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Research on Sampling Inspection Method for Supply Chain Components Based on Binomial Distribution

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Abstract: In the context of accelerated growth in global manufacturing, conventional full inspection faces challenges meeting enterprise demands for effective supplier component quality control, while statistical sampling based on binomial distribution ensures reliability and reduces costs by rationally setting confidence levels and error ranges. This study proposes an autonomous method for determining the minimum sample size based on binomial distribution and hypothesis testing to ascertain the acceptance or rejection of spare parts contingent on the defective rate. Specifically, spare parts should be rejected if the defective rate exceeds the nominal value at 95% confidence, and accepted if it does not exceed this value at 90% confidence. This paper derives the formula for calculating the minimum sample size using test theory based on normal distribution approximation. The findings indicate that when the defective rate constitutes 10% of the nominal value, the minimum sample size necessary to receive the spare parts with 95% confidence is 139, and the minimum sample size required to receive the spare parts with 90% confidence is 98. This significant efficiency enhancement over comprehensive inspections is complemented by sensitivity analysis of the defective rate, confidence level, and random error, supporting rational production decisions under resource constraints.

Keywords: Global Manufacturing, Quality Control, Supply Chain Decision Making, Binomial Distribution, Hypothesis Testing.

1. Introduction

In the contemporary context of accelerated global manufacturing industry expansion and the concomitant increase in the intricacy of supply chain management, enterprises have elevated their expectations concerning the quality control of components supplied by vendors [1]. The efficacy of quality control extends beyond the final quality of the product, directly impacting the production efficiency and economic benefits of the enterprise. Consequently, the assurance of component quality by implementing scientific sampling inspection methods has emerged as a pivotal concern.

The conventional full inspection approach is characterized by its time-consuming and labor-intensive nature, resulting in significant resource wastage [2]. This is particularly problematic in the context of the modern manufacturing industry, where rapid and efficient production is paramount. In contrast, statistical sampling inspection, as a scientific quality control method, has the potential to markedly reduce inspection costs while maintaining the reliability of the inspection process [3]. The application of statistical sampling inspection, founded on the test method of binomial distribution, enables the effective evaluation of batch components' quality. This approach, when coupled with the judicious configuration of confidence levels and allowable error ranges, furnishes enterprises with a reliable statistical foundation for decision-making concerning batch acceptance.

To address this, the present study has developed a hypothesis testing method based on the binomial distribution. This method eliminates subjective judgment through statistical analysis and ensures the objectivity of the assessment. The study first sets the null and alternative hypotheses, which correspond to the component defect rate not exceeding or exceeding the supplier's stated nominal value of 10%, respectively. Utilizing the normal distribution, the binomial distribution is then approximated to streamline the calculation. The critical value Z is selected based on the 95% or 90% confidence level, thereby determining the acceptance or rejection of components. By calculating the minimum sample size n and the allowable error E, the maximum acceptable number of defects can be ascertained at varying confidence levels. The proposed methodology enables the rapid selection of the optimal solution by considering both quantitative and qualitative evaluation criteria, providing a more comprehensive overall assessment framework and filling a gap in existing research.

2. Inspection Theory Based on Binomial Distribution

The data for this study were sourced from https://www.mum.edu.cn. In the domain of enterprise supply chain management, the implementation of sampling inspections on vendor-supplied components constitutes a pivotal quality assurance measure within the manufacturing process [4]. The employment of rigorous statistical sampling methodologies empowers firms to not only assess the aggregate quality of incoming consignments efficiently but also to promptly detect non-compliant items. This approach effectively mitigates the risk of defective units infiltrating the production line and significantly curtails manufacturing delays and the supplementary resource allocation necessitated by substandard components [5].

Research indicates that enterprises endeavor to formulate a sampling inspection strategy that minimizes the number of tests required to determine the acceptance or rejection of vendor-supplied components. This ensures an accurate judgment of the defect rate relative to an established nominal value at an acceptable confidence level. The challenge can be framed as a hypothesis-testing problem grounded in binomial distribution theory, which can be resolved through statistical hypothesis-testing procedures. In this study, the supplier's stated defect rate is subjected to hypothesis formulation, with the outcomes of the sampling inspections informing the decision to accept or reject the component batch.

