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Supply-hub-based Collaborative Replenishment Decisions for Enterprise Supply Logistics Under Demand Uncertainty

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Abstract: To address the collaborative replenishment problem of enterprise supply logistics, a Supply-hub-based collaborative replenishment decision model is constructed from the perspective of horizontal collaboration among suppliers and the introduction of component demurrage costs and product out-of-stock penalty costs. The number of suppliers participating in the system, the replenishment level of suppliers and the impact of demand uncertainty on the operational cost of supply logistics are analysed. The results show that: the participation of multiple suppliers can reduce the increase of supply logistics operating costs to a certain extent; as the service level increases, the supply logistics operating costs increase and then decrease; the impact of demand uncertainty on the supply logistics operating costs can be mitigated to a certain extent by the coordination function of Supply-hub.

Keywords: Supply-hub, Supply logistics, Collaborative replenishment.

1. Introduction

Collaborative replenishment of supply logistics is one of the supply chain management studies, and many manufacturing companies have been strengthening their management of supply chain collaboration. Supply logistics is an important material supply activity that occurs between suppliers and manufacturing enterprises, and is a logistics activity that occurs when raw materials, components or other materials are provided to manufacturing enterprises [1]. Bowersox et al. [2] considered that the Supply-hub is used to store some of the supplied materials and is generally located in the vicinity of the core manufacturing enterprise and the supplier is only paid accordingly when the material is consumed by the manufacturer. Based on Supply-hub's enterprise supply logistics collaborative replenishment optimization problem, Hua-Ming Gui and Shi-Hua Ma [3] introduced the concept of circular pickup (Milk-run) for the first time in the Supply-hub environment, and optimized the batch pickup model. Further, Huang Xiaoling et al [4] allow supply points to be visited multiple times for the purpose of demand unbundling and use the Milk-run method instead of the mainline direct shipping method, combined with the Supply-hub model to optimise the cost of auto parts admission logistics. E. Farsi et al. [5] studied a two-level supply chain for the transportation of perishables, introduced product shelf life into the decision model, and constructed a collaborative replenishment model with supply chain profit maximisation as the objective function. Jianhong Yu et al. [6] study a three-tier supply chain assembly system in which suppliers perform decentralised replenishment, replenishment without coordinated quantities and replenishment with coordinated quantities in three areas to propose optimal decisions under different strategies.

The existing literature has studied the operation mode and characteristics of Supply-hub in depth, proving the advantages of Supply-hub in reducing the replenishment cost of enterprise supply logistics from both qualitative and quantitative perspectives. Most of the research on supply logistics under uncertain demand is focused on the vertical collaboration of different nodes upstream and downstream in the supply chain, while there is little research on replenishment under horizontal collaboration at the same level.

2. Question Description

A supply logistics operation system consisting of multiple suppliers, a Supply-hub and a single core manufacturer is shown in Figure 1. Each supplier meets the manufacturer's JIT demand through bulk distribution under a circular pick-up method, and the Supply-hub adopts a unified deployment measure to supply the parts required by the manufacturer for production. Due to changes in demand, when a certain component is in short supply and out of stock, it will cause other components used in conjunction with it to be held in stock, thus incurring demurrage costs; this in turn will lead to out of stock for the manufacturer and increase product out-of-stock costs.

