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Optimal Design of Decision Problem in The Production Process

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Abstract: Based on the production decision problem of the "Higher Education Society Cup" national College students Mathematical Contest in Modeling, this paper presents an optimization scheme to solve the sampling detection and production decision of two kinds of spare parts in the production process of enterprises. By establishing the production decision model and cost optimization model, the binomial distribution model is used to test the hypothesis, and the relationship between the min imum sampling quantity and the real defective rate is determined, which improves the detection efficiency and reduces the cost. A production decision model with binary code is constructed, which considers many cost factors, optimizes the model by decision tree and dynamic programming, and obtains the optimal strategy and net profit expectation. The whole calculation strategy of multi-process and multi-parts is proposed. The model is further optimized by simulated annealing and genetic algorithm, and the maximum profit of a single finished product is obtained. By integrating all models, a cost optimization model based on sampling inspection defective rate is proposed, which takes the posterior distribution of defective rate as the key basis to provide a more accurate decision analysis and optimization path for enterprises.

Keywords: Sampling detection; Bayesian estimation; Dynamic programming; Simulated annealing; Genetic algorithm.

1. Introduction

An enterprise is committed to producing a best-selling electronic product. In the production process, enterprises need to purchase various key parts (parts 1 and 2) and assemble them into the final product. The quality of products depends on the quality of spare parts, and different spare parts have different defective rates, purchase prices, and testing costs. The finished products assembled by the two kinds of accessories also have costs caused by exchange and disassembly, that is, the following problems need to be solved:

Problem 1: Enterprises purchase spare parts from suppliers, to ensure the quality of spare parts, they need to design a sampling testing program. The program requires that the quality of parts be evaluated with the least number of inspections while ensuring that the defective rate does not exceed the nominal value. The test results will affect whether the enterprise accepts the spare parts provided by the supplier.

(1) Sampling test scheme design: reject the parts with a defective rate exceeding the standard at a 95% confidence level, receive the standard batches at a 90% confidence level, and minimize the number of tests.

(2) Production decision optimization: Based on the defective rate of spare parts and finished products, determine whether to test spare parts/finished products and disassemble unqualified products to maximize profits.

(3) Multi-process production strategy: For the complex process of 2 processes and eight parts, design testing and dismantling strategies to optimize global profits.

(4) Dynamic decision optimization: Based on the sampling test results, the defective rate is updated by the Bayesian method to re-optimize the production decision.

On this basis, enterprises need to re-evaluate and adjust their strategies in parts procurement, assembly of finished products, testing, disassembly, and disposal of unqualified finished products to ensure the minimum cost and maximum profit in the overall production process.

2. Optimal Design of Decision Problem in The Production Process

2.1. Model Assumption

(1) Assume that all production processes are on the assembly line.

(2) It is assumed that the testing process of all parts, semi-finished products, and finished products is independent of each other, and the test results are not affected by other factors.

(3) Assume that cost, market price, adjustment cost, assembly cost, etc., do not change with other factors and remain stable.

3. Model Solution and Result Analysis

3.1. The Establishment and Solution of Problem 1 Model

In the production process of the enterprise, to ensure that the two key parts provided by the supplier meet the quality standards, a sampling inspection must be carried out to evaluate the secondary quality rate. The core objective is to determine accurately whether the defective rate of the parts exceeds the supplier's nominal value by minimizing the number of inspections so as to decide whether to receive the parts. Since the testing cost is borne by the enterprise itself, it is necessary to find the best balance between ensuring the reliability of the results and controlling the cost when designing the testing scheme.

When designing the inspection program, the main goal is to optimize the number of inspections to ensure that the defect rate can be accurately estimated with the minimum sample size and that the requirements of determining whether the defect rate exceeds the nominal value with 95% confidence and whether the defect rate does not exceed the nominal value with 90% confidence are met. Assume a nominal defect rate of 10% and make decisions at a specified level of confidence to avoid adding unnecessary inspection costs.

In the process of scheme design, it is necessary to consider the difference between the actual defective rate and the nominal value. If the actual defective rate is close to the nominal value, a larger sample size is needed to ensure the accuracy of the test results. If the defective rate is significantly higher or lower than the nominal value, the required sample size is reduced accordingly.

3.1.1. Parameter Setting

Nominal defective rate: $\rho_0 = 0.1$

Real defective rate: ρ

Sample size: n

α : Significance level, for 95% confidence α =0.05; With 90% confidence α =0.10

3.1.2. Selection of Statistics

Since there are only two states of qualified and unqualified parts, the state distribution of parts is subject to Bernoulli distribution (binomial distribution), so in this case, we will use binomial distribution for probability modeling to ensure that the correct decision can be made under the given confidence. For random variables $X \sim b(n, p)$. Using the normal approximation for a large sample size, we assume that X follows a normal distribution.

