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Risk Control in Complex Supply Chain Networks Under Sudden Public Emergencies

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Abstract: After the occurrence of sudden public emergencies, risks propagate throughout the entire supply chain network, triggering a domino effect. Therefore, determining appropriate strategies to intervene in and control risks, thereby blocking or mitigating their spread, becomes critically important. This study constructs a risk control model for complex supply chain networks under sudden public emergencies based on the SEIR system dynamics propagation model, incorporating risk immunization strategies and government participation in risk control. The proposed model adopts a target control approach to manage risk dissemination, and the effectiveness of this approach is verified through simulation experiments. The findings reveal that target control can effectively contain risk propagation, and enhancing the risk immunity of supply chain networks during sudden public emergencies significantly reduces the extent of risk diffusion while lowering the cost of government intervention.

Keywords: Sudden public events, supply chain risk, system dynamics propagation model, target control.

1. Introduction

Under the context of economic globalization, supply chain networks have become increasingly complex. While the complexity of supply chain networks brings new markets and business opportunities to enterprises at various levels, it also raises concerns about supply chain vulnerability. When a risk event occurs at a node enterprise within the network, the risk spreads throughout the entire supply chain, triggering a domino effect that threatens the stability of the entire network. This issue has become particularly prominent in recent years due to the frequent occurrence of sudden public emergencies. For instance, during the COVID-19 pandemic, industries with long and complex supply chains, such as the automotive and aerospace sectors, experienced significant supply chain disruptions. Public reports indicate that more than 100 global automobile manufacturers and parts suppliers halted production, while Boeing, one of the world's leading commercial aircraft manufacturers, suspended operations at its plant in Washington State, USA. This, in turn, forced multiple Japanese suppliers of Boeing aircraft components to cease production as well. In response to these supply chain risk issues, the report of the 20th National Congress of the Communist Party of China emphasized the need to pursue high-quality development as the central theme, focusing on improving total factor productivity, enhancing the resilience and security of industrial and supply chains, and promoting substantive improvements in economic performance alongside reasonable quantitative growth.

In existing research, numerous scholars have conducted a series of studies on the emergence and propagation of supply chain risks under sudden events. Sheffi et al. (2001) [1] proposed strategies for minimizing the impact of terrorism-related events on supply chains. Chopra et al. (2004) [2] classified sudden risks into categories such as natural disasters, labor disputes, supplier bankruptcies, wars, and terrorism. Kleindorfer et al. (2005) [3], focusing on the outcomes of risks, categorized sudden event risks as those leading to increased supply chain costs or even collapse due to disruptions. Stephan et al. (2006) [4] examined the relationship between supply chain vulnerability and the occurrence of supply chain risks during sudden public events. Tomlin et al. (2009) [5] analyzed how to balance and select between various preventive and emergency measures in supply chain risk management during sudden events. Lu et al. (2021) [6] identified relevant indicators in the construction industry that can serve as early warnings for supply chain risks caused by sudden public events. Hohenstein et al. (2022) [7] specifically analyzed effective supply chain risk management measures during the COVID-19 pandemic, a significant public emergency. The aforementioned studies provide valuable insights and methodologies for understanding the propagation of supply chain risks and implementing measures to control the spread of such risks.