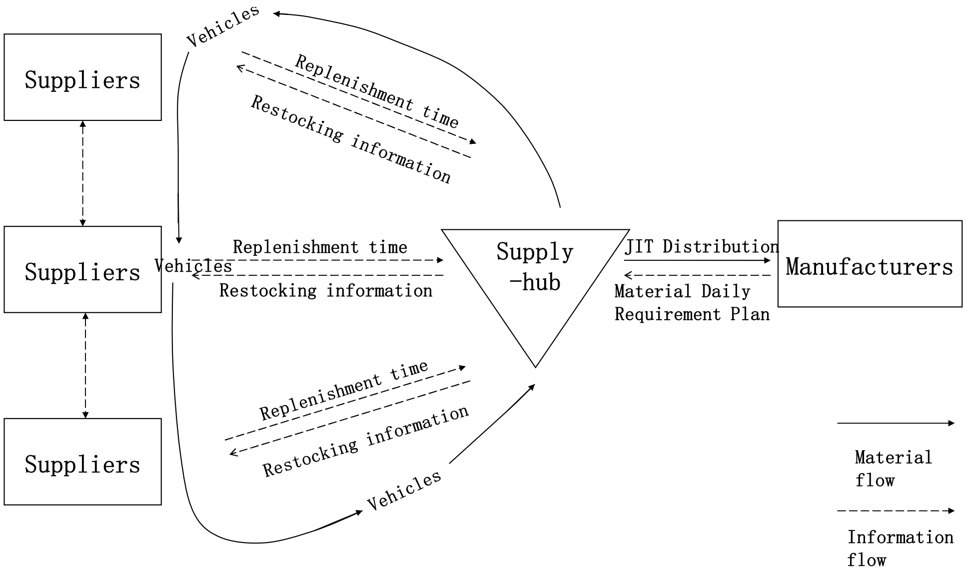

Figure 1. Supply-hub-based collaborative replenishment model for enterprise supply logistics

3. Basic Assumptions and Symbolic Definitions

3.1. Basic assumptions

(1) The quantity of each component in the final product is 1:1.

(2) Suppliers do not take into account production costs, only inventory costs. Only one type of component is supplied by each supplier and one component is supplied by only one supplier.

(3) Production preparation costs and adjustment lead times for suppliers remain unchanged.

(4) Total annual demand D is fixed.

(5) The daily demand for spare parts follows a normal distribution $N(\mu_i, \sigma_i{}^2)$ and is independent of each other.

(6) Supply-hub uses a (Q, R) inventory strategy.

(7) Supply-hub's replenishment of manufacturers does not take into account time costs and is assumed to be instantaneous.

(8) Consider safety stock $SS_i = z_i\sigma_i\sqrt{L_i}$, with z_i being the safety stock factor and $z_i > 0$.

(9) Out-of-stocks are allowed for the final product and the cost of penalties caused by out-of-stocks is borne by the manufacturer.

(10) Service levels remain consistent across suppliers, above 96%.

3.2. Symbolic definitions

n —— Number of suppliers of spare parts.

i —— suppliers and their part codes. i = 1,2,…,n;

D ——The manufacturer's demand for parts for a cycle year.

Q_i ——Replenishment quantity for supplier i (decision variable);

Q_{si} ——the quantity produced by supplier i. $Q_{si} = kQ_i, (k \geq 1)$;

b_i, B_i ——denotes the lead time for a single adjustment of a component produced by supplier i and its corresponding cost, respectively.

SL_i —— The service level of supplier i.

F_i —— A fixed fee for one delivery of replenishment to Supply-hub by supplier i.

h_{Si}、h_{Hi} —— denotes the unit inventory cost of component i at the supplier, Supply-hub, respectively.

π——The cost of out-of-stock penalties per unit of product per unit of time.

f(.) ——The probability density function of demand per unit of time. $f(x) = \frac{1}{\sigma\sqrt{2\pi}} exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$;

L_i —— Lead time for replenishment of part i.

$f_i(x_i)$ ——Probability density function of the demand occurring within the replenishment lead time L_i for part i. $f_i(x_i) = \frac{1}{\sigma_i\sqrt{2\pi L_i}} exp\left(-\frac{(x_i-\mu_i L_i)^2}{2\sigma_i^2 L_i}\right)$;

r_i —— denotes the order point for part i. $r_i = \mu_i L_i + z_i\sigma_i\sqrt{L_i}$.

