

# A study on Scheduling of Human Resource Constrained Projects with A Single Model of Minimised Duration Targets

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**Abstract:** This paper investigates the project scheduling problem with single-mode human resource constraints under the minimised duration target. In civil engineering construction projects, human resource is one of the key factors that constrain the duration. In this paper, an integer planning model considering human resource constraints is constructed with the aim of minimising the project duration by optimising the allocation and scheduling of human resources. By introducing an efficient solution algorithm - CPLEX optimisation technique, this paper successfully solves this complex problem and provides real cases to verify the effectiveness of the model and algorithm. The results show that a reasonable human resource scheduling strategy can significantly shorten the project duration and improve the efficiency of project management under resource-constrained conditions.

**Keywords:** Minimising duration; Human resource constrained project scheduling; Integer planning.

## 1. Introduction

With the increasing scale and complexity of civil engineering construction, the problem of project scheduling has increasingly become one of the core issues in project management. In the process of project implementation, human resources, as one of the most important production factors, its allocation and scheduling are directly related to the progress, cost and quality of the project. However, in actual projects, human resources are often limited by a variety of factors, such as professional skills, experience, quantity, and overtime hours. Therefore, how to minimise the project duration under the constraints of human resources has become an important challenge for project managers.

This paper focuses on the project scheduling problem with single-mode human resource constraints under the minimisation schedule objective, aiming to provide scientific and efficient decision support for project managers by constructing reasonable models and algorithms. This study not only has important theoretical significance, which can enrich and improve the theoretical system in the field of project scheduling; at the same time, it also has important practical value, which can provide effective solutions for human resource scheduling in actual engineering projects.

## 2. Literature Review

Since the middle of the 20th century, the project scheduling problem has been a research hotspot in the fields of operations research and management science. Although traditional project scheduling methods, such as Gantt chart, critical path method (CPM) and programme review technique (PERT), provide powerful tools for project schedule management to a certain extent, these methods tend to ignore the problem of resource constraints. With the continuous development of project management theory and practice, the Resource Constrained Project Scheduling Problem (RCPSPP) has gradually become the focus of research.

In RCPSPP, the problem of limited human resources is particularly prominent. Scholars at home and abroad have conducted a lot of research on this problem and proposed a variety of solution methods. Among them, the integer programming method is one of the commonly used methods to solve such problems. This method transforms the project scheduling problem into an optimisation problem by constructing a mathematical model and solves it using mathematical planning techniques. However, due to the complexity of the RCPSPP problem, it is often difficult for traditional mathematical planning methods to find the optimal solution in a reasonable time. Therefore, heuristic algorithms and meta-heuristic algorithms are gradually gaining attention.

In terms of heuristic algorithms, priority rule-based scheduling methods are the most common class. This type of method guides the scheduling order of tasks by setting different priority rules to obtain an approximate optimal solution. However, the choice of priority rules has a significant impact on the quality of the solution and it is difficult to guarantee the optimality of the solution. Meta-heuristic algorithms, such as genetic algorithms, particle swarm algorithms and ant colony algorithms, on the other hand, search the solution space by simulating the optimisation mechanisms in nature and can find high-quality solutions in a shorter time. These algorithms have been widely used in RCPSPP problems and achieved better results.

In summary, the project scheduling problem with constrained human resources is a complex and challenging research topic. In this paper, based on the previous research, we will further explore the project scheduling problem with single-mode human resource constraints under the minimised schedule objective, and propose effective solution models and algorithms.

## 3. Modelling

With the increasing scale and complexity of civil engineering construction, the problem of project scheduling has increasingly become one of the core issues in project

management. In the process of project implementation, human resources, as one of the most important production factors, its allocation and scheduling are directly related to the progress, cost and quality of the project. However, in actual projects, human resources are often limited by a variety of factors, such as professional skills, experience, quantity, and overtime hours. Therefore, how to minimise the project duration under the constraints of human resources has become an important challenge for project managers.

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### 3.1. Description of the Problem

In the resource-constrained project scheduling problem, minimising the project duration is the most basic element in scheduling decision-making and the most used management index. Studying the solution method of this problem and the optimal scheduling scheme has obvious practical value and wide practical significance. This paper is based on the following assumptions when discussing the problem of minimising duration in scheduling projects with human resources constraints:

(1) A project contains several different activities, which can be represented by a single-code network diagram. The network is directed, where nodes represent activities and arcs represent timing constraints between activities;

(2) There is a timing constraint between activities in the project, i.e., an activity must be ready to start after all its immediately preceding activities have been completed;

(3) Each activity has a fixed duration (work period);

(4) The human resources required by the project for each skilled labour category are limited at each time slot, but are renewable between time slots;

(5) There is a fixed availability (supply) of human resources for each skilled labour category at each time of the scheduling period;

(6) Activities may not be interrupted once they have begun;

(7) There is only one mode of implementation for each activity;

(8) The management objective is to minimise the project completion time while meeting timing constraints and resource constraints.