Due to the similarities in the entities, mediums, and transmission methods between virus propagation and risk dissemination, many scholars argue that the patterns of risk propagation in supply chain networks exhibit similarities to the mechanisms of infectious diseases. In the early 20th century, Kermack and McKendrick introduced the renowned infectious disease model, known as the Kermack-McKendrick model. This model is based on the number of susceptible (S), infected (I), and recovered (R) individuals in a population, and considers the transitions between these categories. Building on this, Zhang et al. (2003) [8] proposed the SEIR model, which includes exposed individuals (E) and demonstrated the threshold role of the basic reproduction number in infectious disease models. When intervening in infectious disease models, it is necessary to account for a variety of factors, making the identification of optimal control strategies critical. Vaccination has long been considered one of the most effective methods. Dietz and Schenzle et al. (1985) [9] noted that real-world infectious disease models must incorporate vaccination programs. Kribs-Zaleta and Martchevab (2002) [10] discussed a simple SIS vaccination model and proved that vaccination can successfully control disease spread when the basic reproduction number is reduced to less than 1. Meng et al. (2022) [11] demonstrated the control effect of vaccination on SEIR models with nonlinear infection rates. Furthermore, in the context of controlling and assisting with infectious disease models, optimal control theory is often applied. Revell et al. (1967) [12] first introduced optimal control theory in the context of infectious disease control, focusing on minimizing the cost of control with optimal strategies. Bruno et al. (2011) [13] discussed how to use optimal control vaccination strategies in SEIR models to minimize both disease burden and intervention costs. Ahmad et al. (2016) [14] applied optimal control theory to the control of the Ebola outbreak using immunization strategies and developed the corresponding SEIR model. Olivares et al. (2021) [15] addressed the optimal control problem for COVID-19 based on evolving vaccination strategies.

Unlike previous studies, this paper utilizes the similarities between supply chain network risk propagation and infectious disease transmission, applying the infectious disease system dynamics model and optimal control theory to the study of supply chain network risk propagation. The model is further improved to construct a theoretical framework for risk propagation and control in supply chain networks under sudden public events. An immunization-based risk control approach is proposed to reduce the propagation of risks within the supply chain network. Through simulation experiments, the paper models the risk propagation process under different control scenarios, recording changes in the number of various types of enterprises. The study adopts a combination of theoretical reasoning and system simulation to analyze the effects of different government control methods, and based on this analysis, suggests measures for preventing and controlling the propagation of risks in supply chain networks. This research aims to provide valuable reference for governmental and other functional departments.

2. Model

2.1. Hypothesis

The complex supply chain network in this paper refers to a supply chain system composed of a large number of interconnected enterprises, forming a network structure. The complexity of this supply chain network is mainly reflected in two aspects: quantity and structure. Specifically, in terms of quantity, the network involves a vast number of enterprises, resulting in a large-scale network. In terms of structure, due to differences in enterprise size and connectivity, different node enterprises exhibit varying levels of resilience to risk, presenting heterogeneous characteristics. The parameters involved in the model are listed in Table 1.

Table 1. Related parameters of SEIR model

Parameter	Description
μ	The entry rate of new enterprises and the exit rate of existing enterprises.
β_1	The contact infection rate of enterprises with strong risk resistance.
β_2	The contact infection rate of enterprises with weak risk resistance.
m	The proportion of enterprises with strong risk resistance.
α	The infection transition rate.
γ	The recovery transition rate.
ρ	The immunization control rate.

Hypothesis 1. The complex supply chain network can be viewed as a relatively open system, where existing enterprises may exit, and new enterprises continually join, resulting in the replacement of some enterprises. It is assumed that the exit rate of existing enterprises is equal to the entry rate of new enterprises. As a result, the supply chain network exhibits a continuous updating characteristic.

Hypothesis 2. The supply chain risk discussed in this paper is characterized by undirected propagation. When an enterprise in the network is affected by risk, this risk can simultaneously spread both upstream and downstream in the supply chain, infecting other enterprises connected to it. Thus, the risk can propagate in an undirected manner throughout the supply chain.

Hypothesis 3. In order to study the risk propagation process, the SEIR system dynamics transmission model is employed. The enterprises in the supply chain network are divided into four categories: susceptible enterprises (S), exposed enterprises (E), infected enterprises (I), and immune enterprises (R). It is also assumed that susceptible enterprises do not possess risk immunity before the government implements an immunization strategy. The four categories of enterprises undergo state transitions according to certain probabilities.

Hypothesis 4. Due to differences in the size and management capabilities of node enterprises, different enterprises in the supply chain network have varying abilities to withstand risks. Larger enterprises generally possess more resources and capabilities, enabling them to better resist risk shocks. In contrast, smaller enterprises are relatively more vulnerable and are more easily affected by the spread of risks. Therefore, susceptible enterprises (S) are further divided into two categories: those with strong risk resistance and those with weak risk resistance. The probability of infection differs when different risk-sensitive groups come into contact with infected enterprises. Thus, different infection rates are adopted to represent the infection levels of these two groups.