4. Mathematical Models

A mathematical model is constructed with the objective of minimising the operational costs of supply logistics, and the analytical equations for the order points and replenishment batches of components are derived using mathematical theory. In order to simplify the calculations and to keep the service levels consistent across suppliers, $SL_i = SL(i = 1,2,\dots,n)$,This gives:$z_i = z(i = 1,2,\dots,n)$. Under demand uncertainty, the components of supply logistics synergy operating costs are considered to include mainly inventory holding costs, replenishment costs, component demurrage costs and penalty costs when the final product is out of stock for each node company. The planned supply logistics operating cost for a cycle (year) is $E(TC)$:

$$E(TC) = \sum_{i=1}^{n}\frac{DB_i}{kQ_i} + \sum_{i=1}^{n}\frac{(k-1)Q_i h_{Si}}{2} + \sum_{i=1}^{n}\frac{D}{Q_i}F_i + \sum_{i=1}^{n} h_{Hi}\left(\frac{Q_i}{2} + z_i\sigma_i\sqrt{L_i}\right) + \sum_{i=1}^{n}\sum_{j=1\, i\neq j}^{n} h_{Hi}\frac{D}{Q_j}\int_{r_j}^{+\infty}(x_i - r_j)f_j(x_j)dx_j + \pi\sum_{i=1}^{n}\frac{D}{Q_i}\int_{r_i}^{+\infty}(x_i - r_i)f_i(x_i)dx_i \quad (1)$$

Taking the derivative of $E(TC)$ with respect to Q_i yields:

$$\frac{\partial E(TC)}{\partial Q_i} = -\sum_{i=1}^{n}\frac{DB_i}{kQ_i^2} + \sum_{i=1}^{n}\frac{(k-1)h_{Si}}{2} - \sum_{i=1}^{n}\frac{DF_i}{Q_i^2} + \sum_{i=1}^{n}\frac{h_{Hi}}{2} - (h_{H1} + h_{H2} + h_{H3} + \cdots + h_{Hn})\sum_{i=1}^{n}\frac{D}{Q_i^2}\int_{r_i}^{+\infty}(x_i - r_i)f_i(x_i)dx_i - \pi\sum_{i=1}^{n}\frac{D}{Q_i^2}\int_{r_i}^{+\infty}(x_i - r_i)f_i(x_i)dx_i \tag{2}$$

$$\frac{\partial^2 E(TC)}{\partial Q_i^2} = \sum_{i=1}^{n}\frac{2DB_i}{kQ_i^3} + \sum_{i=1}^{n}\frac{2DF_i}{Q_i^3} + (h_{H1} + h_{H2} + h_{H3} + \cdots + h_{Hn})\sum_{i=1}^{n}\frac{2D}{Q_i^3}\int_{r_i}^{+\infty}(x_i - r_i)f_i(x_i)dx_i + \pi\sum_{i=1}^{n}\frac{2D}{Q_i^3}\int_{r_i}^{+\infty}(x_i - r_i)f_i(x_i)dx_i \tag{3}$$

From (3), the supply logistics operating cost $E(TC)$ is a strictly downward convex function with respect to Q_i such that its first-order partial derivative $\frac{\partial E(TC)}{\partial Q_i} = 0$, which gives:

$$Q_i^* = \sqrt{\frac{2D\left\{\frac{B_i}{k} + F_i + [(h_{H1} + h_{H2} + \cdots + h_{Hn}) + \pi]\int_{r_i}^{+\infty}(x_i - r_i)f_i(x_i)dx_i\right\}}{(k-1)h_{Si} + h_{Hi}}} \tag{4}$$

$$\sum_{i=1}^{n} h_{Hi} = M,\quad \sum_{i=1}^{n} h_{Hi} + \pi = I,\quad \int_{r_i}^{+\infty}(x_i - r_i)f_i(x_i)dx_i = N,$$
$$(k-1)h_{Si} + h_{Hi} = A_i:$$

$$Q_i^* = \sqrt{\frac{2D(\frac{B_i}{k} + F_i + IN)}{A_i}} \tag{5}$$

Substituting Q_i^* into (1), we get:

$$E(TC) = \sum_{i=1}^{n}\sqrt{2DA_i(\frac{B_i}{k} + F_i + IN)} + \sum_{i=1}^{n} h_{Hi} z\sigma_i\sqrt{L_i} \tag{6}$$