To more accurately mirror real-world engineering conditions, this research establishes two scenarios. These scenarios are premised on a supplier's assertion that the defect rate of a component batch will not surpass 10% [6]. The enterprise intends to employ a sampling inspection approach to determine the acceptability of the components sourced from the supplier, with the inspection process being overseen by the company. In Scenario 1, the batch is rejected if the defect rate is statistically determined to be above the nominal value at a 95% confidence level. Conversely, in Scenario 2, the batch is accepted if the defect rate does not exceed the nominal value at a 90% confidence level. The binomial distribution is a discrete probability model in statistical theory, characterized by its core parameters: the total number of independent trials, n, and the success probability of an individual trial, p. To evaluate whether the defect rate of a component batch exceeds the nominal value p_0 , this challenge can be conceptualized as a hypothesis testing problem grounded in binomial distribution principles. According to this model, the overall defect rate of the components is represented by p (an unknown parameter), while the supplier-provided nominal defect rate is denoted as p_0 (a known parameter). The planned sample size for testing is indicated as n, and X represents the count of defective units within the sample. It is noteworthy that X adheres to a binomial distribution, as depicted below:

$$X \sim Binomial(n,p) \tag{1}$$

Table 1 presents the common critical values of Z corresponding to the standard normal distribution.

Table 1. Corresponding Z for common confidence levels

Confidence level	80%	90%	95%	97.5%	99%	99.5%
Critical value Z of the standard normal distribution	0.842	1.282	1.645	1.96	2.326	2.576

The defect rate p of the entire batch is estimated by calculating the sample defect rate $\hat{p} = \frac{X}{n}$, where X is the number of defective units observed in the sample. The construction and testing of hypotheses are based on this estimate. The null hypothesis, designated as H_0, posits that the defect rate, designated as p, is less than or equal to the specified limit, denoted by p_0. This hypothesis indicates that the defect rate does not exceed the specified limit. Conversely, the alternative hypothesis, designated as H_1, suggests that the defect rate exceeds the specified limit, denoted by p_0.

When the sample size is sufficiently large, the binomial distribution can be approximated by a normal distribution, according to the central limit theorem. This approximation simplifies the calculations. Specifically, the binomial distribution can be approximated by a normal distribution with mean np_0 and variance $np_0(1-p_0)$, such that:

$$X \sim N(np_0, np_0(1-p_0)) \tag{2}$$

The critical values corresponding to the desired confidence intervals can be utilized to determine whether to accept or reject the null hypothesis. For a 95% confidence level, the critical value is approximately $Z_{0.95/2} \approx 1.96$, and for a 90% confidence level, it is approximately $Z_{0.90/2} \approx 1.645$.

3. Calculation Analysis of Sample Size and Defective Products

Based on the confidence interval calculation formula, the sample size n required to meet specific criteria can be determined as:

$$n = (Z_{(\alpha/2)}/E)^2 p_0(1-p_0) \tag{3}$$

Where E represents the permissible error, reflecting the discrepancy between the expected and actual defect rates; is the nominal defect rate of 10% and $Z_{(\alpha/2)}$ denotes the critical value from the standard normal distribution [7]. To determine the minimum sample size for sampling tests, this study incorporates a permissible random error E, consistent with the practicality of random errors in manufacturing processes. Setting the allowable error E to 0.05, this study found that under a 95% confidence level, the minimum sample size n is 139, while at a 90% confidence level, it is 98.

The determination of the minimum sample sizes required for 95% and 90% confidence levels was achieved through a systematic approach, whereby the number of defective units within the samples was incrementally increased to ascertain the maximum number of defects that could be tolerated at each confidence level. As illustrated in Figure 1, the analysis indicates that, under a 95% confidence level, with a sample size of n=139, the maximum number of defective items that can be accommodated is 20; whereas, under a 90% confidence level, with a sample size of n=98, the maximum number of defective items that can be tolerated is 14.