Hypothesis 5. Considering the characteristics of sudden public events, which typically have a significant impact and widespread effects, it is assumed that after the occurrence of such an event, multiple infected enterprises coexist simultaneously within the supply chain network, leading to a scenario where risks propagate from multiple sources at the same time. This assumption more accurately reflects the impact of sudden public events on the supply chain network and provides a reasonable foundation for subsequent research and analysis.

Hypothesis 6. To control the propagation of risks, the government implements immunization measures for certain risk-sensitive enterprises, providing financial and policy support to enhance the immunity of these enterprises against risks. The government incurs control costs, which, similar to previous studies, are positively correlated with the square of the immunization rate. This implies that as the immunization rate increases, the control costs rise disproportionately.

Under the immunization strategy, the conceptual model of risk propagation in the complex supply chain network is shown in Figure 1.

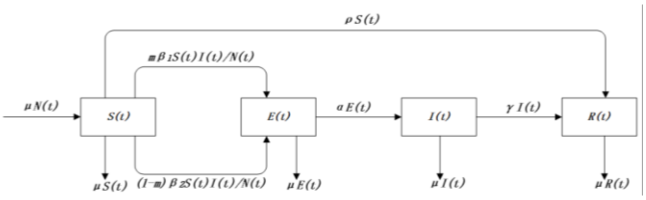

Figure 1. Risk propagation model of supply chain network

2.2. Model Construction

Based on the above assumptions, the SEIR system dynamics risk propagation model for complex supply chain networks under sudden public events is constructed, as shown in Equation (1). Equation (1) represents the relationship between the number of enterprises in each risk state and the time t.

$$\begin{cases} \dfrac{dS(t)}{dt} = \mu N(t) - \dfrac{[m\beta_1 + (1-m)\beta_2]S(t)I(t)}{N(t)} - \rho S(t) - \mu S(t) \\ \dfrac{dE(t)}{dt} = \dfrac{[m\beta_1 + (1-m)\beta_2]S(t)I(t)}{N(t)} - \mu E(t) - \alpha E(t) \\ \dfrac{dI(t)}{dt} = \alpha E(t) - \gamma I(t) - \mu I(t) \\ \dfrac{dR(t)}{dt} = \gamma I(t) + \rho S(t) - \mu R(t) \end{cases} \tag{1}$$

After the occurrence of a sudden public event, some node enterprises are affected and become risk-infected enterprises, i.e., the sources of risk infection. These enterprises then propagate the risk throughout the supply chain network, leading to its spread. The number of various types of enterprises evolves over time according to a system of nonlinear differential equations (referencing Equation (1)). Since solving nonlinear differential equations analytically is generally quite difficult and obtaining precise analytical expressions is challenging, the focus is placed on the steady-state characteristics of the system over long time scales, rather than on precisely capturing each instantaneous state. In system dynamics propagation theory, the concepts of equilibrium points and their stability are commonly applied to study the trends and characteristics of the differential equation system.

The following Equation (2) implies that $dS/dt = 0, dE/dt = 0. dI/dt = 0, dR/dt = 0$, at which point the system is in a steady state.

$$\begin{cases} \dfrac{dS(t)}{dt} = \mu N(t) - \dfrac{[m\beta_1 + (1-m)\beta_2]S(t)I(t)}{N(t)} - \rho S(t) - \mu S(t) = 0 \\ \dfrac{dE(t)}{dt} = \dfrac{[m\beta_1 + (1-m)\beta_2]S(t)I(t)}{N(t)} - \mu E(t) - \alpha E(t) = 0 \\ \dfrac{dI(t)}{dt} = \alpha E(t) - \gamma I(t) - \mu I(t) = 0 \\ \dfrac{dR(t)}{dt} = \gamma I(t) + \rho S(t) - \mu R(t) = 0 \end{cases} \tag{2}$$

