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Research on Risk Propagation in Complex Supply Chain Networks Based on SEIR Model

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Abstract: The complexity of supply chain network structure makes the risk propagation among the enterprises in the supply chain more and more complicated. This paper constructs a SEIR risk propagation model for complex supply chain networks based on the analytical idea of contagion model, which reveals the risk propagation mechanism of supply chain networks from a macro perspective. The article verifies the existence of risk propagation threshold and the stability of different equilibrium points by analyzing the basic regeneration number, equilibrium point and stability of the model; explores the effectiveness of different parameters on the basic regeneration number by combining sensitivity analysis; and verifies them by using MATLAB simulation software. The results show that the basic regeneration number is a key factor affecting the risk propagation, and the risk diffusion in this system can be effectively curbed by controlling the basic regeneration number.

Keywords: supply chain network, risk propagation, SEIR model, fundamental regeneration number, simulation.

1. Introduction

In recent years, due to trade protectionism, natural disasters, COVID-19, regional conflicts, energy crisis and other factors, the global supply chain risk events are frequent, leading to an increasingly serious problem of risk transmission in the supply chain. In the context of economic and trade globalization, risk factors are ubiquitous. Especially in the information age, the interaction between various actors in the supply chain becomes increasingly close and complex, which makes the supply chain system increasingly large. The traditional chain structure of the supply chain gradually develops into a complex network structure. Risk events will quickly spread to the whole supply chain network through various ways, which poses a serious threat to the benign development of supply chain and the smooth operation of economy. In this context, it is of great theoretical and practical significance to study the transmission mechanism of risk in the supply chain network, analyze the key factors of risk diffusion, and seek effective measures to inhibit risk transmission for improving the security and stability of the supply chain and promoting the healthy development of the supply chain architecture.

LinFR et al. (1998) [1] first introduced network science into the field of supply chain. They believed that the supply chain network was a complex "supply-production-marketing" network composed of suppliers, manufacturers and distributors, which depended on each other in terms of strategy, resources, capabilities and business. Choi et al. (2001) [2] studied supply chain network from the perspective of complex Adaptive system theory (CAS) for the first time, and created a new perspective and direction for the study of complex supply chain network. Thadakamalla et al. (2004) [3] first proposed the structural model of supply chain network. Zhao et al. (2011) [4] established a Hybrid and Tunable (hybrid adjustable) supply chain network growth model. Chen-Yang et al. (2014) [5] analyzed the structural complexity of supply chain networks and argued that the uncertainty of supply chain networks increases with the increase of complexity. Some scholars have noted that the diffusion behavior of infectious diseases in social networks is very similar to that of risks in supply chain networks in system dynamics. They have studied the risk transmission behavior in supply chain networks by combining the infectious disease model with complex network and dynamic characteristics theories. For example, Mcfarland et al. (2008) [6] verified for the first time that the phenomenon of "contagion" also exists in the supply chain. He believed that the contagion behavior between enterprises in the supply chain is a process of transmission from a bilateral node to adjacent bilateral nodes, and adjacent nodes have imitability and infectivity. Cheng (2008) [7] et al. studied the evolution process and complexity of risks in supply chain network. He believed that the transmission of risks in supply chain network is mainly composed of four elements: preconditions, risk events, propagation path and influence. Qiu Yinggui [8] simulated the risks of supply chain network with Petri Nets and studied the problems of risk control, risk assessment and structure optimization. Yang Kang et al. (2013) [9] applied the transmission dynamics theory of complex networks to the study of supply chain risk transmission for the first time. Based on the SIS model of infectious diseases, the risk transmission model of supply chain network was established, and the risk elements of supply chain network were analyzed. Zhao Gang et al. (2015) [10] studied the change of risk diffusion characteristics under the evolution of supply chain network structure by considering the probability of risk transmission among supply chain network agents and setting the threshold of subject infection risk. Yi Huini (2015) [11] applied the SIR Model to the study on the risk transmission and control of the automobile supply chain. She believed that the risk transmission mechanism of the automobile supply chain was similar to that of infectious diseases, and the transmission process was mainly influenced by infectious rate, node degree and recovery rate. Ganyi (2019) [12] combined the complex network theory with the characteristics of supply chain risk transmission in the automobile manufacturing industry, conducted an in-depth study on the infection rate and recovery rate based on the SIR Model, and considered several risk immunity strategies. Based on the average field theory, HUO LA et al. [13] (2019) conduct numerical simulation through the SIR Model to analyze the influence of herd psychology and risk preference on risk transmission in the supply chain network. Zhao Meng (2020) [14] established a BSR-RP model reflecting the characteristics of enterprise supply chain risk communication, conducted simulation analysis on various influencing factors of risk communication from the perspective of the whole supply chain, and the results showed that the degree of risk communication was closely related to the enterprise's own ability.

Because the transmission behavior of risk in a complex supply chain network is very similar to the transmission behavior of infectious disease in the population, the similarity of this transmission behavior makes the infectious disease model have strong applicability in the risk transmission model of supply chain network, and some scholars have carried out research on this. Yang Kang et al. (2013) analyzed the similarities of the two communication environments and processes. Zhao Gang et al. (2015) investigated and analyzed the characteristics of supply chain network domain groups. Zhao Meng et al. (2020) noted that both propagation directions were undirected radially. Therefore, the transmission behavior of risk in supply chain network is highly similar to that of virus in social network. This similarity of transmission behavior makes the application of dynamic infectious disease model in the study of supply chain network risk transmission of theoretical significance.

2. Model

2.1. Hypothesis

The complex supply chain network in this paper refers to the supply chain which is formed by the cross connection of many enterprises and presents the form of network structure. Its complexity is reflected in the quantity and structure and so on. Before modeling, make the following assumptions:

Hypothesis 1. Nodal enterprises in the supply chain network have no significant difference in their ability to resist risks, and all nodal enterprises are not immune to risks. All nodal enterprises have the same probability of being infected after contact with risk-infected enterprises.

Hypothesis 2. Complex supply chain network is a relatively open system with a network structure, and the number of its nodes changes dynamically. Some new enterprises enter the system in pursuit of profits, while others exit the system due to bankruptcy, strategic transformation and other reasons. In this paper, the total number of nodal enterprises in the supply chain network is in a dynamic equilibrium state. Suppose that the entry rate of the new firm and the exit rate of the original firm are $\mu\ (0 < \mu < 1)$.