3.1.3. Conditions of Rejection and Acceptance

For case (1), with 95% confidence, parts are rejected if the test results meet the following conditions:

$$H_0 : \rho \leq 0.1 \quad H_1 : \rho > 0.1 \tag{1}$$

The decision rule is: if ρ <=0.1, then accept, otherwise reject.

For case (2), at 90% confidence, parts are received if the results of the inspection meet the following conditions:

$$H_0 : \rho \geq 0.1 \quad H_1 : \rho < 0.1 \tag{2}$$

The decision rule is: if ρ <0.1, then accept, otherwise, reject.

3.1.4. Model Solving

Using the normal approximation, the central limit theorem, we can make:

$$\hat{\rho} \approx N\left(\rho, \sqrt{\frac{\rho(1-\rho)}{n}}\right) \tag{3}$$

For rejection conditions with 95% confidence, we need to ensure that:

$$P\left(\hat{\rho} > 0.1 \middle| H_0\right) \leq 0.05 \tag{4}$$

For a 90% confidence acceptance condition, we need to ensure that:

$$P\left(\hat{\rho} < 0.1 \middle| H_1\right) \leq 0.10 \tag{5}$$

Since the test statistic is:

$$Z = \frac{\hat{\rho} - \rho_0}{\sqrt{\frac{\rho_0(1-\rho_0)}{n}}} \tag{6}$$

According to the different confidence levels, the corresponding sample size n can be calculated by using the cumulative probability of the normal distribution.

1) For 95% confidence:

$$n = \left(\frac{Z_\alpha^{\ 2} \bullet \rho_0 \bullet (1-\rho_0)}{(\rho_0 - \rho)^2}\right) \tag{7}$$

$Z_\alpha \approx 1.96$ is the Z value corresponding to the 95% confidence level.

2) For 90% confidence:

$$n = \left(\frac{Z_\alpha^{\ 2} \bullet \rho_0 \bullet (1-\rho_0)}{(\rho_0 - \rho)^2}\right) \tag{8}$$

$Z_\alpha \approx 1.645$ is the Z value corresponding to the 90% confidence level.

Based on the above two scenarios, we obtain the minimum sampling quantity N under different real defective rates, and the specific results are shown in the figure below:

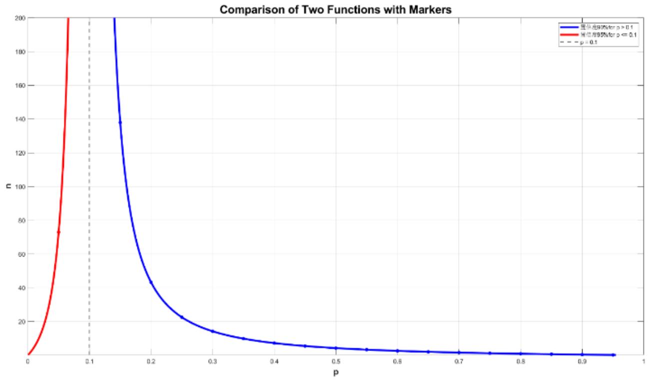

Figure 1. The relationship between different defective rates and the minimum sample size

3.1.5. Result of Problem 1

Table 1. Minimum sample size under different confidence levels

Comparison of Two Functions with Markers			
	P	n1	n2
90% confidence	0.00	0.00	
	0.01	3.31	
	0.02	8.29	
	0.03	16.07	
	0.04	28.86	
	0.05	51.41	
	0.06	95.39	
	0.07	195.74	
	0.08	497.91	
	0.09	2216.23	
95% confidence	0.10		Inf
	0.11		3760.93
	0.12		1014.18
	0.13		482.76
	0.14		289.08
	0.15		195.92
	0.16		143.42
	0.17		110.62
	0.18		88.60
	0.19		72.99
	0.20		61.47

According to the relationship between different defective rates and minimum sampling samples, the minimum sampling quantity n under different confidence levels is obtained, and the specific results are shown in the following table.

3.1.6. Question 1 Results Are Discussed and Analyzed

For the case of 90% confidence, the following is brought into formula (8):

When ρ =0.06, n=95, if the number of products X>5, reject this batch of samples, otherwise, receive.

For the case of 95% confidence, it is brought into formula (7) as follows:

When ρ =0.18, n=88, if the number of products X>15, reject this batch of samples, otherwise, receive.