If $E = I = 0$ at this point, it means that the risk in the system will have disappeared, and this equilibrium point is referred to as the zero equilibrium point. It can be easily derived that:

$$X^{(0)} = (S^{(0)}, E^{(0)}, I^{(0)}, R^{(0)}) = (\frac{\mu N}{\mu + \rho}, 0, 0, \frac{\rho N}{\mu + \rho}) \tag{3}$$

Conversely, if $E \neq 0, I \neq 0$ at this point, it indicates that the number of risk-infected enterprises in the system remains constant, and the risk does not disappear. In this case, a non-zero equilibrium point $X^* = (S^*, E^*, I^*, R^*)$ is obtained.

$$S^* = \frac{\mathrm{N}(\gamma+\mu)(\alpha+\mu+\rho)}{[\mathrm{m}\beta_1 + (1\text{-m})\beta_2]\alpha} \tag{4}$$

$$E^* = \frac{(\gamma+\mu)(\mu N - \rho S^* - \mu S^*)}{\alpha[m\beta_1 + (1-m)\beta_2]S^*} \tag{5}$$

$$I^* = \frac{N(\mu N - \rho S^* - \mu S^*)}{[m\beta_1 + (1-m)\beta_2]S^*} \tag{6}$$

$$R^* = \frac{N\gamma(\mu N - \rho S^* - \mu S^*)}{[m\beta_1 + (1-m)\beta_2]S^*\mu} + \frac{\rho S^*}{\mu} \tag{7}$$

At this point, it indicates that once the system reaches the non-zero equilibrium point, the system is at the critical point of risk propagation. The number of infected enterprises remains stable, and the risk will neither spread further nor fade away.

2.3. Target Control

In target control, transforming susceptible enterprises into immune enterprises requires a cost, which is positively correlated with the square of the immunization control rate and is given by $A\rho^2/2$, where A is the cost coefficient. On one hand, increasing the immunization control rate reduces the basic reproduction number, thereby reducing risk propagation; on the other hand, increasing the immunization control rate leads to a sharp rise in control costs. The optimal target control decision is to find the immunization control rate that minimizes the sum of control costs and the basic reproduction number. The objective function expression is as follows:

$$J = \frac{A}{2}\rho^2 + R_0 \tag{8}$$

The latter part of the objective function represents the basic reproduction number R_0, and the magnitude of the cost coefficient A reflects both the size of the control costs and the relative importance of control costs compared to the basic reproduction number. The larger the value of A, the higher the control costs, and at the same time, the more significant the control costs become relative to the basic reproduction number. This paper uses the next-generation matrix method to calculate the basic reproduction number, and the result for the basic reproduction number of the model is:

$$R_0 = \frac{[\mathrm{m}\beta_1 + (1-m)\beta_2]\mu\alpha}{(\mu+\rho)(\alpha+\mu)(\gamma+\mu)} \tag{9}$$

Equation (9) reflects the average number of enterprises that one infected enterprise in the system dynamics propagation model will infect. It is crucial in the risk propagation process, as whether the basic reproduction number is greater than 1 determines whether the risk will continue to spread within the supply chain network or gradually dissipate.

Property 1 In target control, the optimal immunization control rate ρ^* satisfies $A\rho^*(\mu + \rho^*)^2 = [m\beta_1 + (1-m)\beta_2]\mu\alpha/[(\alpha + \mu)(\gamma + \mu)]$.

Proof: Taking the first derivative of Equation (8) with respect to ρ gives:

$$\frac{dJ}{\partial\rho} = A\rho - \frac{[\mathrm{m}\beta_1 + (1-m)\beta_2]\mu\alpha}{(\mu+\rho)^2(\alpha+\mu)(\gamma+\mu)} \tag{10}$$

Taking the second derivative with respect to ρ gives:

$$\frac{d^2J}{d\rho^2} = A + \frac{2(m\beta_1 + (1-m)\beta_2)\mu\alpha}{(\mu+\rho)^3(\alpha+\mu)(\gamma+\mu)} \tag{11}$$

According to Equation (11), it can be concluded that the second derivative is always greater than 0. Since ρ^* is the point that satisfies $J'(\rho) = 0$, ρ^* is a minimum point, and $J'(\rho^*) = 0$ represents the minimum value.