Hypothesis 3. According to the risk status of each node enterprise, it can be divided into the following four categories: (1) The number of risk susceptible enterprises at moment t remains $S(t)$; (2) The number of risk exposed enterprises at moment t remains $E(t)$. (note: risk prone enterprises will not be infected immediately after contact with risk infected enterprises, but will use their own elastic and redundant production capacity to fight against them, but have not shown symptoms for the time being, and are in risk exposed stage); (3) The number of risk infected enterprises at moment t remains $I(t)$; (4) The number of risk recovered enterprises at moment t remains $R(t)$. At moment t, the total number of nodal enterprises in the whole supply chain network remains $N(t)$. (P.S. Due to the total number of node enterprises in supply chain network remain unchanged, so $N(t)$ can also be referred to N). The total number of node enterprises at moment t is

$$N(t) = S(t) + E(t) + I(t) + R(t) \tag{1}$$

Hypothesis 4. Due to the correlation and complexity of the structure among the main bodies of the supply chain network, risks will be transmitted to each other due to direct supply and marketing or business transactions. However, the transmission of risks among the main bodies of the supply chain network is not inevitable, but conducted with a certain probability.

Hypothesis 5. Enterprises under the four risk states will transform with a certain probability. At the time T, the total number of enterprises N remains unchanged. After a certain risk infected enterprise contacts with the risk infected enterprise, it transforms into a risk latent enterprise with a certain probability. The conversion rate is expressed by $\beta\ (0 < \beta < 1)$. A risk latent enterprise is converted into a risk infected enterprise with a certain probability, and the conversion rate is denoted as $\alpha\ (0 < \alpha < 1)$. A risk infected enterprise recovers from risk infection with a certain probability and transforms into a risk recovered enterprise. The conversion rate is denoted as $\gamma\ (0 < \gamma < 1)$. The parameters are described as follows

Table 1. Related parameters of SEIR model

Parameter	Description
$\boldsymbol{\alpha}$	Risk infection rate of risk latent enterprise
$\boldsymbol{\beta}$	The probability that the risk is latent
$\boldsymbol{r}$	Recovery rate of infected enterprises
$\boldsymbol{\mu}$	The entry rate and exit rate of enterprises

Hypothesis 6. The nodal enterprises in the supply chain network with risk infection have antibodies to become risk recovery enterprises after recovery from risk, and have immunity during the T period, will not be infected by risk again, and will not have risk transmission.

Hypothesis 7. $P_0 = (S_0, E_0, I_0, R_0)$ was set as the zero equilibrium point (ZEP) of risk transmission in the system, indicating that the number of risk infected enterprises in the supply chain network gradually decreased and finally cleared to zero. $P^* = (S^*, E^*, I^*, R^*)$ is the non-zero equilibrium point (N-ZEP) of risk transmission, indicating that the number of risk infected enterprises in the supply chain network remains constant and cannot be eliminated.

2.2. SEIR System Dynamics Model

According to the above assumptions, the risk propagation process based on SEIR model in this complex supply chain network is shown in Figure 1.

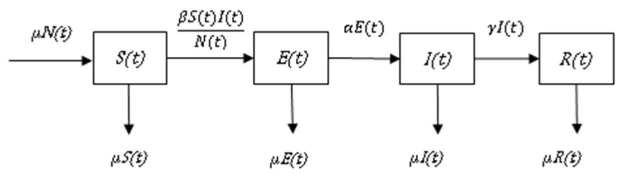

Figure 1. Risk propagation model of supply chain network

On this basis, the SEIR system dynamics differential equation of risk propagation in complex supply chain network is constructed as follows:

$$\begin{cases} \frac{dS(t)}{dt} = \mu N(t) - \frac{\beta S(t)I(t)}{N(t)} - \mu S(t) \\ \frac{dE(t)}{dt} = \frac{\beta S(t)I(t)}{N(t)} - \alpha E(t) - \mu E(t) \\ \frac{dI(t)}{dt} = \alpha E(t) - \gamma I(t) - \mu I(t) \\ \frac{dR(t)}{dt} = \gamma I(t) - \mu R(t) \end{cases} \tag{2}$$

It can be seen from the actual situation that the four groups of S, E, I and R are all positive numbers, that is, the initial conditions of the system $S(t) \geq 0$、$E(t) \geq 0$、$I(t) \geq 0$、$R(t) \geq 0$. Its state space $\Omega = \{S(t), E(t), I(t), R(t):$ $S(t) \geq 0$、$E(t) \geq 0$、$I(t) \geq 0$、$R(t) \geq 0\}$. Since Ω is a positive invariant set, we only need to consider the solution whose initial condition is in Ω, which has existence and uniqueness in Ω. Equation (2) represents the relationship between the number of enterprises in each risk state and time t. N represents the total number of enterprises in the supply chain network at time t, $0 < \alpha$、β、γ, $\mu < 1$, adding the four formulas in $N = S(t) + E(t) + I(t) + R(t)$, to obtain $\frac{d(S+E+I+R)}{dt} = \frac{dN}{dt} = 0$.

2.3. Zero equilibrium and non-zero equilibrium

chain network, at t=0, nodal enterprises with the number of I_0 are infected by the risk and become the source of infection. Meanwhile, the infection risk spreads along the risk-prone enterprises associated upstream and downstream of the supply chain network according to a certain conversion rate. In system dynamics, the equilibrium point and its stability theory are often used to study the trend and characteristics of differential equations. The equilibrium point of the equation set (2) in this paper means that in the final state of risk propagation and evolution, the system is in an equilibrium state, that is, the number of enterprises in the four risk states will no longer change. Then, if the terms of the equation (2) are equal to 0, it can be obtained

$$\begin{cases} \frac{dS(t)}{dt} = \mu N(t) - \frac{\beta S(t)I(t)}{N(t)} - \mu S(t) = 0 \\ \frac{dE(t)}{dt} = \frac{\beta S(t)I(t)}{N(t)} - \alpha E(t) - \mu E(t) = 0 \\ \frac{dI(t)}{dt} = \alpha E(t) - \gamma I(t) - \mu I(t) = 0 \\ \frac{dR(t)}{dt} = \gamma I(t) - \mu R(t) = 0 \end{cases} \tag{3}$$

The solution of the formula (3) is the balance point of risk transmission.

In the formula (3), if E=0 and I=0, the zero equilibrium P_0 can be calculated as

$$P_0 = (\mathrm{S_0}, \mathrm{E_0}, \mathrm{I_0}, \mathrm{R_0}) = (N, 0,0,0) \tag{4}$$

In the formula (3), if $E \neq 0, I \neq 0$, calculate the non-zero equilibrium point $P^* = (S^*, E^*, I^*, R^*)$, where

$$S^* = \frac{N(\alpha + \mu)(\mu + \gamma)}{\alpha\beta} \tag{5}$$

$$E^* = \frac{N\mu[-\mu^2 - (\gamma + \alpha)\mu - (\gamma - \beta)\alpha - \gamma]}{\alpha\beta(\alpha + \mu)} \tag{6}$$

$$I^* = \frac{N\mu[\alpha\beta - \mu^2 - (\gamma + \alpha +)\mu - \gamma\alpha^2]}{\beta(\alpha + \mu)(\mu + \gamma)} \tag{7}$$

$$R^* = \frac{N[(\alpha\beta - \mu^2 - (\gamma + \alpha)\mu - \gamma\alpha^2)(\mu + \gamma\alpha)]}{\beta(\alpha + \mu)(\mu + \gamma)\alpha} \tag{8}$$

2.4. Basic Regeneration Numberl (R_0)

In the infectious disease model, the basic regeneration number R_0 [15] refers to the average number of individuals infected by an infected individual in the average infection period, which is the threshold to judge whether the infectious disease will die out. In this paper, the basic regeneration number refers to the average number of enterprises infected by a risky enterprise when all enterprise nodes are vulnerable to risk infection at the initial stage of risk transmission in the entire supply chain network. $R_0 = 1$ is the threshold to distinguish risk transmission and determine whether the result of risk transmission is at zero equilibrium point or non-zero equilibrium point. When $R_0 < 1$, it indicates that risk will gradually disappear in the supply chain network. On the contrary, when $R_0 > 1$, the risk will always exist and cannot be eliminated.