This will be given by$r_i = \mu_i L_i + z_i\sigma_i\sqrt{L_i}$:

$$N(r_i) = \int_{\mu_i L_i + z_i\sigma_i\sqrt{L_i}}^{+\infty}(x_i - \mu_i L_i - z_i\sigma_i\sqrt{L_i})\frac{1}{\sigma_i\sqrt{2\pi L_i}}exp\left(-\frac{(x_i - \mu_i L_i)^2}{2\sigma_i^2 L_i}\right)dx_i$$

$$N(r_i) = \sigma_i\sqrt{L_i}[\varphi(z) - z(1 - \emptyset(z))] \tag{7}$$

Substituting (7) into (6), the solution yields the cost of supply logistics operations under optimal distribution Q_i^*:

$$E(TC) = \sum_{i=1}^{n}\sqrt{2DA_i\{\frac{B_i}{k} + F_i + I\sigma_i\sqrt{L_i}[\varphi(z) - z(1 - \emptyset(z))]\}} + \sum_{i=1}^{n} h_{Hi} z\sigma_i\sqrt{L_i} \tag{8}$$

Taking the derivative of (8) with respect to z yields:

$$\frac{dE(TC)}{dz} = \sum_{i=1}^{n}\frac{-DA_i I\sigma_i\sqrt{L_i}(1 - \emptyset(z))}{\sqrt{2DA_i\left\{\frac{B_i}{k} + F_i + I\sigma_i\sqrt{L_i}[\varphi(z) - z(1 - \emptyset(z))]\right\}}} + \sum_{i=1}^{n} h_{Hi}\sigma_i\sqrt{L_i}$$

$$\frac{d^2E(TC)}{dz^2} = \sum_{i=1}^{n}\frac{DA_i I\sigma_i\sqrt{L_i}\varphi(z)}{\sqrt{2DA_i\left\{\frac{B_i}{k} + F_i + I\sigma_i\sqrt{L_i}[\varphi(z) - z(1 - \emptyset(z))]\right\}}} + \frac{[DA_i I\sigma_i\sqrt{L_i}(1 - \emptyset(z))]^2}{\sqrt{\{DA_i\left\{\frac{B_i}{k} + F_i + I\sigma_i\sqrt{L_i}[\varphi(z) - z(1 - \emptyset(z))]\right\}\}^3}} > 0$$

It follows from $\frac{d^2E(TC)}{dz^2} > 0$ that the operating cost $E(TC)$ per unit time of supply logistics is a strictly downward convex function about z. Letting $\frac{dE(TC)}{dz} = 0$ yields z^*:

$$\sum_{i=1}^{n} h_{Hi}\sigma_i\sqrt{L_i} - \sum_{i=1}^{n}\frac{DA_i I\sigma_i\sqrt{L_i}(1 - \emptyset(z))}{\sqrt{2DA_i\left\{\frac{B_i}{k} + F_i + I\sigma_i\sqrt{L_i}[\varphi(z) - z(1 - \emptyset(z))]\right\}}} = 0 \tag{9}$$

Solving for (9) gives a safety stock factor z^*, which gives the service level of each supplier.

5. Example Analysis

Assume that the number of suppliers involved in this supply logistics operation system is 2-4, and that Supply-hub is required to deliver the supporting parts directly to the workstation to meet the manufacturer's demand, and that the service level of the supplier is not less than 96%. Let the manufacturer's total demand in one cycle D=500; the market penalty cost per unit of shortage for the final product π=50; the demand for part i per unit of time obeys a normal distribution $N(\mu_i, \sigma_i^2)$; and the parameters are set as shown in Table 1.

Table 1. Setting of each parameter for part i

Suppliers i	B_i	F_i	h_{Hi}	h_{Vi}	h_{Si}	L_i	μ_i	${\sigma_i}^2$
1	200	100	20	22	17	9	11	9
2	250	140	30	25	11	18	15	16
3	220	180	25	28	22	10	20	4
4	280	210	35	30	17	15	16	9

Based on the above model and solved in python, the optimal distribution lot size, order points and the operational costs of supply logistics can be obtained. The influence of different factors on the operational cost of supply logistics is analysed in the following aspects.