This property indicates that under the target control strategy, as long as the government's assistance cost is within the critical value range, there will inevitably be an optimal control rate that minimizes the sum of the assistance cost and the risk diffusion degree. This provides a basis for the government to formulate the optimal target control assistance strategy.

3. Simulation Experiment

In order to thoroughly investigate the effectiveness of different control methods on supply chain risk propagation, simulation experiments were conducted using MATLAB software based on relevant data. The trends of enterprise states over time under three scenarios—no intervention, target control, and process control—were analyzed to assess the degree of risk development and to examine the impact of government intervention measures on risk propagation within the supply chain network. The focus of the analysis was on the relative values of parameters and results, rather than their absolute values, to highlight the practical significance of the simulation outcomes. In the simulation experiment, the key parameters for the supply chain network were set as follows: the entry rate of new enterprises and the exit rate of existing enterprises μ=0.02, the latent conversion rate for enterprises with high risk resistance β1=0.3, the latent conversion rate for enterprises with low risk resistance β2=0.4, the proportion of enterprises with high risk resistance m=0.5, the infection transition rate α=0.45, and the recovery rate γ=0.15. The cost coefficient for target control was set to A=20. The number of enterprises in the supply chain network was set to N=1200. In the initial stage after the occurrence of the sudden public event, 150 enterprises were infected by risk and became risk-infected enterprises, and the risk then began to spread from these enterprises to other associated enterprises.

In the simulation experiment, the supply chain network was simulated under three different control methods: no control, target control, and process control. The control rates and basic reproduction numbers corresponding to each control method are shown in Table 2.

In the case of no control, the basic reproduction number of the supply chain network is greater than 1, indicating that in the absence of government intervention, the risk within the supply chain network will spread and eventually stabilize at a certain level. However, if the target control method is applied to intervene in the supply chain network, the optimal control rate can be calculated, which shows that the government should continuously provide assistance to approximately 11% of the risk-sensitive enterprises, transforming them directly into risk-immune enterprises. Under this scenario, the basic reproduction number of the supply chain network with target control is less than 1, and the risk within the network will gradually dissipate due to the intervention.

Table 2. Optimal control rate and basic reproduction number under different control strategies

Risk Control Methods	ρ	R_0
No Control	0	1.971
Target Control	0.11	0.303

3.1. Risk Enterprises under Different Control Schemes

Risk-sensitive enterprises are the main target of government assistance. The government adopts a series of measures to directly convert a portion of the risk-sensitive enterprises into risk-immune enterprises. By reducing the number of risk-sensitive enterprises, the number of risk-exposed enterprises generated from contact with risk-infected enterprises is indirectly reduced, which in turn lowers the number of risk-infected enterprises and ultimately eliminates the risk. Therefore, the changes in the number of risk-sensitive enterprises reflect the effectiveness of different control strategies. The specific situation is shown in Figure 2. Under the no-control scenario, the number of risk-sensitive enterprises rapidly decreases due to the continuous spread and expansion of risk in the network, reaching a minimum of 420 enterprises by period 34. Afterward, the number fluctuates slowly and eventually stabilizes at around 608 enterprises by period 300. In the case of target control, a fixed proportion of risk-sensitive enterprises are directly converted into risk-immune enterprises. As a result, the decrease in the number of risk-sensitive enterprises under target control occurs more rapidly compared to the no-control scenario. By period 60, the number stabilizes at around 180 enterprises, with a shorter time required to reach the stable state.