In this paper, the "next generation matrix method" [16] is adopted to isolate the newly infected population and calculate the basic regeneration number R_0 according to the system dynamics model (2), denoted P={(S,E,I,R)}^T, then the formula (2) can be abbreviated as

$$\frac{dP}{dt} = \tilde{F}(X) - \tilde{V}(X) \tag{9}$$

$\tilde{F}(X)$与$\tilde{V}(X)$分别为

$$\tilde{F}(X) = \begin{bmatrix} 0 \\ \frac{\beta S(t)I(t)}{N} \\ 0 \\ 0 \end{bmatrix}$$
$$\tilde{V}(X) = \begin{bmatrix} -\mu N(t) + \frac{\beta S(t)I(t)}{N(t)} + \mu S(t) \\ \alpha E(t) + \mu E(t) \\ -\alpha E(t) + \gamma I(t) + \mu I(t) \\ -\gamma I(t) + \mu R(t) \end{bmatrix} \tag{10}$$

$\tilde{F}(X)$ represents the number of newly infected enterprises among the four risk types of enterprises. Since only category E and category I contain infected companies, the government ordered

$$F(X) = \begin{bmatrix} \frac{\beta S(t)I(t)}{N(t)} \\ 0 \end{bmatrix}$$
$$V(X) = \begin{bmatrix} \alpha E(t) + \mu E(t) \\ -\alpha E(t) + \gamma I(t) + \mu I(t) \end{bmatrix} \tag{11}$$

Compute the Jacobian matrix of $F(X)$ and $V(X)$ and substitute the values of the zero equilibrium point P_0 into the Jacobian matrix of the sum

$$F = \begin{bmatrix} 0 & \beta \\ 0 & 0 \end{bmatrix}$$
$$V = \begin{bmatrix} \alpha + \mu & 0 \\ -\alpha & \gamma + \mu \end{bmatrix} \qquad (12)$$

Obtain the regeneration matrix FV^{-1}

$$FV^{-1} = \begin{bmatrix} \frac{\beta\alpha}{(\alpha+\mu)(\mu+\gamma)} & \frac{\beta}{\gamma+\mu} \\ 0 & 0 \end{bmatrix} \qquad (13)$$

It can be obtained that the spectral radius of the matrix FV^{-1} is $\frac{\beta\alpha}{(\alpha+\mu)(\mu+\gamma)}$, and $R_0 = \rho\left(\mathrm{FV}^{-1}\right)$ is the maximum eigenvalue of FV^{-1} when calculating the basic regeneration number

$$R_0 = \frac{\beta\alpha}{(\alpha+\mu)(\mu+\gamma)} \qquad (14)$$

The basic regeneration number R_0 is the threshold for distinguishing the prevalence of infectious diseases. According to R_0 we get the following property.

Property 1 : If the basic regeneration number $R_0 = \frac{\beta\alpha}{(\alpha+\mu)(\mu+\gamma)} < 1$, the SEIR system dynamics model(2) of risk propagation will evolve to the zero equilibrium point, where it is locally asymptotically stable.

Proof: The *Jacobian* matrix of equation (2) at the ZEP P_0 is

$$J_{P_0} = \begin{bmatrix} -\mu & 0 & -\beta & 0 \\ 0 & -(\alpha+\mu) & \beta & 0 \\ 0 & \alpha & -(\gamma+\mu) & 0 \\ 0 & 0 & \gamma & -\mu \end{bmatrix} \qquad (15)$$

The corresponding characteristic equation of this matrix is：

$$\lambda^4 + (4\mu + \gamma + \alpha)\lambda^3 + (-\alpha\beta + \alpha\gamma + 3\alpha\mu + 3\gamma\mu + 6\mu^2)\lambda^2 + (-2\mu\alpha\beta + 2\alpha\gamma\mu + 3\alpha\mu^2 + 3\gamma\mu^2 + 4\mu^3)\lambda + \mu^2(-\alpha\beta + \alpha\gamma + \alpha\mu + \gamma\mu + \mu^2) = 0$$

Where,

$$b_1 = 4\mu + \gamma + \alpha \qquad (17)$$

$$b_2 = \alpha(3\mu - \beta + \gamma) + 3\gamma\mu + 6\mu^2 \qquad (18)$$

$$b_3 = \mu(-2\alpha\beta + 2\alpha\gamma + 3\mu\alpha + 3\gamma\mu + 4\mu^2) \qquad (19)$$

$$b_4 = \mu^2\left(\mu^2 + (\gamma+\alpha)\mu - \alpha(\beta - \gamma)\right) \qquad (20)$$

The ZEP P_0 is locally asymptotically stable if and only if all eigenvalues of the characteristic equation have negative real parts. According to *Routh–Hurwitz stability criterion*, if $b_1 > 0, b_4 > 0, b_1b_2 > b_3, b_3(b_1b_2 - b_3) > {b_1}^2b_4$, then all the eigenvalues of the eigenequation have negative real parts. By calculation, when $R_0 < 1$, the condition $\beta\alpha < (\alpha+\mu)(\mu+\gamma)$ is obtained. Under this condition, $b_1 > 0, b_2 > 0, b_3 > 0, b_4 > 0, b_1b_2 - b_3 > 0, b_1b_2b_3 > {b_3}^2 + {b_1}^2b_4$. Therefore, when $R_0 < 1$, the system model is locally asymptotically stable at the ZEP P_0.

Property 2 If the basic regeneration number $R_0 = \frac{\beta\alpha}{(\alpha+\mu)(\mu+\gamma)} > 1$, the (2) equation of the *SEIR system dynamics model* of risk propagation will evolve to a N-ZEP P^*, where it is locally asymptotically stable.