Among the above assumptions, the most important ones are assumption (2) and assumption (5). Assumption (2) indicates that the execution of different activities in the project has sequential constraints, in which the immediately preceding activity means: activity B can only start after activity A finishes, and there is no other activity between activities A and B. Activity A is said to be the immediately preceding activity of activity B. Assumption (2) is usually referred to as the timing constraint of the problem. Assumption (5) indicates that the use of human resources cannot exceed the supply of resources (availability) at all times that the project is in progress, and is often referred to as the resource constraint of the problem. The essence of the minimisation duration problem for human resource constrained project scheduling is

a combinatorial optimisation problem under timing constraints and resource constraints, i.e., how to determine the start times of all activities to minimise the project duration while satisfying these two constraints.

For the convenience of describing the problem and the need to establish a mathematical model, some numerical symbols and their significance are given below:

K: Total number of human resource categories for different skilled trades;

$S_i$ : start time of activity  $i$  (integer),  $i=1, 2, \dots, N$ ;

$d_i$ : duration of activity  $i$ ;

$P_i$ : the set of immediately preceding activities of activity  $i$ ;

$r_{ik}$ : demand of activity  $i$  for resource  $k$  at time  $t$ ,  $i=1, 2, \dots, N$ ;  $k=1, 2, \dots, K$ ;

$R_k$ : availability of resource  $k$  at time  $t$ ,  $k=1, 2, \dots, K$ ;

$A_t$ : the set of all activities performed in the time period  $(t-1, t]$ .

### 3.2. Modelling

Since the completion time of the project is to be determined, a pure integer planning model is used to describe the minimisation duration problem in human resource constrained scheduling.

The mathematical model of the minimisation of the schedule target single mode human resource constrained project scheduling problem is

obj.

$$\min S_N \quad (1)$$

s.t.

$$S_j + d_j \leq S_i \quad \forall j \in P_i \quad (2)$$

$$\sum_{i \in A_t} r_{ik} \leq R_k \quad k = 1, 2, \dots, K \quad (3)$$

$$S_i \in \{0\} \cup I \quad i=1, 2, \dots, N \quad (4)$$

Objective equation (1) indicates that the goal of project scheduling is to minimise the project duration, i.e., the start time of the end activity is minimised; objective equation (2) indicates the timing constraints between project activities, i.e., any activity must be completed before it can be started after the completion of all of its immediate preceding activities; objective equation (3) indicates the resource capacity constraints, i.e., human resources must not be used more than they are supplied at any one time period; and objective equation (4) indicates that the start time of the activity takes on a non-negative integer value.

## 4. Model Solving

The core solution to the single-mode resource-constrained scheduling problem of minimising project cycle time focuses on two main areas: the construction of precise mathematical models to accurately describe the problem and devise solution representations, and the development of heuristic algorithms aimed at exploring and approximating the optimal solution. Early explorations failed to achieve the desired results by relying on integer programming techniques alone, and new heuristics such as the branch-and-bound method proposed by Demeulemeester and Herroelen have emerged. Although these methods perform well in solving small or medium-sized projects, they are not sufficient for large-scale and complex projects, which further motivates the need for the development of efficient heuristic algorithms.

An extensive literature review reveals that most heuristic

algorithms adopt priority rules in the scheduling decision-making process, which assign priorities to activities based on factors such as time urgency, resource demand, or position in the project network to resolve resource allocation conflicts. The effectiveness of priority rules is strongly influenced by the structure of the project network, the number of activities, the types of constraints and the degree of resource tension.

For the models in this paper, there are two key challenges that need to be addressed:

The first is to rationalise the order of implementation of activities while ensuring that the logic of chronology is not broken;

Secondly, the earliest possible start or end time for each activity is precisely calculated and scheduled, given the available resources.

To address the above challenges, we can employ a heuristic scheduling strategy based on priority rules, a traditional but powerful tool that can effectively manage and resolve inter-activity conflicts arising from shared resources. Despite its long history, the priority rule-based scheduling approach still occupies an irreplaceable position in the field of scheduling resource-constrained projects. The implementation of this method is divided into two steps: first, based on predefined selection criteria or heuristic logic, the activities to be scheduled are filtered and assigned a corresponding priority to form an orderly scheduling sequence; then, under the premise of ensuring that the timing and resource constraints of the activities are satisfied, the starting time of each activity is determined one by one, thus completing the scheduling planning of the entire project.

