

# Pet Industry Development Forecast Based on Gray GM (1, 1) Modeling

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**Abstract:** This study aims to forecast the development of China's pet industry based on a gray GM (1,1) model, focusing on the current state of the industry in the last five years and the global demand for pet food in the next three years. Through hierarchical analysis, this paper builds multiple models to capture market dynamics at different levels to enhance forecasting accuracy. The study shows that the popularity of pet cats continues to rise, while the pet dog market is becoming saturated, reflecting changes in consumer preferences. In forecasting the global demand for pet food, this study compares the annual growth rate and the average annual growth rate, utilizing the performance of the GM (1,1) model versus the NGM (1,1, k, c) model, and ultimately selects the model with better forecasting capabilities. The analysis shows that the global demand for pet food will continue to grow over the next three years, highlighting the potential and challenges of the pet food market. This study provides a scientific basis for strategy development in the pet industry through a gray forecasting methodology to cope with data uncertainty and changing market dynamics.

**Keywords:** Pet Industry, Stratified Analysis, GM (1, 1), NGM (1, 1, k, c).

## 1. Introduction

With the rapid economic development and the improvement of per capita income, China's pet industry [1] has shown a remarkable growth trend in the past five years, and pets have gradually become an indispensable part of family life. In order to analyze this phenomenon in depth, this paper comprehensively applies algorithms such as GM (1, 1) [2] and NGM (1, 1, k, c) [3] to construct a mathematical model in order to segment the market demand and growth trend of different pet types [4]. By digging deeper into the data, this paper not only analyzes the distribution of various types of pets, but also explores the multiple factors affecting the development of the industry. In a global perspective, this paper also examines the dynamics of the pet industry in Europe and the United States [5] and forecasts the demand for pet food with the help of corresponding mathematical models, an analysis that provides valuable international market insights for Chinese pet industry participants. Through a hierarchical approach to analysis [6], this paper not only reveals the unique challenges and opportunities of China's pet industry, but also provides a scientific basis for forecasting the industry's development over the next three years.

## 2. Pet Type Distribution and Growth Trends

### 2.1. Development of China's Pet Industry in the Last Five Years

To describe the development of China's pet industry in the last five years, this section presents the annual growth rate, average annual growth rate, and analyzes the volume development trend.

The annual growth rate indicates the growth rate of one year:

$$r = \frac{N_{t+1} - N_t}{N_t} \times 100\% \quad (1)$$

Where  $R$  is the annual growth rate,  $N_{t+1}$  is the volume for the next year, and  $N_t$  is the transaction volume for the current year.

This can be obtained by calculation:

**Table 1.** Annual growth rate of cats and dogs

Pets/year	2019-2020	2020-2021	2021-2022	2022-2023
Cat	10.20%	19.42%	12.57%	6.79%
Dog	-5.11%	3.96%	-5.71%	1.09%

From Table 1:

(1) The annual growth rate of cats shows a rapid growth trend in 2019-2021 but slows down in 2022-2023.

(2) The annual growth rate of dogs rebounded after being negative in 2019-2020, but generally showed a fluctuating trend with no significant growth trend.

The average annual growth rate is the average annual growth rate of the indicator over a period. It helps in understanding the growth trend of the indicator and comparing it with other indicators.

$$\bar{r} = \left( \frac{N_t}{N_0} \right)^{\frac{1}{t}} - 1 \quad (2)$$

Where:

$\bar{r}$  is the average annual growth rate,  $N_t$  is the number of target years,  $N_0$  is the number of base years, and  $t$  is the time horizon (years).

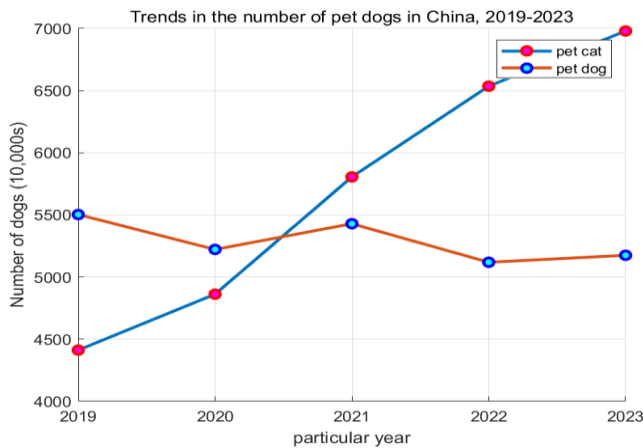
This can be calculated:

**Table 2.** Average annual growth rate for cats and dogs

Pets/year	2019-2023
Cat	9.61%
Dog	-1.22%

As can be seen in Table 2:

- (1) The average annual growth rate for cats is approximately 11.25%, indicating that the cat population has shown a steady increase over the past five years.
- (2) The average annual growth rate for dogs is approximately -0.57%, indicating that the number of dogs has shown a slight downward trend over the past five years.



