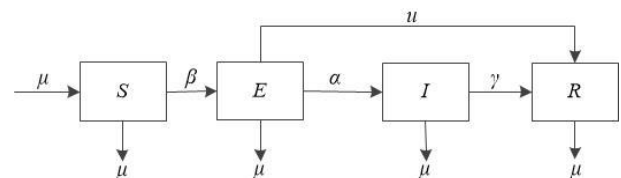


Figure 1. Process of government intervention in complex supply chain network risk

According to the construction idea of SEIR model, the mathematical model of differential equation is established, and the equation of state (1) of the system is obtained as follows:

$$\begin{cases} \frac{dS(t)}{dt} = \mu N(t) - \frac{\beta S(t)I(t)}{N(t)} - \mu S(t) \\ \frac{dE(t)}{dt} = \frac{\beta S(t)I(t)}{N(t)} - \alpha E(t) - uE(t) - \mu E(t) \\ \frac{dI(t)}{dt} = \alpha E(t) - \gamma I(t) - \mu I(t) \\ \frac{dR(t)}{dt} = \gamma I(t) + uE(t) - \mu R(t) \end{cases} \quad (1)$$

In formula (1), μ is the entry rate of new enterprises and the exit rate of original enterprises; β is latent conversion rate; α is the infection rate; γ is recovery conversion rate; u is the government control rate; N is the total number of enterprises. The equation represents the relationship between the number of firms in each risk state and time t .

In order to further explore how government intervention can better control risk transmission in the supply chain network, the risk transmission threshold of system equation of state (1) will be analyzed below.

3.3. Risk Transmission Threshold

According to the theory of communication dynamics, there is a key point in any communication, that is, the basic regeneration number R_0 , which is crucial in the process of risk communication and determines the final result of risk communication. For the purpose of this study, the basic regeneration number is the average of how many firms a risk-infected firm will infect. When $R_0 < 1$, the risk will gradually disappear during the transmission process, and the number of risk-infected enterprises is zero. On the contrary, the risk will gradually spread in the transmission process, and the number of risk-infected enterprises is not zero. Therefore, R_0 can be used as a threshold for whether the risk spreads during transmission. Using the "next generation matrix method [18]" to solve the risk propagation threshold, the R_0 expression can be obtained as follows:

$$R_0 = \frac{\alpha\beta}{(\alpha + \mu + u)(\mu + \gamma)} \quad (2)$$

As can be seen from equation (2), R_0 is a decreasing function of u , and the larger u is, the smaller R_0 is. The government controls R_0 by regulating the size of u , which is an effective means to block the risk transmission and restore the supply chain network. However, the larger the u , the higher the cost of government intervention, and it is not feasible economically to intervene regardless of cost in order to block the spread of risk. Therefore, in order to balance this contradiction, the following will seek an optimal control measure.

3.4. Optimal Control

In risk control, considering that R_0 has a great impact on the risk transmission of the supply chain, the smaller R_0 is, the slower the risk transmission will be. Therefore, this section establishes the following objective function:

$$J(u) = R_0 + \frac{C}{2}u^2 \quad (3)$$

Where, C is the weight coefficient, which can also be regarded as the balance coefficient between R_0 and government control cost. Our goal is to solve for an optimal u such that:

$$J(u^*) = \min_{0 \leq u \leq 1} J(u) \quad (4)$$

Substituting the expression for R_0 into the objective function (3) yields:

$$J(u) = \frac{\alpha\beta}{(\alpha + \mu + u)(\mu + \gamma)} + \frac{C}{2}u^2 \quad (5)$$

To get the threshold of risk control u^* , we need to know:

$$\frac{\partial J}{\partial u} = -\frac{\alpha\beta}{(\alpha + \mu + u)^2(\mu + \gamma)} + Cu = 0 \quad (6)$$

As can be seen from the above equation (6), when $\alpha\beta = Cu(\mu + \gamma)(\alpha + \mu + u)^2$ is satisfied, u^* minimizes the value of the objective function, that is, when $u = u^*$, the government can optimize the control cost and risk control effect when it is economically feasible.

In order to study which factors, affect the government's decision on the control rate, the sensitivity analysis of u^* is carried out to provide the direction of risk control for government departments.