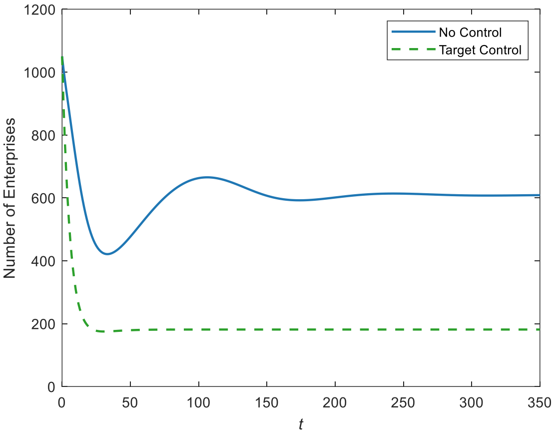

Figure 2. Risk-Sensitive Enterprises under Different Control Scenarios

The generation of risk-exposed enterprises occurs because of the interactions between risk-sensitive enterprises and risk-infected enterprises, where the risk spreads to a portion of the risk-sensitive enterprises, converting them into risk-exposed enterprises. At this point, although these enterprises have been infected and will eventually turn into risk-infected enterprises, risk-exposed enterprises do not infect risk-sensitive enterprises. The number of risk-exposed enterprises determines the number of risk-infected enterprises in subsequent periods and, to some extent, represents the level of risk in the network. The changes in the number of risk-exposed enterprises under different control scenarios are shown in Figure 3. In the case of no control, with the initial spread of risk, the number of risk-exposed enterprises reaches its peak in period 11, at 75 enterprises. Following this, as the number of risk-sensitive enterprises decreases, the number of risk-exposed enterprises begins to rapidly decline, dropping to 16 enterprises by period 76. As risk continues to spread through the supply chain network, the number fluctuates, ultimately reaching a stable state in period 300, with around 24 risk-exposed enterprises remaining. When target control is applied, the number of risk-exposed enterprises reaches its peak in period 4, at 50 enterprises. After reaching this peak, the number of risk-exposed enterprises starts to decrease, eventually reaching zero by period 60.

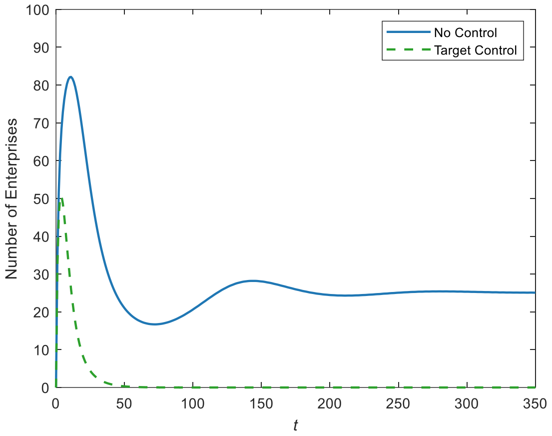

Figure 3. Risk- Exposed Enterprises under Different Control Scenarios

The number of risk-infected enterprises is the most intuitive reflection of the risk propagation situation within the supply chain network. After the occurrence of a sudden public event, risk-infected enterprises begin to exist, initially numbering 150. These risk-infected enterprises will infect risk-sensitive enterprises, and a portion of the transformed risk-sensitive enterprises will turn into risk-infected enterprises. The number of risk-infected enterprises represents the extent of risk diffusion. The changes in the number of risk-infected enterprises under different control scenarios are shown in Figure 4. In the case of no control, the number of risk-infected enterprises in the supply chain network will briefly decrease, then rapidly rise to the highest point. When risk propagation starts, the number of newly infected risk-sensitive enterprises is initially low, resulting in fewer risk-sensitive enterprises being converted into risk-infected enterprises. As the number of risk-sensitive enterprises increases, the number of risk-infected enterprises increases as well, reaching a peak of 201 enterprises in period 16. After this point, the number of risk-infected enterprises follows a pattern similar to that of risk-sensitive enterprises, first decreasing, then fluctuating, and finally stabilizing at a non-zero equilibrium point around period 300, with 67 risk-infected enterprises remaining. Under target control, the number of risk-infected enterprises shows a brief, rapid decline, followed by a slowdown in the decline rate and then another rapid decrease. Ultimately, under target control, the risk dissipates, and the number of risk-infected enterprises reaches zero in period 70, remaining stable thereafter with no further fluctuations.