Proof: The *Jacobian* matrix of equation (2) at the N-ZEP $X^{(0)}$ is

$$J_{P^*} = \begin{bmatrix} -\beta I^* + \mu & 0 & -\beta S^* & 0 \\ \beta I^* & -(\alpha+\mu) & \beta S^* & 0 \\ 0 & \alpha & -(\gamma+\mu) & 0 \\ 0 & 0 & \gamma & -\mu \end{bmatrix} \qquad (21)$$

The corresponding eigenequation of this matrix is: $\lambda^4 + (2\mu + \gamma + \alpha\beta I^*)\lambda^3 + \left((-\beta S^* + \beta I^* + \gamma + \mu)\alpha + (3\mu + \gamma)\beta I^* + \gamma\mu\right)\lambda^2 + (-2\mu^3 + (-\alpha - \gamma + 3\beta I^*)\mu^2 + 2\beta I^*(\gamma+\alpha)\mu + \alpha\gamma\beta I^*)\lambda + \mu\left(-\mu^3 + (-\alpha - \gamma + \beta I^*)\mu^2 + \left((\beta S^* + \beta I^* - \gamma)\alpha + \beta I^*\gamma\right)\mu + \alpha\gamma\beta I^*\right) = 0$

Where,

$$\lambda^4 + c_1\lambda^3 + c_2\lambda^2 + c_3\lambda + c_4 = 0 \qquad (22)$$

$$c_1 = 2\mu + \gamma + \alpha\beta I^* \qquad (23)$$

$$c_2 = (-\beta S^* + \beta I^* + \gamma + \mu)\alpha + (3\mu + \gamma)\beta I^* + \gamma\mu \qquad (24)$$

$$c_3 = -2\mu^3 + (-\alpha - \gamma + 3\beta I^*)\mu^2 + 2\beta I^*(\gamma+\alpha)\mu + \alpha\gamma\beta I^* \qquad (25)$$

$$c_4 = \mu\left(-\mu^3 + (-\alpha - \gamma + \beta I^*)\mu^2 + \left((\beta S^* + \beta I^* - \gamma)\alpha + \beta I^*\gamma\right)\mu + \alpha\gamma\beta I^*\right) \qquad (26)$$

The N-ZEP P^* is locally asymptotically stable if and only if all eigenvalues of the eigenequation have negative real parts. According to *Routh–Hurwitz stability criterion*, if $c_1 > 0, c_4 > 0, c_1c_2 > c_3, c_3(c_1c_2 - c_3) > {c_1}^2c_4$, then all the eigenvalues of the eigenequation have negative real parts. We can see that $d_1 > 0, d_2 > 0$, and when $R_0 > 1$, the condition $\beta\alpha > (\alpha+\mu)(\mu+\gamma)$ is obtained. Under this condition, $c_3 > 0, c_4 > 0, c_1c_2 - c_3 > 0, c_1c_2c_3 > {c_3}^2 + {c_1}^2c_4$. Therefore, when $R_0 > 1$, the system model is locally asymptotically stable at the N-ZEP P^*.

The above results show that if the basic regeneration number $R_0 < 1$, then (2) is asymptotically stable at the zero equilibrium point P_0, that is, the number of enterprises infected by the risk in the supply chain network gradually decreases until zero. If $R_0 > 1$, then (2) is locally asymptotically stable at the non-zero equilibrium point P^* that is, the number of enterprises infected by the risk in the supply chain network remains constant and the risk is still spreading. Therefore, necessary measures should be taken to control the basic regeneration number R_0 within a reasonable range.

2.5. Sensitivity analysis of R_0

The propagation of risks in the supply chain network largely depends on the basic regeneration number R_0. In order to control the propagation of risks in the supply chain network, it is necessary to consider the influence factors in the process of risk propagation, that is, to analyze the influence of relevant parameters on the basic regeneration number. In this paper, sensitivity analysis is used to calculate the influence index of each parameter on R_0. According to R_0, the stability of basic regeneration number $R_0 = \frac{\beta\alpha}{(\alpha+\mu)(\mu+\gamma)}$ depends on the validity of parameter α、β、γ、μ.

Property 3: $\frac{\partial R_0}{\partial \mu} < 0, \frac{\partial R_0}{\partial \gamma} < 0$, indicating that with the increase of the enterprise recovery rate γ and the increase of the entry (exit) rate μ of new enterprises in the supply chain network, the basic regeneration number R_0 gradually decreases. Furthermore, if $\beta\alpha - \beta\alpha\big(D(\alpha)(\gamma+\mu) + D(\mu)(\gamma+\mu)\big) < 0$, then $\frac{\partial R_0}{\partial \mu} < \frac{\partial R_0}{\partial \gamma}$, then the effect of recovery rate γ on the basic regeneration number R_0 is stronger than that of enterprise entry (exit) rate μ.

Proof: $\frac{\partial R_0}{\partial \mu} = -\frac{\beta\alpha\left(D(\alpha)(\gamma+\mu)+D(\mu)(\gamma+\mu)\right)}{\left((\alpha+\mu)(\gamma+\mu)\right)^2} < 0$,

if $\frac{\partial R_0}{\partial \gamma} = -\frac{\beta\alpha(\gamma+\mu)}{\left((\alpha+\mu)(\gamma+\mu)\right)^2} < 0$,

$\frac{\partial R_0}{\partial \mu} - \frac{\partial R_0}{\partial \gamma} = -\frac{\beta\alpha\left(D(\alpha)(\gamma+\mu)+D(\mu)(\gamma+\mu)\right)+\beta\alpha}{\left((\alpha+\mu)(\gamma+\mu)\right)^2} < 0$,

then $\frac{\partial R_0}{\partial \mu} < \frac{\partial R_0}{\partial \gamma}$.

Property 3 shows that with the increase of the number of new enterprises and the decrease of the number of original enterprises, the enterprises in the supply chain network are maintaining a dynamic balance of upgrading, and the increase of μ can inhibit the spread of risks. As the number of risk recovery enterprises in the supply chain network increases, such enterprises can effectively block the spread of risk in the supply chain network because they are risk resistant and will not re-infect the risk for a certain period of time, so the risk will gradually die out with the increase of γ. When $\frac{\partial R_0}{\partial \mu} < \frac{\partial R_0}{\partial \gamma}$, the basic regeneration number R_0 decreases faster with the increase of recovery rate γ than with the increase of entry rate μ. At this point, in order to more effectively control the spread of risks in the supply chain network, measures should be taken to increase the recovery rate γ.

Property 4: $\frac{\partial R_0}{\partial \beta} > 0, \frac{\partial R_0}{\partial \alpha} > 0$, represents that with the increase of latent conversion rate β and latent infection α, the basic regeneration number R_0 gradually increases. Furthermore, if $\alpha^2 + \alpha\mu - \mu\beta > 0$, then $\frac{\partial R_0}{\partial \beta} > \frac{\partial R_0}{\partial \alpha}$, then the effect of latent conversion rate β on basic regeneration number R_0 is stronger than that of latent infection rate α.