Property 1 $\partial u^*/\partial\beta > 0$, $\partial u^*/\partial\gamma < 0$, that is, with the decrease of latent conversion rate β , the increase of recovery conversion rate γ , the optimal government control rate u^* gradually decreases.

Proof

$$\frac{\partial u^*}{\partial\beta} = \frac{\alpha(\alpha + \mu + u^*)}{C(\alpha + \mu + u^*)^3(\mu + \gamma) + 2\alpha\beta} > 0;$$

$$\frac{\partial u^*}{\partial\gamma} = -\frac{\alpha\beta(\alpha + \mu + u^*)}{C(\alpha + \mu + u^*)^3(\mu + \gamma)^2 + 2\alpha(\mu + \gamma)\beta} < 0;$$

Property 2 If $\alpha < \mu + u^*$, then $\partial u^*/\partial\alpha > 0$, that is, with the increase of infection conversion rate α , the optimal government control rate u^* gradually increases. On the contrary, if $\alpha > \mu + u^*$, then $\partial u^*/\partial\alpha < 0$, that is, with the increase of infection conversion rate α , the optimal government control rate u^* gradually decreases.

Proof

$$\frac{\partial u^*}{\partial\alpha} = \frac{\beta(\mu - \alpha + u^*)}{C(\alpha + \mu + u^*)^3(\mu + \gamma) + 2\alpha\beta},$$

therefore, if $\alpha < \mu + u^*$, then $\partial u^*/\partial\alpha > 0$. On the contrary, if $\alpha > \mu + u^*$, then $\partial u^*/\partial\alpha < 0$.

4. Simulation

In order to explore the optimal measure of government intervention in supply chain network node enterprises under public emergencies, MATLAB(R2016a) software was used for numerical simulation. Let $\mu = 0.02$, $\beta = 0.3$, $\alpha = 0.45$, $\gamma = 0.2$ and $C = 5$.

In the absence of government intervention, substituting the above parameter values into R_0 , $R_0 = 1.3056 > 1$ is obtained. Therefore, the risk will spread in the supply chain network. The number of four types of enterprises without government intervention is shown in Figure 2.

As can be seen from Figure 2, the number of risk susceptible enterprises experienced an oscillation process of decreasing and increasing, and finally stabilized at 922 in 365 days. The number of risks exposed enterprises went through

the oscillating process of increasing and decreasing, and finally stabilized at 12 in 290 days. After the oscillating process of decreasing and increasing, the number of risk infected enterprises finally stabilized at 24 in 313 days. After an oscillating process of increase and decrease, the number of risk recovery enterprises finally stabilized at 242 in 372 days.

According to Section 3.4, the optimal control rate u^* is 0.2421, where $R_0 = 0.8617 < 1$. It can be seen that government intervention prevents the risk from spreading in the supply chain network. The number of four types of enterprises under optimal control is shown in Figure 3.

As can be seen in Figure 3, the number of risk-susceptible enterprises showed a trend of rapid decline at first, and then a slow rise, reaching the lowest point of 838 on the 21st day, recovering to 1200 after 375 days, and the risk transmission disappeared and was cleared. The number of enterprises at risk of infection was controlled by the basic regeneration number less than 1, which showed a gradually decreasing trend, and the infection scale did not expand further, and gradually approached 0 from the 145th day. The number of risk latent enterprises peaked at 36 on the third day, then decreased, and approached zero after 117 days. The number of risk recovery enterprises first increased and then decreased, reaching 324 on the 26th day and approaching zero after 400 days.

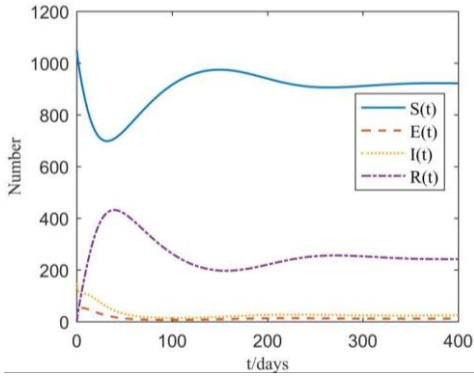


Figure 2. The number of four types of enterprises without government intervention

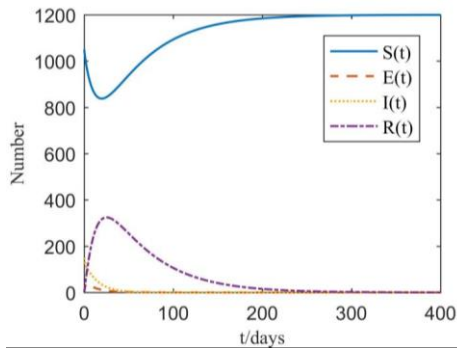


Figure 3. The number of four types of enterprises under optimal control

From Property 1 and Property 2, it can be known that β , α and γ all have an effect on u^* . The sensitivity analysis experiment will be done below.