5.1. The impact of the number of supplier participants on the operational costs of supply logistics

The operating costs of supply logistics and the corresponding service levels are calculated when the number of suppliers in the supply logistics operation is 2, 3 and 4 respectively. The results of the calculation are shown in Table 2:

Table 2. Results of model calculations with varying number of suppliers n

Number of suppliers n	suppliers i	Distribution lot Q_i	Order points r_i	Safety stock factor z	Service Level SL	Supply logistics operating costs TC
2	1	463.66	116.09	1.90	97.13%	7596.08
	2	541.53	302.22			
3	1	462.51	117.70	2.08	98.12%	11783.06
	2	539.68	305.26			
	3	547.53	213.14			
4	1	463.61	118.51	2.17	98.50%	17145.37
	2	541.44	306.79			
	3	548.17	213.71			
	4	607.69	265.19			

According to the calculations in Table 2, when the number of suppliers involved in the supply logistics operation system increases from 2 to 3, the increase in the cost of supply logistics operation is $\Delta TC_1 = \frac{11783.06-7596.08}{7596.08} \times \% = 55.12\%$. When the number of suppliers involved in the supply logistics operation system increases from 3 to 4, the increase in the cost of supply logistics operations is$\Delta TC_2 = \frac{17145.37-11783.06}{11783.06} \times \% = 45.51\%$. $\Delta TC_1 > \Delta TC_2$, As the number of participating suppliers increases, supply logistics operating costs and service levels increase, but the increase in supply logistics operating costs decreases. The reason for this is that the increase in the number of suppliers makes it easier to meet the manufacturer's component requirements through centralised management and distribution by Supply-hub, resulting in economies of scale in the supply process, which not only reduces the cost of the supply logistics operation system, but also increases the level of supplier service.

5.2. The impact of suppliers' service levels on supply logistics operating costs

Assuming that the service level varies between 90% and 100% and taking a step of 0.05, the calculation yields the operational cost of supply logistics at different service levels. The results are shown in Figure 2.

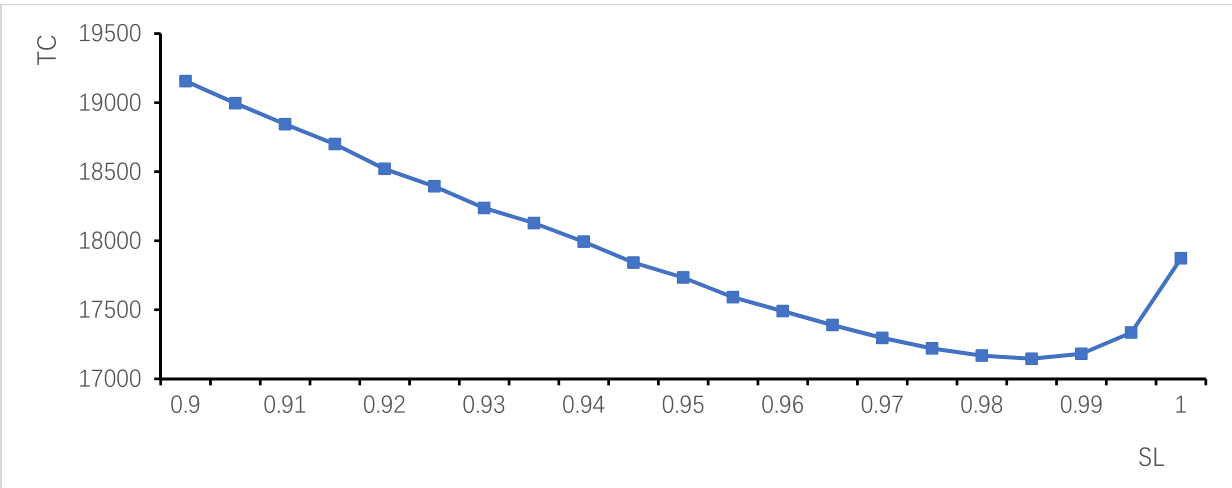

Figure 2. Impact of changes in supplier service levels on supply logistics operating costs

As can be seen from Figure 2, the operational cost of supply logistics decreases and then increases as the service level of the supplier gradually increases. The reason for the decrease in supply logistics operating costs with increasing service levels is that the increase in supplier and Supply-hub inventory costs is smaller than the decrease in supplier replenishment costs, the stranded cost of parts in the Supply-hub and the cost of product out-of-stock penalties. In particular, when SL ≥ 98.5%, the operational costs of Supply-hub-based collaborative replenishment of supply logistics increase. This is because the replenishment and inventory costs incurred by suppliers to achieve higher service levels are much higher than their reduced costs when considering co-replenishment. Therefore, higher service levels are not better, but should be kept within a certain range.