**Figure 1.** Trend Chart on the Number of Pet Cats and Dogs

As can be seen in Figure 1:

- (1) The number of pet cats in China has continued to grow during the period 2019-2023, while the number of pet dogs has remained generally stable with a slight decline.
- (2) The popularity of pet cats is accelerating, while the market for pet dogs is becoming saturated.

## 2.2. Hierarchical Analysis to Analyze Weights

Hierarchical analysis is used below to analyze the factors affecting the development of China's pet industry.

### 2.2.1. Relationship analysis of factors

- (1) Per capita disposable income in China.
- (2) Pet safety.
- (3) Social happiness.
- (4) Social ratio of men and women.
- (5) Other factors.

### 2.2.2. Judgment Matrix Construction

These two comparisons were made using 1-9 in Table 3 to indicate the level of importance.

### 2.2.5. Analysis of Results

According to the weighted results, China's per capita disposable income has the largest weight, pet safety is next, social happiness is next, followed by the ratio of men and

**Table 3.** Inter-factor significance scale

Scale	Meaning
1	Indicates that two factors are equally important compared to each other
3	Indicates that one factor is slightly more important than the other
5	Indicates that one factor is more important than the other
7	Indicates that one factor is more important than the other
9	Indicates that one of the two factors is more important than the other
2, 4, 6, 8	Median of the above two neighboring judgments
Inverse	If A is 3 compared to B, then B is 1/3 compared to A

**Table 4.** Judgment Matrix

	a	b	c	d	e
a	1	5	4	2	8
b	1/5	1	2	5	6
c	1/4	1/2	1	3	7
d	1/2	1/5	1/3	1	5
e	1/8	1/6	1/7	1/5	1

Note: Table 4 Judgment Matrix data were obtained by rounding the mean of the survey results for the 30 students who had pets in their homes to the nearest whole number.

### 2.2.3. Calculation of Relative Weights

The above matrix can be calculated to pass the consistency test and weights.

Calculate the consistency indicator  $CI$ :

$$CI = \frac{\lambda_{\max} - n}{n - 1} \quad (CI = 0.1030) \quad (3)$$

Find the corresponding Average random consistency measure  $RI$  according to the Average random consistency metric  $RI$  table.

Calculation of Consistency Ratio:

$$CR = \frac{CI}{RI} \quad (4)$$

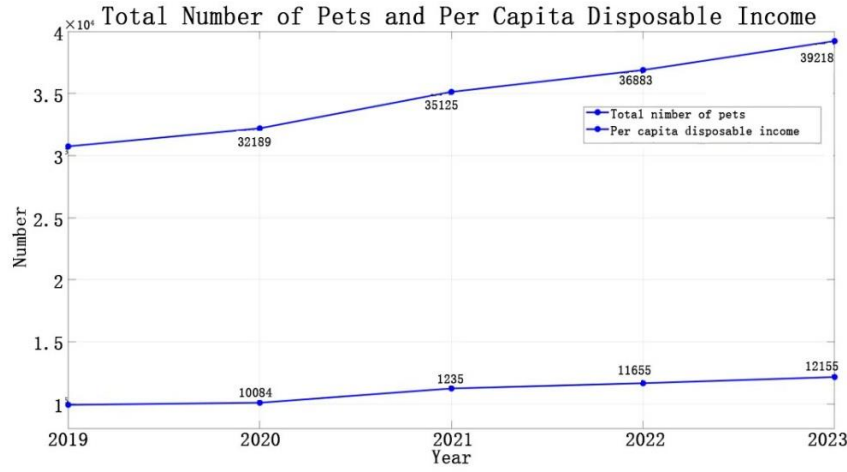
Because the evaluation results are only favorable to the development of the pet industry and unfavorable to the development of the industry two, so the matrix rankings must have a multiplicative relationship between, then also represents a certainty that the consistency test will be passed, belonging to the consistency matrix.