Proof: $\frac{\partial R_0}{\partial \beta} = \frac{\alpha}{(\alpha+\mu)(\mu+\gamma)} > 0, \frac{\partial R_0}{\partial \alpha} = \frac{\mu\beta}{(\alpha+\mu)^2(\mu+\gamma)} > 0$,

$\frac{\partial R_0}{\partial \beta} - \frac{\partial R_0}{\partial \alpha} = \frac{\alpha}{(\alpha+\mu)(\mu+\gamma)} - \frac{\mu\beta}{(\alpha+\mu)^2(\mu+\gamma)} = \frac{\alpha^2+\alpha\mu-\mu\beta}{(\alpha+\mu)^2(\mu+\gamma)}$,

if $\alpha^2 + \alpha\mu - \mu\beta > 0$, then $\frac{\partial R_0}{\partial \beta} > \frac{\partial R_0}{\partial \alpha}$.

Property 4 shows that the enterprises in the risk infection in the supply chain network are spreading the risk to the upstream and downstream related enterprises through business transactions, making the number of enterprises with potential risks increasing, and then the number of enterprises with latent risks turning into enterprises with risk infection increases, and the risk is in the stage of diffusion. At this time, relevant departments or enterprises should take timely countermeasures to control latent conversion rate β and latent infection rate α, so as to reduce the basic regeneration number R_0 and inhibit the spread of risks in the supply chain network. When $\frac{\partial R_0}{\partial \beta} > \frac{\partial R_0}{\partial \alpha}$ is used, the basic regeneration number R_0 increases with the increase of latent conversion rate β faster than the increase of infection rate α. At this time, in order to more effectively control the spread of risks in the supply chain network, measures should be taken to reduce the latent conversion rate β.

3. Simulation

In order to visually describe the communication rules of supply chain network risk and the role of government intervention under public emergencies, MATLAB was used to conduct simulation experiments.

3.1. Parameter setting

When the risk breaks out in the supply chain network, it is assumed that in the initial state, there are I_0 enterprises infected with the risk and become the source of risk transmission. During the period T, the risk is spread in the supply chain network through the upstream and downstream related enterprises. The parameters are shown in Table 2.

Table 2. Related parameters of simulation experiment

Parameter	Definition	Estimated value
α	Infection rate	0.25
β	Latent conversion rate	0.40
r	Risk recovery rate	0.38
μ	Entry (exit) rate	0.03
N	Number of enterprises in period T period	1000
T	Time (day)	300
I_0	Number of enterprises initially infected	150

3.2. Zero Equilibrium Point simulation experiment

The basic regeneration number $R_0 = 0.37037 < 1$ of (2) in this case is calculated according to the parameter values in Table 3. It can be seen that the risk will not spread continuously, and there is a zero equilibrium point in the system.

Simulation Experiment 1: According to (2), simulation experiments are conducted on parameters in Table 3, and simulation results of the system with zero equilibrium point are obtained, as shown in Figure 2. It shows that in period T, the number of risk-susceptible enterprises experienced a rapid decline in a short period, and gradually rose steadily to 1000 when the number reached about 700. The number of latent enterprises increased rapidly in a short period of time, and the number began to decline until zero at 80; From the initial number of 150, the number of enterprises with infection risk showed a gradual decline, indicating that because $R_0 = 0.37037 < 1$, the risk did not continue to break out in the system, but gradually dissipated. The number of recovered enterprises increased rapidly to 280 within 0-30 days, and then gradually decreased to zero. The above results show that when $R_0 < 1$, ZEP P_0 exists in the system. In this case, although there is risk diffusion among enterprises in the supply chain network in a certain period of time, enterprises with risk infection and enterprises with latency will gradually recover from the risk at a certain rate until they break away from the risk. This is consistent with the result in property 1 that the ZEP P_0 of the system is locally asymptotically stable on T when$R_0 < 1$.

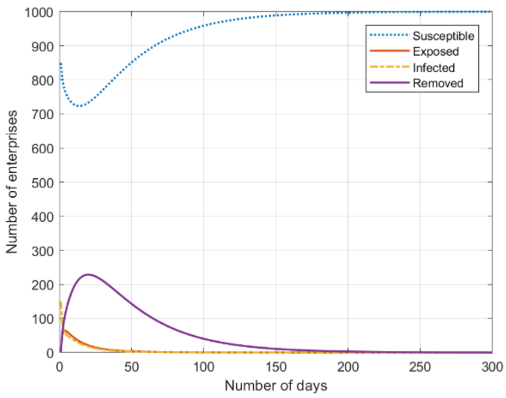

Figure 2. Simulation experiment of ZEP

Simulation experiment 2: In order to observe the impact of the change of infection rate on risk transmission and R_0, the infection rate α of latent enterprises was adjusted on the basis of experiment 1, where $\alpha = 0.25$ was the initial value.

Table 3. Simulation experiment

$\boldsymbol{\alpha}$	0.175	0.25	0.5	0.75
$\boldsymbol{R_0}$	0.296	0.370	0.457	0.563

In experiment 2, the above adjustment is made on α respectively, and the corresponding R_0 is obtained, and R_0 is less than 1. The simulation results are shown in Figure 3 to Figure 6. It can be seen that with the increase of α from $\frac{1}{2}\alpha$ to 3α, R_0 also increases, and the minimum value of the number of risk-sensitive enterprises decreases from 750 to 630. The incubation period for the number of enterprises to fall to 0 was shortened from 53 days to 25 days. The decline in the number of infected enterprises also slowed down from fast; The number of recovered businesses peaked at 340, up from 200, and peaked 15 days earlier. The above results show that the higher the infection rate of latent enterprises α, the faster the risk spreads in the supply chain network and the wider the spread range. However, in general, α has a weak influence on the basic regeneration number R_0.