5.3. The impact of demand uncertainty on supply logistics operating costs

Assuming that the supplier service level remains constant at 96%, the change in supply logistics operating cost TC is calculated from the perspective of demand (i.e. ${\sigma_i}^2$ as the independent variable) when ${\sigma_i}^2$ takes a change step of 1. As shown in Figure 3.

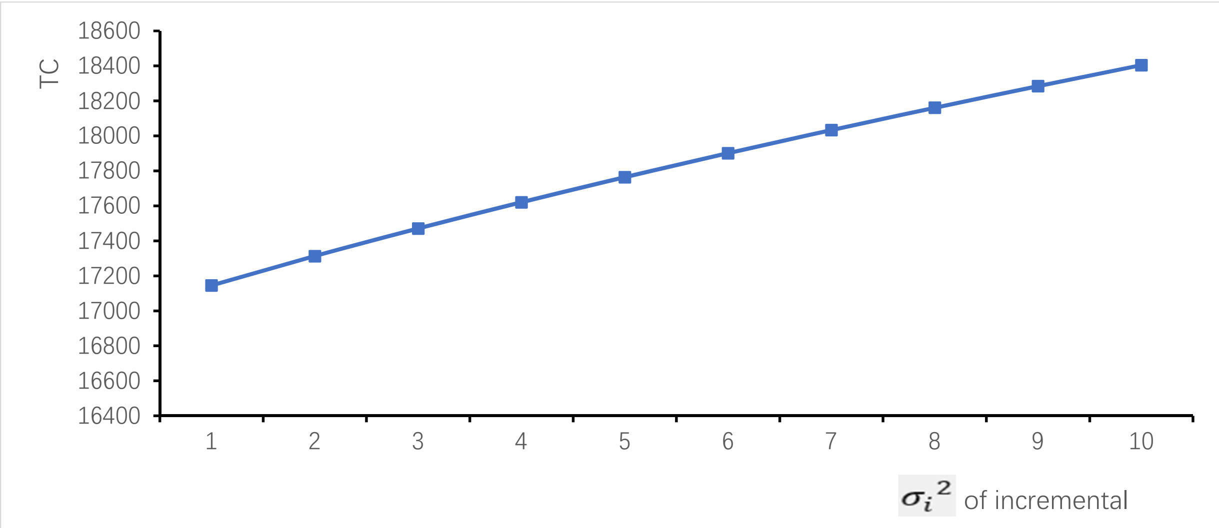

Figure 3. Impact of demand uncertainty on supply logistics operating costs

As can be seen from Figure 3, market demand uncertainty has a significant impact on supply logistics operating costs. This is because the greater the uncertainty of market demand, the more likely it is that the probability of stock-out in each segment will increase, thus affecting the inventory strategy adopted by each segment of supply logistics and increasing the operating costs. However, as the unit volume of ${\sigma_i}^2$ increases, the increase in supply logistics operating costs decreases, indicating that the centralised deployment of Supply-hub can alleviate the cascading effect of the bullwhip effect to a certain extent.

6. Conclusion

This paper investigates the problem of collaborative replenishment decisions in the Supply-hub model under demand uncertainty and analyses the impact of the number of suppliers involved, the level of supplier service and demand uncertainty on the operational costs of supply logistics, leading to the following conclusions.

(1) Supply-hub minimises the cost of supply logistics operations by coordinating multiple suppliers in a (Q,R) model with an inventory strategy that allows for optimal distribution lots and order points for each supplier.

(2) The number of suppliers involved in the supply logistics system has a significant impact on the operating costs. As the number of suppliers increases, the increase in supply logistics operating costs decreases, indicating that there is a certain scale effect when multiple suppliers collaborate to replenish goods, and the cost is controlled under the unified allocation management of Supply-hub.

(3) The Supply-hub model has a certain degree of integration and coordination that can mitigate this effect to a certain extent.

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