### 2.2.4. Calculate the Synthesis Weights

The result of the eigenvalue method used to find the weights is:

$$(a, b, c, d, e)^T = (0.5360, 0.2177, 0.1449, 0.0718, 0.0297)^T \quad (5)$$

women, and other factors have less influence on the pet industry, querying data related to the per capita disposable income with the largest weight, and exploring the linear relationship with China's pet industry development.



**Figure 2.** Collapsed chart of total number of pets and per capita disposable income in China from 2019 to 2023

According to Figure 2, the correlation between the development of China's pet industry and per capita disposable income is large, and the nature of the fit is good, which is in line with the expectations of the hierarchical analysis method of the highest-weighted influencing factors.

### 2.3. Gray Forecasting Model Forecasting Data

Due to the small sample size, a gray prediction model can be built to predict the development of China's pet industry in the next three years, and this section uses the model GM (1, 1) to study it.

#### 2.3.1. Data Processing

The original data series  $(x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n))$  is summed once  $(1 - AGO)$  to obtain a new data series  $(x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)}(n))$  that exhibits an approximate exponential pattern that allows the application of the gray model for prediction.

Where:

$$x^{(1)}(k) = \sum_{j=1}^k x^{(0)}(j) \quad (6)$$

#### 2.3.2. Linear differential equation of the model

Linear differential equation (LDE):

$$\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = b \quad (7)$$

This is called the whitening equation of the model.

The linear difference equation is obtained by integration and other calculations:

$$x^{(0)}(k) + az^{(1)}(k) = b \quad (8)$$

#### 2.3.3. Parameter Estimation

The parameters  $a$  and  $b$  of the above equation are estimated using the least squares method and the difference equation is:

$$[a, b]^T = (X^T X)^{-1} X^T Y \quad (9)$$

Where:

$$X = \begin{bmatrix} -z^{(1)}(2) & 2 & 1 \\ -z^{(1)}(3) & 3 & 1 \\ \vdots & \vdots & \vdots \\ -z^{(1)}(n) & n & 1 \end{bmatrix}, Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix} \quad (10)$$

#### 2.3.4. Introducing the prediction of the Original Series

Given the initial conditions, the response function of the as model can be obtained by solving the linear differential equation GM (1, 1).

$$\hat{x}^{(1)}(k) = \left[ x^{(0)}(1) + \frac{b}{a} \right] e^{-a(k-1)} + \frac{b}{a} \quad (11)$$

The response function is used to predict  $1 - AGO$  the sequence  $\hat{x}^{(1)}$ . By predicting the obtained  $\hat{x}^{(1)}$ , the predicted value of the original sequence can be introduced as:

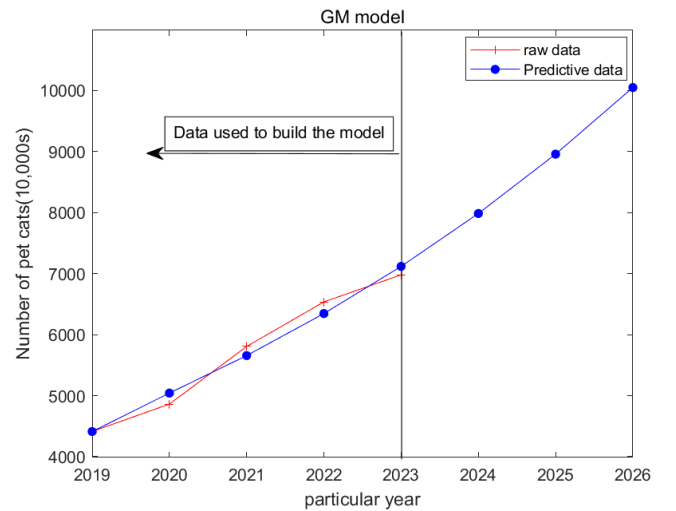
$$\hat{x}^{(0)}(k) = \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1) \quad (12)$$

In order to assess the effectiveness of the grayscale prediction model, the mean relative error (ARE) and the maximum relative error (MRE) are introduced to assess the overall performance of the prediction model.

$$ARE = \frac{1}{r} \sum_{k=1}^r \left| \frac{\hat{x}^{(0)}(k) - x^{(0)}(k)}{x^{(0)}(k)} \right| \times 100 (\%), \quad (13)$$

$$MRE = \max \left| \frac{\hat{x}^{(0)}(k) - x^{(0)}(k)}{x^{(0)}(k)} \right| \times 100 (\%). \quad (14)$$

The data can be obtained by substituting the data, and some of the results are shown in Figure 3 below.