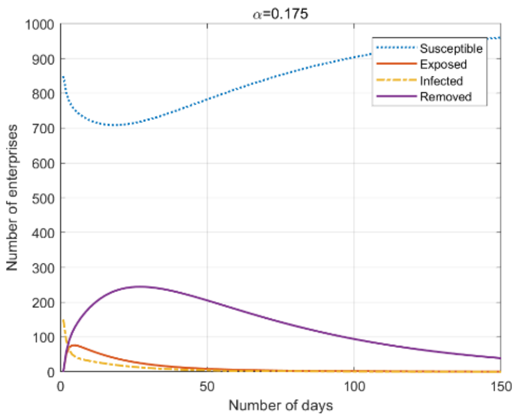

Figure 3. Simulation experiment of α

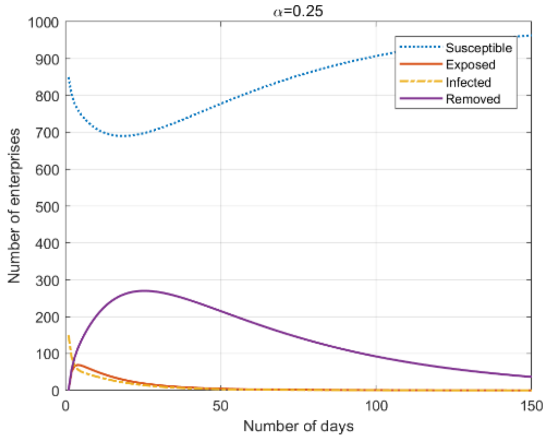

Figure 4. Simulation experiment of α

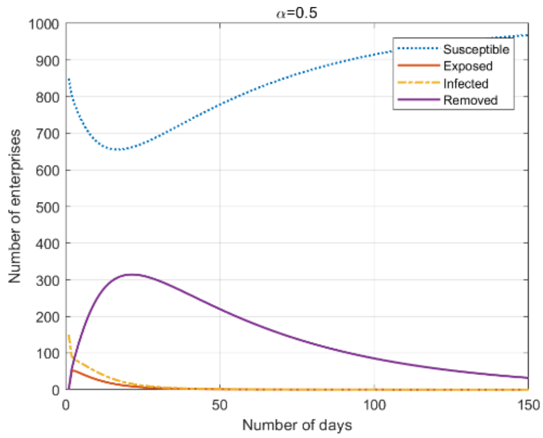

Figure 5. Simulation experiment of α

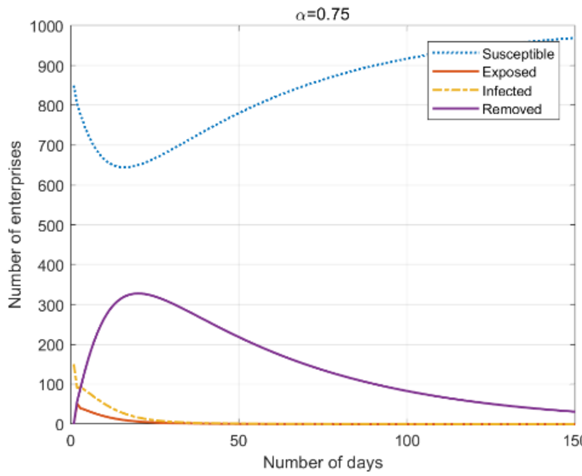

Figure 6. Simulation experiment of α

Simulation experiment 3: In order to observe the influence of the change of latent conversion rate on risk transmission and R0, the latent conversion rate β is adjusted on the basis of experiment 1, where $\beta = 0.4$ is the initial value.

Table 4. Simulation experiment

$\boldsymbol{\beta}$	0.2	0.4	0.54	0.8
$\boldsymbol{R_0}$	0.236	0.370	0.997	1.693

In experiment 3, the above adjustment is made on β and the corresponding R0 is obtained. The results show that when $\beta = 0.54$, R_0=0.967, the system is at the critical value of risk transmission. It can be seen that with β increasing from 0.2

to 0.8, R_0 also increases and breaks the threshold at β=0.55. The simulation results are shown in the figure below. Figure 7 and Figure 8 show that as the infection rate of susceptible firms increased from 0.2 to 0.4, the number of susceptible firms showed a rapid decline from 800 to 680 in 0-25 days. Figure 9 shows that when β=0.54, the system is at the critical value of risk transmission, which shows that the number of risk-infected enterprises always fluctuates around 0 after the 50th day, while the number of susceptible enterprises and recovering enterprises starts to fluctuate significantly after reaching the initial extreme value, indicating that the risk will not stop spreading and has a tendency to continue spreading. Figure 10 shows that when β=0.8, R_0>1, the system is in a state of risk diffusion. The quantitative characteristics of susceptible enterprises and recovering enterprises have more obvious fluctuations as shown in Figure 9, and the number of latent enterprises and the number of infected enterprises gradually reach a dynamic balance, and there is a N-ZEP in the system at this time. The above results show that the change of latent conversion rate β has more effect on the basic regeneration number R_0 than the change of latent enterprise infection rate α. When R_0>1, N-ZEP exists in the system, and the risk propagation phenomenon will reach a dynamic equilibrium at ZEP, which is consistent with the conclusion of Property 2.

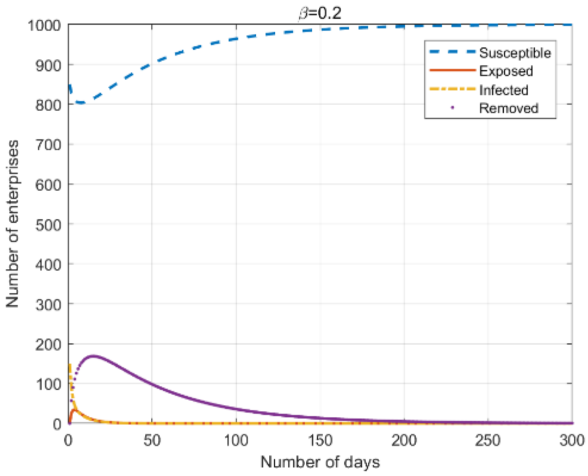

Figure 7. Simulation experiment of β

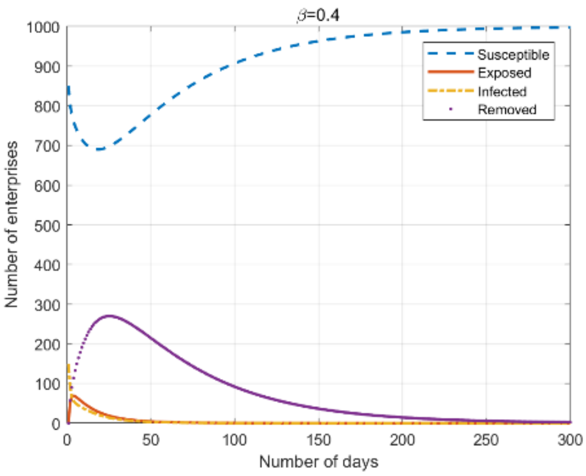

Figure 8. Simulation experiment of β

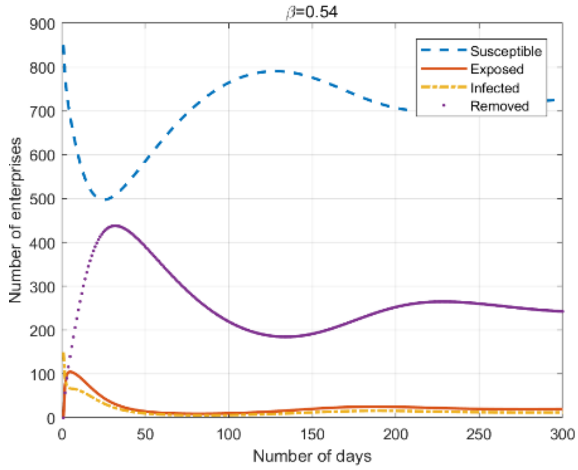

Figure 9. Simulation experiment of β

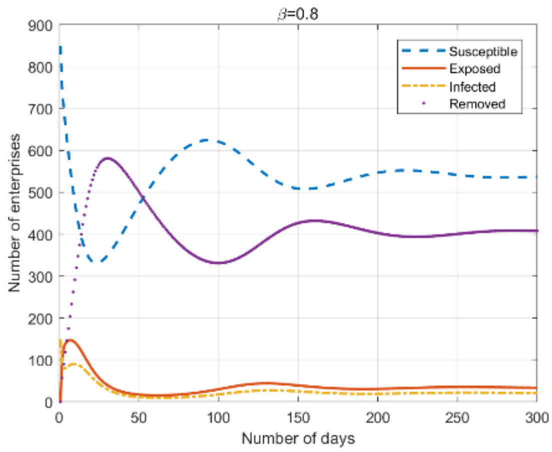

Figure 10. Simulation experiment of β

Simulation experiment 4: In order to observe the impact of recovery rate change on risk propagation and R_0 recovery rate \gamma was adjusted on the basis of experiment 1, where γ =0. 38 was the initial value.