The operational steps of the priority scheduling method are:

$$\alpha_j = \max\{t | t > \alpha_{jmin} \text{ and } R_{kt}, R_{k(t+1)}, \dots, R_{k(t+d_j)} \geq r_{jk}, k = 1, 2, \dots, K\} \quad (5)$$

Where,

$$\alpha_j \min = \max\{\beta | i \in pj\} \text{ and } \beta_j = \alpha_j + d_j \quad (6)$$

From the above priority scheduling approach it is clear that once the priority values of the activities in the project are given, a complete scheduling arrangement can be derived. In the minimisation duration problem, where the nodes are determined in a hierarchical manner, in order to find an optimal scheduling, an optimal sequence of activity priority values must first be found.

## 5. Case Studies

Assuming that there are nine activities in the Human Resources Constrained Project Scheduling Issue and that three types of human resources are required to complete them, the project network diagram is shown in Figure 1, and the information on the duration of the project activities and the supply and demand of human resources is shown in Table 1.

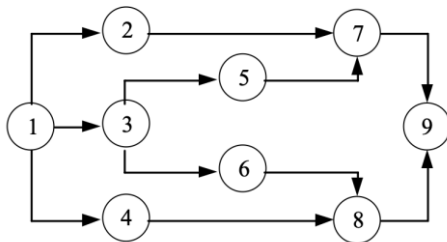


Figure 1. Project Network Diagram

Step 1 Prioritise the activities. Let the current time  $t_{now}=0$ . Initialise the supply vector  $[r_{i1}, r_{i2}, \dots, r_{ik}, \dots, r_{i(K)}]$  of K human resources for activity i at  $t_{now}$ ;

Step 2 If all activities have been dispatched, stop; otherwise, re-establish the set of activities to be selected, E, i.e., determine whether the immediately preceding activities of the activities to be selected at  $t_{now}$  have been completed and whether the demand for one of its human resources is greater than the supply. If E is empty, go to step 3; otherwise, go to step 4;

Step 3 Reset  $t_{now}$  to the latest end time of the current completed activity, update the resource availability, and go to step 2;

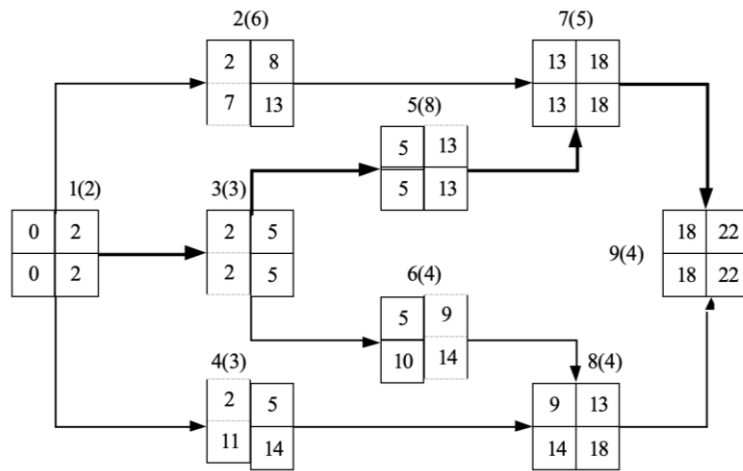
Step 4 Select the activity with the highest priority value from E, schedule its start and end times, update the resource availability, and go to step 2.

The HR-constrained project scheduling problem can essentially be viewed as a class of sequencing problems subject to certain constraints, since once the order of activities is given, the earliest possible start time for each activity can be easily determined based on the available resources. For a particular set of feasible activity orderings  $V_k = \{i_1, i_2, \dots, i_j, \dots, i_{(N)}\}$ , the start time of each activity can be computed from left to right based on the given activity ordering and available resources. Let  $\alpha_i$  denote the earliest possible start time of activity i,  $\beta_i$  denote the earliest possible end time of activity i,  $d_i$  denote the continuation time of activity i, and  $R_{kt}$  denote the availability of resource k in the time period  $(t-1, t)$ . Then the earliest possible start time  $\alpha_j$  is determined as follows for an activity sequence of  $V_k$  and an activity j at any position:

Table 1. Information on Project Activity Duration and Resource Availability and Demand

Activity Number	Duration of activities	Human resource requirements by skilled labour category		
		Cement Worker (R1)	Plumber (R2)	Electrician (R3)
1	2	4	3	1
2	6	3	1	2
3	3	2	0	3
4	3	3	2	1
5	8	5	1	0
6	4	3	1	1
7	5	2	2	2
8	4	3	2	3
9	4	2	1	2
Human resource availability		6	7	7

If the supply of human resources can meet the resource requirements of each activity executed in the order of scheduling according to the critical path method, i.e. the supply of human resources for each type of work is greater than its peak demand in each time period, then the project duration can be projected according to the project network diagram using the CPM method as shown in Figure 2.