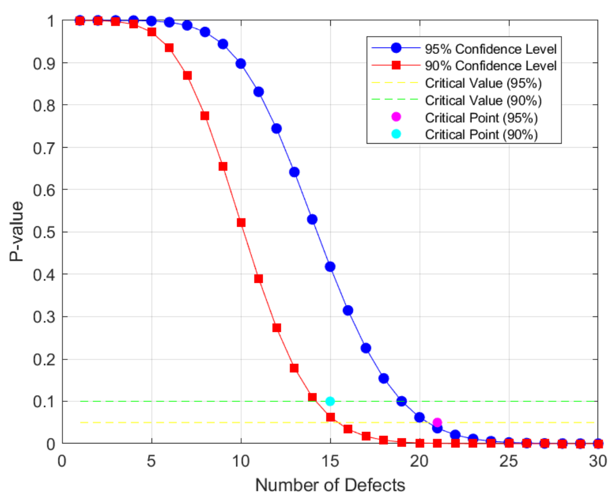

Figure 1. Changes in consistency corresponding to an escalation in defect numbers

4. Sensitivity Analysis of Parameters' Impact on Sample Size

The present study aims to assess the model's performance and enhance the accuracy and reliability of evaluating the effectiveness of the decision-making scheme. To this end, the model's attributes are analyzed and evaluated by employing a single-variable control approach. Specifically, the significance level and random error E are adjusted separately [8]. The sample size, n, is determined using the formula $n = (Z_{(\alpha/2)}/E)^2 p_0 (1 - p_0)$. This methodological approach enables a systematic examination of how variations in these parameters influence the model's attributes, thereby providing a robust foundation for improving decision-making efficacy.

The relationship between the change in sample size and the change in significance level is demonstrated in Figure 2, which shows a line chart illustrating how sample size changes with the level of significance.

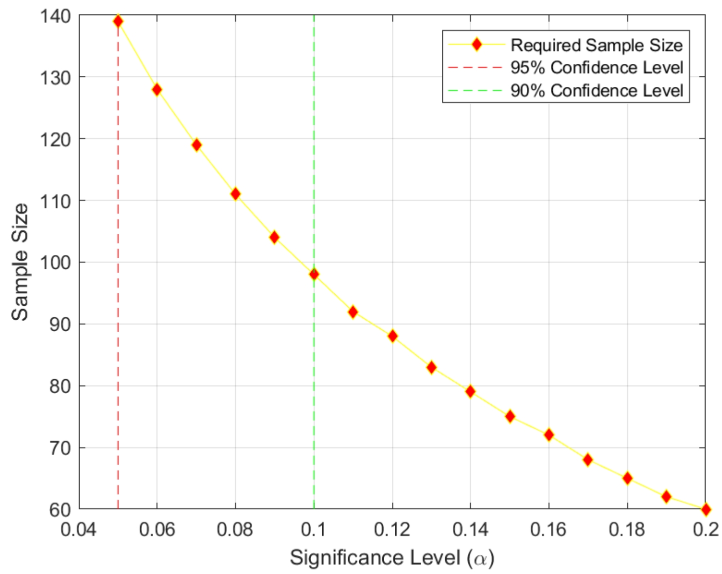

Figure 2. Variation in Sample Size Across Different Significance Levels

It is evident from the figure that as the level of significance increases, the minimum required sample size gradually decreases. For instance, when the confidence level is 95%, corresponding to a level of significance of 0.05, the required sample size is 139. Conversely, when the confidence level is reduced to 90%, corresponding to a significance level of 0.10, the required sample size is reduced to approximately 98. This demonstrates that higher significance levels result in lower minimum sample size requirements. Depending on the specific requirements, an appropriate sample size can be selected at different confidence levels to ensure the accuracy of test results.

Furthermore, the relationship between sample size and error range is depicted in Figure 3.