Table 5. Simulation experiment

γ	0.19	0.3	0.38	0.76
R_0	1.526	0.987	0.370	0.203

In experiment 4, we adjusted γ as above and obtained the corresponding R_0. The results show that R_0=0.987 when γ=0.3, the system is at the critical value of risk propagation. As can be seen, when γ increased from 0.19 to 0.76, R_0 also decreased and broke the threshold at γ=0.3. The simulation results are shown in the figure below. Figure 11 shows that when γ=0.19, R_0>1, the system is in a state of risk diffusion, the number characteristics of susceptible enterprises and recovered enterprises fluctuate significantly in Figure 11, and the number of latent enterprises and infected enterprises gradually reach a dynamic balance, and there is a non-zero equilibrium point in the system at this time. Figure 12 shows that when γ increases to 0.3, the system is at the critical value of risk transmission, which shows that the number of risk infected enterprises and latent enterprises fluctuates around 0 from the 50th day. At this time, the number of susceptible enterprises and recovered enterprises still fluctuates greatly after reaching the initial extreme value, indicating that the degree of risk transmission in the system is weakening and tends to be stable. Figure 13 and Figure 14 show that the enterprise recovery rate γ increased from 0.38 to 0.76, which allowed the risk to be cleared more quickly and the time for the number of infected enterprises to fall to 0 was shortened from 50 days to 10 days. The latency period for the number of enterprises to fall to zero was reduced from 50 days to 18 days. The above results show that the increase of enterprise recovery rate γ can effectively control the speed and scope of risk propagation in the system.

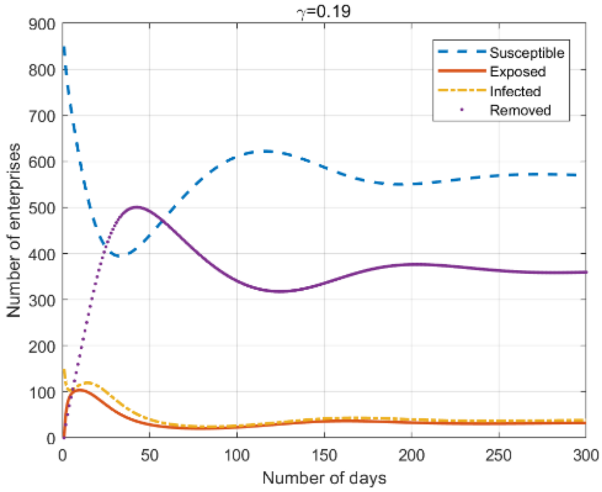

Figure 11. Simulation experiment of γ

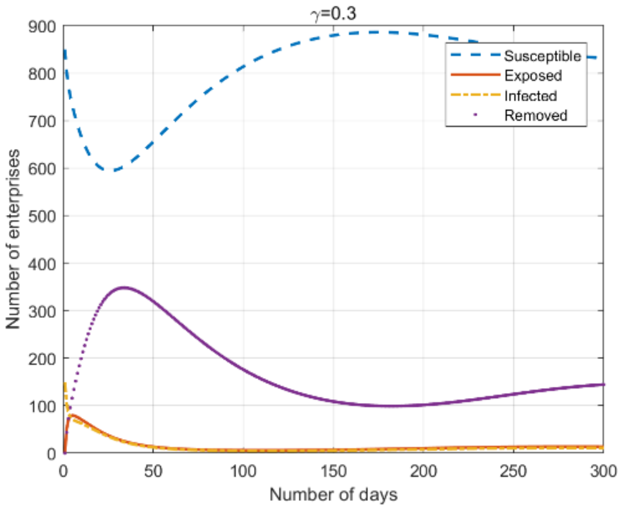

Figure 12. Simulation experiment of γ

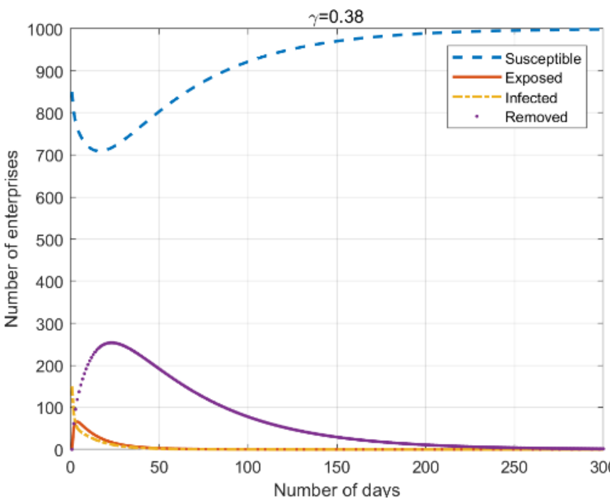

Figure 13. Simulation experiment of γ

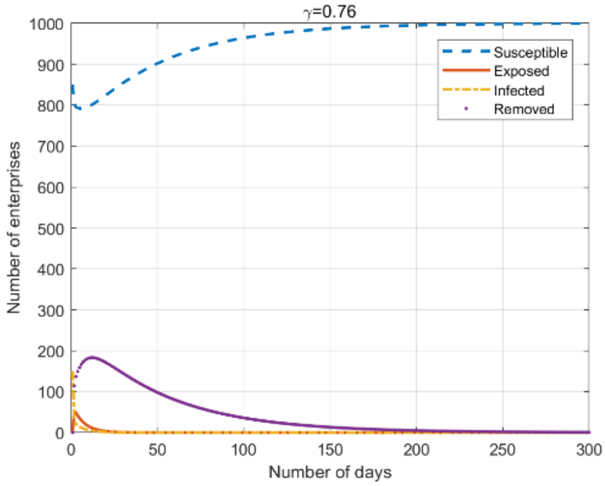

Figure 14. Simulation experiment of γ

Simulation experiment 5: In order to observe the influence of the change of entry (exit) rate on risk propagation and R_0, the entry rate μ is adjusted on the basis of experiment 1, where μ=0. 03 is the initial value.