The effect of β on u^* is shown in Figure 4. As can be seen from Figure 4, with the increase of latent conversion rate β , the optimal control rate u^* also increases.

The effect of α on u^* is shown in Figure 5. As can be seen from Figure 5, with the increase of infection conversion rate α , the optimal control rate u^* shows a trend of first increasing

and then decreasing.

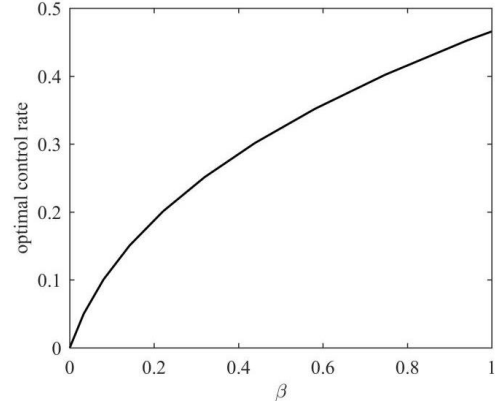


Figure 4. The effect of β on u^*

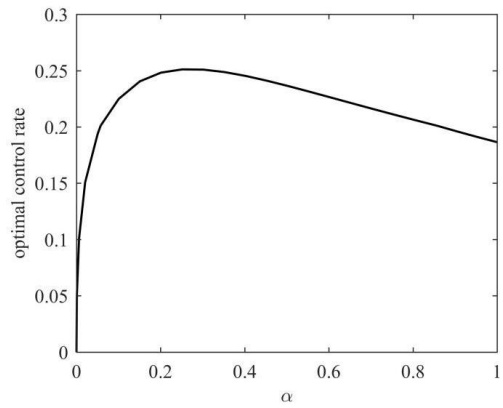


Figure 5. The effect of α on u^*

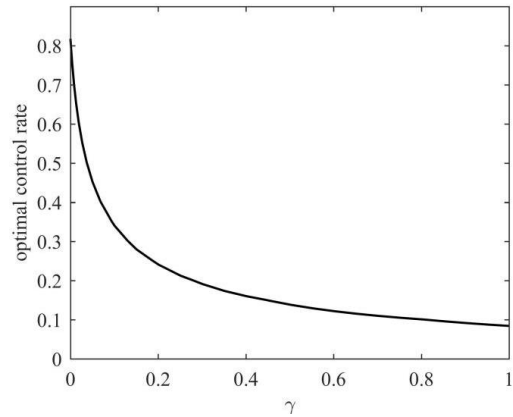


Figure 6. The effect of γ on u^*

The effect of γ on u^* is shown in Figure 6. As can be seen from Figure 6, the greater the recovery conversion rate γ , the smaller the optimal control rate u^* , and with the increase of the recovery conversion rate γ , the speed of the decrease of the optimal control rate u^* becomes slower and slower. In summary, the effect of recovery conversion rate γ on optimal control rate u^* is more significant.

5. Conclusion

This paper studies the optimal risk control of complex supply chain network under government intervention when public emergencies break out. Based on the SEIR model, the differential equation mathematical model is established, and the basic regeneration number theory is used to analyze the risk transmission mechanism of supply chain risk and explore

the best intervention measures of the government.

It is found that there is a threshold for risk propagation, that is, the basic regeneration number R_0 . When R_0 is less than 1, the risk of public emergencies disappears in the supply chain network. When R_0 is greater than 1, the risk of public emergencies spreads in the supply chain network, leading to disasters such as fund break and logistics interruption. In addition, government intervention has an obvious restraining effect on the spread of risks in the supply chain network, and the latent conversion rate β , the infection conversion rate α , the recovery conversion rate γ all have different degrees of influence on the government's risk control measures

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