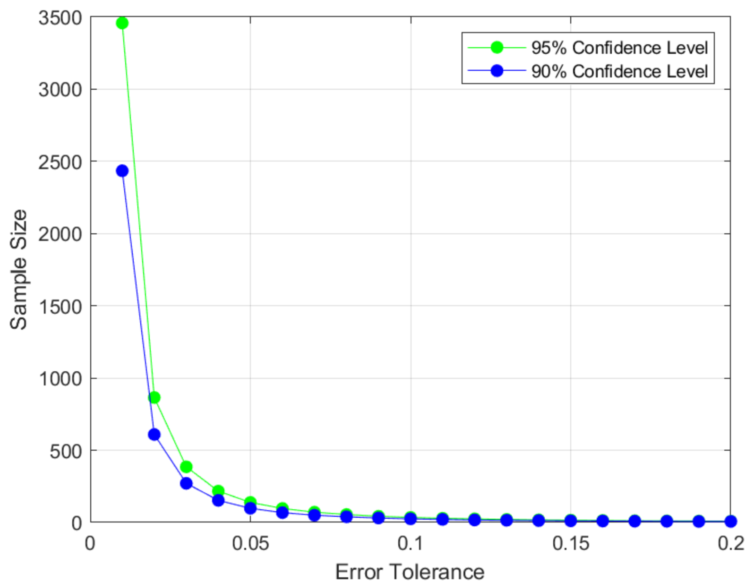

Figure 3. The relationship between sample size and error Tolerance

Within the error range [0, 0.05], a significant decrease in the required sample size is observed as the error range increases. Conversely, when the error range approaches zero, a sharp increase in the sample size is evident. As the error range further increases to around 0.05, a rapid decrease in the sample size is observed, which stabilizes. Finally, when the error range is close to 0.20, the sample size remains at a low level.

For comprehensive analysis, it is necessary to perform these calculations and evaluations by varying parameters such as the nominal value, confidence level, and permissible error. This approach aims to derive a sampling inspection scheme that minimizes the number of inspections, thereby effectively optimizing the inspection process.

5. Conclusions

This study proposes a novel hypothesis testing method based on the binomial distribution, with the objective of addressing the challenges associated with ensuring effective supplier component quality control in the context of rapid global manufacturing growth. The research aims to autonomously determine the minimum sample size for statistical sampling inspections, thereby balancing cost reduction with inspection reliability. The proposed methodology enables the acceptance or rejection of spare parts based on defect rates at specified confidence levels (95% and 90%). Key findings indicate that for a defect rate of 10%, the minimum sample sizes required are 139 for 95% confidence and 98 for 90% confidence. Sensitivity analysis supports rational production decisions under resource constraints. While the study assumes a constant defect rate across batches and uses normal distribution approximations, it acknowledges these limitations may not fully reflect real-world variability, especially for small samples or extreme proportions.

The study proposes a robust framework applicable to operations research and related fields, demonstrating the feasibility of using binomial distribution and hypothesis testing for quality control in supply chain management. It underscores the importance of integrating statistical principles into operational practices to enhance overall supply chain performance. By optimizing the quality assurance process, enterprises are equipped with a robust tool to maintain high standards of product quality in the context of rapid global manufacturing growth. Subsequent research endeavors will entail the exploration of more intricate statistical models in order to address batch variability, the investigation of machine learning techniques for the purpose of enhancing accuracy, and the integration of real-time data analytics to optimize quality control processes.

References

[1] Jahani Mw, Raji F, Zojaji Z .Securing supply chain through blockchain-integrated algorithmic system: ensuring product quality and counterfeiting tags detection [J]. Cluster Computing, 2024, 28(1):51-51.

[2] Rai U, Oluleye G, Hawkes A .Stochastic optimisation model to determine the optimal contractual capacity of a distributed energy resource offered in a balancing services contract to maximise profit [J]. Energy Reports, 2024, 115800-5818.

[3] Russell D M, Galloway B J .Driving down the cost of biologics: lessons from a nationalised health-care system [J]. Lancet (London, England), 2024, 404(10464):1723-1724.

[4] Du L, Zhang Y, Lu K .Pricing and Financing Strategies in a Dual-Channel Low-Carbon Supply Chain for Bilateral Capital-Constrained Retailers [J]. International Journal of Computational Intelligence Systems, 2024, 17(1):272-272.

[5] Barman A, Chakraborty K A, Sana S S, et al. Pricing Strategy and Risk-Averse Flexibility in Sustainable Supply Chain: A Dual-Channel Logistics Process Under Reward Contracts and Demand Uncertainty [J]. Global Journal of Flexible Systems Management, 2024, 25(4):733-762..

[6] Rajabzadeh H, Rabiee M, Sarkis J .Sourcing from risky reverse channels: Insights on pricing and resilience strategies in sustainable supply chains [J]. International Journal of Production Economics, 2024, 276109373-109373.

[7] K. A G, K. J J. Pricing strategy with quality improvement in a dual collection channel closed-loop supply chain under return uncertainty [J]. Operational Research, 2024, 24(2).

[8] SHI Biao, LI Yu Xia, YU Xhua, YAN Wang. Short-term load forecasting based on modified particle swarm optimizer and fuzzy neural network model [J]. Systems Engineering-Theory and Practice, 2010, 30(1): 158-160.