The analysis results show that in practical applications, choosing the appropriate confidence level and sample size is a tradeoff process that needs to consider inspection costs and quality control needs. High confidence, while providing more reliable decision support, can lead to higher detection costs and time delays. By adjusting confidence levels and sample sizes, businesses can manage the risks associated with receiving defective products. For example, for high-value or performance-critical parts, higher confidence levels and stricter sampling standards may be required to ensure quality.

3.2. The Establishment and Solution of Problem Two Model

The essence of generative decision is to maximize expected returns. In practice, manufacturers usually have an expected production volume \(n\). Companies need to maximize the expected return on each qualified product. Given the constancy of the selling price, the above problem is equivalent to minimizing the expected cost of producing each qualified product. By establishing binary coding and a decision set, the abstract decision scheme is transformed into an easy-to-understand code form. Then, to maximize total profit, the production decision model is established. Finally, by introducing the decision tree model, the phased decision is classified, analyzed, and calculated, and the optimal decision scheme is obtained by using dynamic programming. The specific process is shown as follows:

3.2.1. Problem Modeling

Enterprises need to solve the following four decision-making problems: spare parts, whether to detect; Parts 2, Whether to test; Whether the finished product is tested; and Whether the unqualified finished product is disassembled.

Therefore, there are 16 decision schemes. The best scheme is defined as the scheme with the maximum profit expectation, and the profit expectation of each qualified item produced by 16 decision schemes is obtained strictly by using the properties of the binomial distribution and geometric distribution. Then, the specific values of six cases are substituted into the profit expectation formula, and the optimal decision scheme and decision index value of each case are obtained.

3.2.2. Result Solving

Combined with the production decision model, the sorted data are traversed through 16 decision schemes in 6 cases respectively, and the net profit expectation of a single finished product in each case is summarized and drawn into a line chart by visualization algorithm so that the corresponding optimal strategy can be obtained more intuitively.

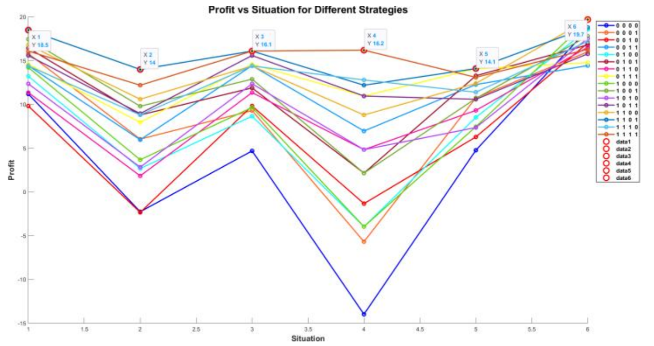

Figure 2. The profitability of the decision scheme in different situations

3.2.3. Result Analysis

By analyzing the line chart, we find that there are significant differences in the optimal strategy for different production situations. For example, case 6 performs best with a net profit of 20.9524, while in case 1, the X1 strategy is optimal with a net profit of 19.04. This shows that enterprises need to consider the specific conditions of a particular situation when choosing a production strategy. The fluctuation of net profit also suggests that enterprises should conduct detailed risk assessments and cost-benefit analyses before implementing strategies. In general, data-driven decision-making methods help companies optimize production processes and maximize profits.

3.3. The Establishment and Solution of Problem Three Model

Compared with question 2, in question 3, the production process of the enterprise has changed. Spare parts need to be processed into semi-finished products before they can be assembled into finished products, and the production strategy model of the enterprise can be adjusted according to the change in the production process. First, we focus on the total profit calculation of the finished product and do not calculate the local optimal solution of the semi-finished product. Then, based on the idea of problem two modeling, we extend binary coding and introduce a pruning strategy. Then, simulated annealing and genetic algorithms were used to optimize the model, and the model was solved. Finally, the results are analyzed.

3.3.1. Problem Modeling

The core of the multi-process, multi-part problem is how to combine inspection and disassembly strategies in the production and assembly process to maximize the total profit of the final product. To this end, we first set the key parameters related to each process, then construct a decision tree covering all possible strategies, reduce the calculation of invalid strategies by pruning strategies, and finally optimize the total profit by simulated annealing and genetic algorithms.

In question 3, we divide the production process into several stages: part processing, semi-finished product processing, finished product assembly, and final inspection. To represent each possible production strategy, we extend the problem by binary coding. In this extension, we represent each production, inspection, and disassembly operation with a single binary bit, forming a 16-bit binary string. These binary bits represent:

The first 8 bits indicate whether each part is tested; the 9-11 bits indicate whether each semi-finished product is tested; the 12-14 bits indicate whether each semi-finished product is disassembled; the 15 bits indicate whether the finished product is tested; the 16 bits.