**Figure 2.** CPM Network Diagram

From the resource requirements by time period, the project duration can be reduced to 22 days if the supply of the three resources is greater than 11, 3 and 6 respectively. In this case, the actual supply of R1 is less than 11, resulting in a corresponding increase in the project duration.

A heuristic algorithm based on priority rules is used to solve the problem of minimising the duration scheduling of the resource-constrained project in the example, and the increase in duration is greatly reduced compared to that without any optimal scheduling strategy, and the results are shown in Table 2:

**Table 2.** Scheduling of project activities

Activity Time	1	2	3	4	5	6	7	8	9
Starting time	0	2	2	8	11	5	19	19	24
end time	2	8	5	11	19	9	24	23	28

The minimum duration of the project is 28 days.

The use of the three human resources in the example is shown in table 3.

**Table 3.** Three types of human resources utilization after optimized scheduling

Time	Human resources		
	Cement Worker (R1)	Plumber (R2)	Electrician (R3)
1	4	3	1
2	4	3	1
3	5	1	5
4	5	1	5
5	5	1	5
6	6	2	3
7	6	2	3
8	6	2	3
9	6	3	2
10	3	2	1
11	3	2	1
12	5	1	0
13	5	1	0
14	5	1	0
15	5	1	0
16	5	1	0
17	5	1	0
18	5	1	0
19	5	1	0
20	5	4	5
21	5	4	5
22	5	4	5
23	5	4	5
24	2	2	2
25	2	1	2
26	2	1	2
27	2	1	2
28	2	1	2

From the results obtained, it is shown that the heuristic method used in this paper for solving the minimisation duration problem is an effective algorithm.

## 6. Conclusion

This paper investigates how to minimise project duration by optimising the allocation and scheduling of human resources under constrained conditions in civil engineering construction projects. This is a problem with practical application value because human resources are one of the key factors constraining the duration of civil engineering projects.

In this paper, the problem is investigated using model construction, solution algorithms, and validation. Among them, model construction is the construction of an integer planning model considering human resource constraints, which is based on a series of assumptions including the timing constraints of activities, the fixed continuation time of activities, the finiteness and updatability of human resources, and the singularity of activity execution modes, etc., and mathematical symbols and variables are used to accurately describe the problem, such as the total number of human resource types for the skilled trades ( $K$ ), the start time of activity  $i$  ( $S_i$ ), etc.; the solution algorithm introduces the CPLEX optimisation technique to solve the project scheduling problem under resource-constrained conditions and obtains a solution to minimise the project duration; the validity and practicability of the model and algorithm are verified through real cases.

The research results show that a reasonable human resource scheduling strategy can significantly shorten the project duration and improve the project management efficiency under resource constrained conditions. By constructing an integer planning model and adopting CPLEX optimisation technique, this paper successfully solves the project scheduling problem with single-mode human resource constraints.

The shortcoming of this paper is that the research mainly focuses on the project scheduling problem with single-mode human resource constraints and does not consider the case of multi-mode scheduling, where the same activity may be executed in multiple ways. Although CPLEX is a powerful solver, computational time and resource consumption may become an issue when dealing with large-scale projects, and the limited number of real-world examples in the paper may not be sufficient to fully validate the generalisability of the model and algorithm.

Combining the above problems, the later extensible studies in this paper are: first, multi-modal human resource constrained project scheduling, which investigates how to optimise the allocation and scheduling of human resources in the presence of multiple execution modes in order to further shorten the project duration. The second is the combination of heuristic and meta-heuristic algorithms, which explores the combination of priority rule-based scheduling methods with meta-heuristic algorithms in order to find higher quality

solutions in a reasonable time. The third is optimisation of large-scale projects, which investigates how to improve the solution algorithms to deal with larger-scale project scheduling problems and reduce computation time and resource consumption. Fourth, multi-project scheduling, which extends the research to multi-project environments by considering how to rationally allocate limited human resources among multiple projects in order to optimise the performance of the overall project portfolio.

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