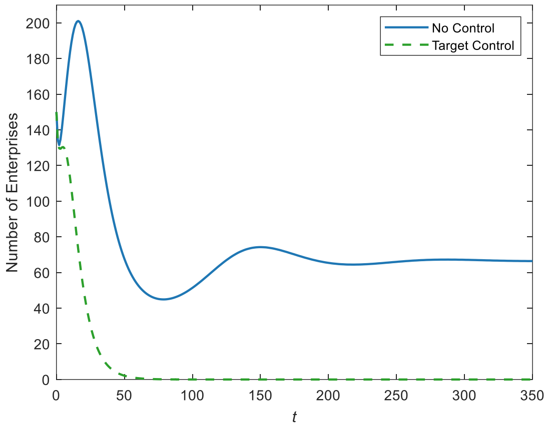

Figure 4. Risk- Infected Enterprises under Different Control Scenarios

Risk-recovered enterprises consist of two parts: one part recovers by itself after being infected, while the other part is directly converted into risk-immune enterprises through government assistance from risk-sensitive enterprises. The changes in the number of risk-recovered enterprises reflect the recovery of risk-infected enterprises and the extent of government aid. The variations in the number of risk-immune enterprises under different control scenarios are shown in Figure 5. Under the no-control scenario, risk-recovered enterprises rely entirely on self-recovery, and their number changes are directly related to the number of risk-infected enterprises. Both exhibit a pattern of initially rising to a peak, followed by a decline and fluctuation, eventually stabilizing near a non-zero equilibrium point. When the system reaches the non-zero equilibrium point, the number of risk-immune enterprises stabilizes at around 500, with a time span of 300 periods. Under the target control scenario, risk-immune enterprises consist of both self-recovered and government-aided types. In the early stages, since both risk-infected enterprises recover and government assistance is provided, the number of risk-immune enterprises rises rapidly. It peaks at 1,018 enterprises in period 60 and then stabilizes.

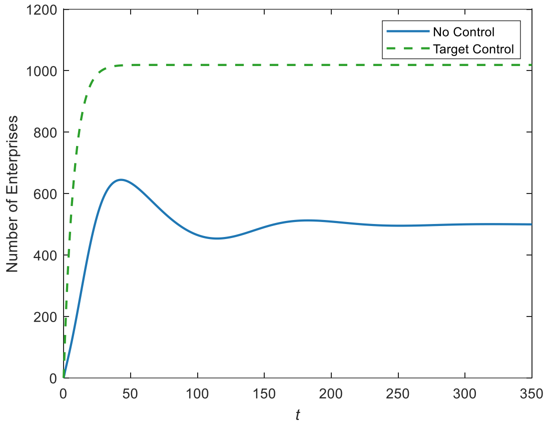

Figure 5. Risk- Recovered Enterprises under Different Control Scenarios

Considering the four types of risk enterprises mentioned above, under the no-control scenario, the supply chain network reaches a non-zero equilibrium point around period 300. In this state, the risk within the supply chain network still exists and remains stable. However, under the target control scenario, the supply chain network reaches a disease-free equilibrium point around period 60, and the risk within the supply chain network no longer exists.

4. Conclusion

Through the simulation experiments and sensitivity analysis of the supply chain network models under different aid strategies in response to sudden public events, it is clear that when the government provides assistance to risk-sensitive enterprises, target control proves to be effective in suppressing risk propagation. Therefore, enhancing the overall risk resilience of the supply chain network before the occurrence of a sudden public event can effectively reduce both the spread of risk and the level of government intervention required, thus lowering costs. This approach not only strengthens the recovery capacity of the supply chain network but also reduces the cost of intervention after the event occurs, allowing for greater government intervention. Consequently, establishing an aid mechanism with low costs and high benefits is essential to minimize control costs and more effectively manage the spread of risks within the supply chain network.

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