In experiment 5, μ is adjusted above respectively, and corresponding R_0 is obtained, and R_0 is less than 1. Figure 15-Figure 18 shows the simulation results. It can be seen that R_0 increases as μ increases from 0.01 to 0.05. The minimum number of risk-susceptible firms increased from 630 to 780; The number of latent enterprises and the number of infected enterprises were not obvious. The time for the number of recovered enterprises to peak increased from 40 days to 15 days, and the peak decreased from 340 days to 160 days. The above results show that the entry and exit rate μ has a great influence on susceptible enterprises and recovering enterprises. With the increase of the number of new enterprises and the decrease of the number of original enterprises, the number of enterprises in the supply chain network maintains a dynamic balance of renewal, and the increase of μ can inhibit the spread of risks.

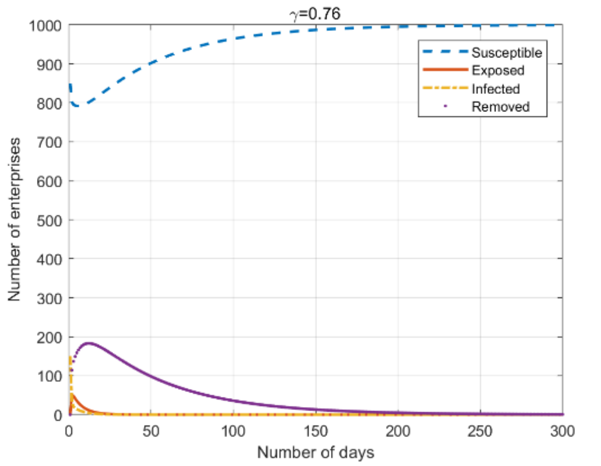

Figure 15. Simulation experiment of μ

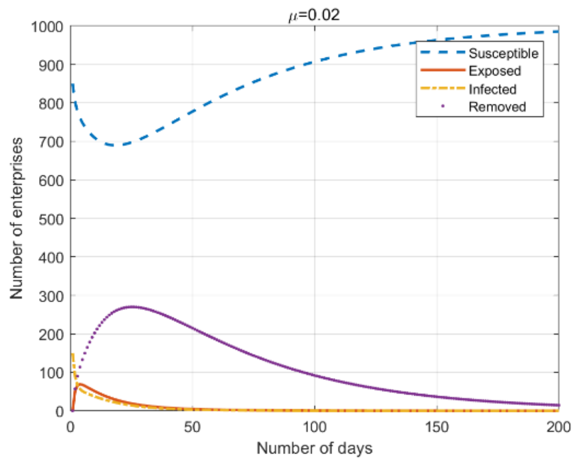

Figure 16. Simulation experiment of μ

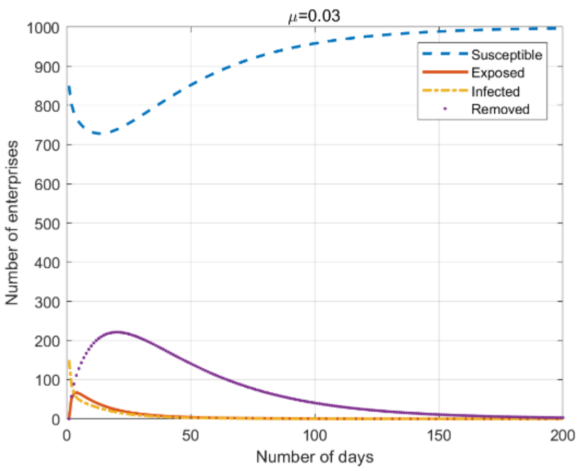

Figure 17. Simulation experiment of μ

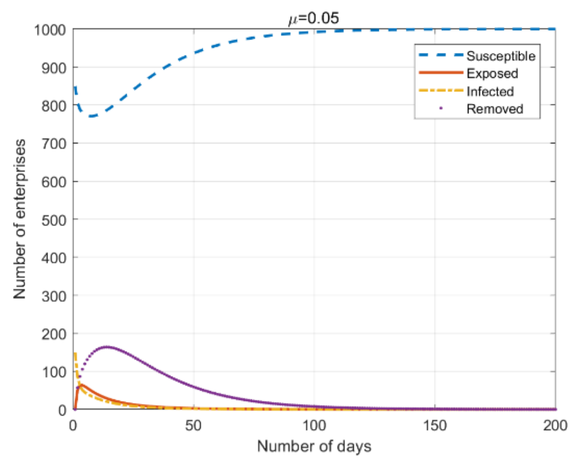

Figure 18. Simulation experiment of μ

4. Conclusion

The SEIR-based risk propagation model of complex supply chain network constructed in this paper is shown by theoretical research and simulation analysis. In the supply chain network, the basic regeneration number R_0 is the key factor to determine the state of risk propagation, and $R_0 = 1$ is the threshold to judge whether the risk will die out: when $R_0 < 1$, there is zero equilibrium point in the system, and the risk will gradually die out. When $R_0 > 1$, the system finally enters the non-zero equilibrium point, and the risk cannot be eliminated. Through simulation, it is found that the value of basic regeneration number R_0 depends on infection rate α, latent conversion rate β, recovery rate γ, entry and exit rate μ, among which latent conversion rate β and recovery rate γ have the best effect on R_0. To sum up, this paper puts forward the following suggestions on risk transmission and control in supply chain network from the macro level of supply chain and the micro level of enterprises:

From the perspective of the supply chain as a whole, when there is a certain risk in the supply chain system, the risk enterprise will spread the risk to other enterprises with a certain probability, so that it has potential risks. At this time, if the risk latent enterprise does not discover and take countermeasures in time, the risk scale will gradually expand unconsciously. When the risk is revealed and erupted, The intensity and speed of transmission will be hard to control. Therefore, relevant departments or associations should play a good role as a bridge between the government and the market, play a powerful service and management function, actively guide the industry to introduce the concept of supply chain management, improve the risk handling ability, industry economic technology and management level, and promote the stability of the whole supply chain network.

From the perspective of enterprises, in a huge and complex supply chain system, each nodal enterprise has a certain risk resistance ability. When the impact of risks exceeds the control scope of the enterprise itself, the risk will break the resistance of the enterprise, and then pass to its upstream and downstream business enterprises. Therefore, enterprises should strengthen their risk awareness, improve risk early warning management, investigate, analyze and identify supply chain bottlenecks, and formulate corresponding containment measures. Make a reasonable forecast scheme based on the original business plan, grasp the dynamic trend of supply and marketing, and reduce the possibility of self-caused risks. At the same time, strengthen the contact with the upstream and downstream business enterprises, break through the information islands, realize the intelligence, interconnection and visualization of the information system, and timely and effective communication with external enterprises can ensure that enterprises can quickly discover and cope with external risks.

The study of this paper reflects the risk transmission mechanism in the supply chain network to a certain extent, and explores the impact of some key elements on risk transmission, but many main elements still need to be further studied. For example, the research object of this paper is not detailed into the specific supply chain situation, but only simulates the risk transmission process of the supply chain system from the macro level. Therefore, I intend to further refine the subject attributes and build a supply chain network model more in line with the actual situation.

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