With this coding, we can enumerate all possible strategies and lay the foundation for subsequent profit calculations and optimizations.

3.3.2. The Introduction of Pruning Strategy

To optimize the calculation process and reduce the calculation of ineffective strategies, we introduce pruning strategies. Specifically, we do pruning according to the following rule:

If a semi-finished product has not been tested, it is not allowed to be dismantled.

The pruning strategy reduces the computational complexity by eliminating strategies that do not meet the constraints in advance.

In order to solve the problem of profit optimization for a single finished product, we first consider all possible production and inspection strategies. By calculating the corresponding profits of each strategy one by one, we can find the optimal strategy, that is, the strategy of profit maximization. As shown in the figure below, when only the finished product is tested and disassembled, the maximum profit value is 73.20.

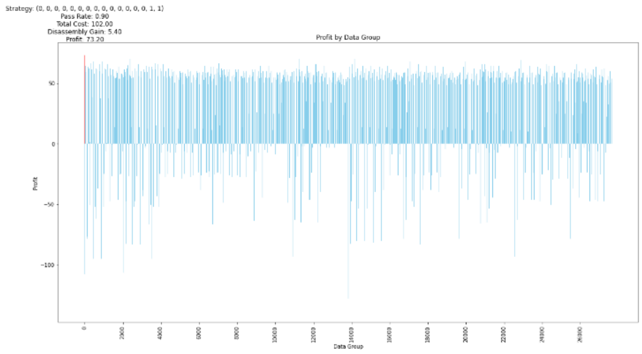

Figure 3. Results under different strategies

3.3.3. Result Analysis

The picture shows that under the strategy of only testing and disassembling the finished product, the maximum profit that the enterprise can achieve is 73.20. This result highlights that focusing on quality control of the final product is an effective profit maximization strategy in a specific production decision model. By optimizing the inspection process and reducing unnecessary intermediate tests, enterprises can reduce costs and improve profits. This suggests that targeted quality control measures are more cost-effective than comprehensive inspection in production management and may also improve the overall efficiency of the production process.

3.4. The Establishment and Solution of Problem 4 Model

Question 4 requires that Question 2 and Question 3 be re-completed on the assumption that the defective rates of all parts, semi-finished products, and finished products in Question 2 and Question 3 are obtained by sampling inspection methods. To solve this problem systematically, we need to combine the sampling detection scheme designed in Problem 1 and apply it to the decision-making process of each link in Problem 2 and Problem 3.

3.4.1. Problem Modeling

At present, we have obtained the defective rate data of the above products through sampling inspection. To further improve the accuracy of the data, we will apply Bayes' theorem.

$$P\left(B_i \middle| A\right)=\frac{P\left(B_i\right)P\left(A\middle|B_i\right)}{P\left(A\right)}=\frac{P\left(B_i\right)P\left(A\middle|B_i\right)}{\sum_{j=1}^{n}P\left(B_j\right)P\left(A\middle|B_j\right)}$$

Make in-depth inferences to recalculate defective rates for various parts.

3.4.2. Result Solving

Re-solving result of Problem 3: The updated defective rate was input into the model of Problem 2 and Problem 3, and the profit increased to 76.18 yuan (Figure 4).

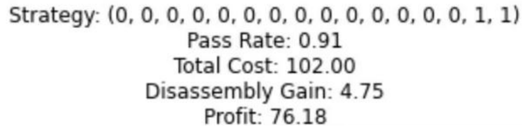

Figure 4. Problem 3 Recalculate the result graph

3.4.3. Result Analysis

The above results show a significant improvement over previous data, which not only proves the significant advantage of Bayesian estimation in providing a more accurate prediction of defective rates but also enables the decision model to optimize production strategies more efficiently, thus significantly improving overall profits. At the same time, this further verifies the practical application value of the Bayes method in the field of production decision-making. It can dynamically adjust according to sampled data, flexibly cope with uncertainties in the production process, and ensure continuous optimization and efficiency improvement of the production process.

4. Conclusion Analysis

This study examines the impact of defective parts rates and inspection costs on production decisions, showing that both factors reduce profits, with inspection costs having a diminishing effect. The proposed model integrates statistical methods, operations research, and optimization algorithms to enhance decision-making in electronic manufacturing. Despite limitations, such as data dependency and process assumptions, the model offers scalability and potential for broader industrial applications, promoting cost-effective and intelligent production processes.

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