**Figure 3.** Plot of predicted number of pet cats

The prediction can be obtained by modeling GM, as shown in Table 5.

**Table 5.** Number of pet cats in China, 2024-2026 (unit: 10,000 pets)

Pets/Years	2024	2025	2026
Cat	7985.19	8957.83	10048.9

It can be seen that China's pet cat market will continue to show an upward trend in the future.

### 3. Comparison of NGM and GM Models

When it is necessary to forecast the global demand for pet food in the next three years, this section needs to take into account the increase in the number of factors, the incomplete or small amount of data obtained, or the upward trend in the data series may not necessarily be present, the overall considerations of this section introduces a non-Cavi gray NGM (1, 1, k, c) model for forecasting, of course, without simulation, it is not certain that the model's performance NGM (1, 1, k, c) will be better than that of the GM model.

NGM (1, 1, k, c) model is different from model GM (only

$$\hat{x}^{(1)}(k) = \left[ x^{(0)}(1) + \frac{b}{a^2} - \frac{b}{a} - \frac{c}{a} \right] e^{-a(k-1)} + \frac{c}{a} + \frac{bk}{a} - \frac{b}{a^2} \quad (18)$$

The algorithmic process is almost the same as that of GM model. The main difference lies in the three points mentioned above and will not be expanded here to write the algorithmic process of model NGM (1, 1, k, c). In the following, we will refer to NGM (1, 1, k, c) as NGM (1, 1, k, c) model.

The performance of the model is still evaluated using ARE and MRE. By comparing the errors of model GM and model NGM, it can be seen that model GM outperforms model NGM, which means that the chi-square grayscale model is more effective than the non-chi-square grayscale model in this data simulation, which means that the collected data are basically sufficient for predicting the data series with an upward trend, and also shows that higher complexity does not necessarily imply a better simulation result.

Therefore, model GM was used to forecast the global demand for pet food for the next three years.

**Table 6.** Global Pet Food Demand 2024-2026

Years	2024	2025	2026
Sales (billions of dollars)	133.961	143.953	154.691

As shown in Table 6, the global demand for pet food will reach 133.961 (billions of dollars) in 2024, 143.953 (billions of dollars) in 2025, and 154.691 (billions of dollars) in 2026, and the global demand for pet food will continue to rise.

### 4. Conclusions

In this study an in-depth analysis of the development of the pet industry in China is conducted based on a gray GM (1, 1) model, focusing on market dynamics over the past five years and trends over the next three years. Through stratified analysis, this paper identifies market changes in pet cats and pet dogs and finds that the popularity of pet cats has risen significantly, while the pet dog market is becoming saturated.

the main differences are listed below).

(1) Linear Difference Equations of the Model NGM (1, 1, k, c)

$$x^{(0)}(k) + az^{(1)}(k) = bk + c \quad (15)$$

(2) Parameter Estimation

The parameters are estimated using least squares and the difference equation is:

$$[a, b, c]^T = (X^T X)^{-1} X^T Y \quad (16)$$

Where:

$$X = \begin{bmatrix} -z^{(1)}(2) & 2 & 1 \\ -z^{(1)}(3) & 3 & 1 \\ \vdots & \vdots & \vdots \\ -z^{(1)}(n) & n & 1 \end{bmatrix}, Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix} \quad (17)$$

(3) Response Function

The response function NGM (1, 1, k, c) of the model is:

This phenomenon reflects a shift in consumer preferences and suggests that industry players need to adapt their products and services to meet market demand. On a global scale, the study utilizes the GM (1, 1) and NGM (1, 1, k, c) models for comparison, and predicts that global demand for pet food will continue to grow over the next three years. This forecast provides an important basis for the internationalization of China's pet industry. This study not only provides scientific decision-making support for pet industry participants, but also serves as a reference for policy makers in promoting the healthy development of the industry.

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