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Third-Party Retailer's Lot-Sizing Policy Under Conditions of Trade Credit, Uncertain Demand and Asymmetric Initial Budget Information

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Abstract: Large retail platform has begun to provide trade credit to the capital-constrained and start-up third party retailer (3PR). Most of the 3PRs face uncertain demands with few historical data, which makes it difficult for retail platform to evaluate the rationality of 3PR's lot-sizing decision and to design trade credit contracts. In addition, and 3PR's initial budget is unknown to the retail platform, which further increases the risk of trade credit. There- fore, this paper considers a two-echelon supply chain consisting of a retail- platform and a 3PR, and investigates the 3PR's lot-sizing decision and the design of trade credit contract based on uncertain demand and asymmetric initial budget information. We find that a) empirical uncertainty distribution is an effective tool for modeling uncertain demand with few historical data. b) The 3PR will place an irrational purchasing order regardless of the market condition when the retail platform does not interfere with 3PR's decision mak- ing. 3) The 3PR will overstate its initial budget to get more loan when the retail platform interferes with 3PR's decision making.

Keywords: Trade credit, Newsvendor problem, Uncertain demand.

1. Introduction

Trade credit is an essential financing tool for small and medium- sized enterprises (SMEs). In developing countries, such as China, there are 40 million SMEs, which accounting for 99 percent of China's enterprises, contributing 80 percent to China's job and 60 percent to China's GDP, yet, over fifty percent of SMEs in developing countries suffer from financing difficulty (Buzacott and Zhang [6]). Caused by lack of collateral, poor management and absence of stable cashflow, the SMEs face an obstacle to get access to bank credit (Beck [4]), which severely limited the development of SMEs.

In addition to bank credit, trade credit is an alternative financing mode in online financing, where the retail platform (e.g., JD Finance) provides trade credit to the capital-constrained retailer. JD Finance, who is an essential department of Chinese largest retail platform JD.com.Inc, provides various kinds of trade credits for its third-party retailer (3PR). One of them is called JingXiaoDai (JXD) whose opera- tion process can be seen in Fig. 1. Unlike bank credit, the 3PRs do not need any collateral, the applicants should only meet some easily satisfied conditions, such as no bad credit history and at least three month operation history. According to the 3PRs' sales history, JD Finance will determine the amount of loan to the 3PRs. After getting trade credit, the 3PR places an order from its supplier, and oper- ates on JD's retail platform. The money received from the customer can be used for repaying the loan. Even the capital-constrained 3PR can realize normal opera- tions through JXD, and therefore, JXD partially solves 3PRs' cash-flow operation problems.

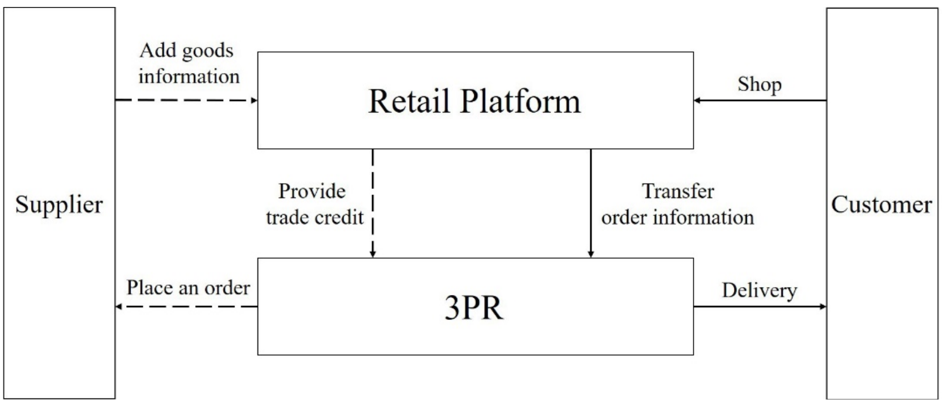

Figure 1. Operational process of JXD

One can see that although the trade credit partially solves the 3PR's cash-flow problems, the retail platform shares the sales risk through trade credit [20]. If the 3PR experiences poor sales, then the retail platform can not recover their money in time, which will severely harm retail platform. The reason for this problem is that most of the 3PRs are start-up companies, so they do not have historical data to predict their future sales, and their lot-sizing policies may be irrational; Hence, estimating the 3PR's uncertain demand is important for the retail platform to manage its risk. This leads to our first research question: How to estimate the uncertain demand of the 3PR when there is few historical data?

Based on the 3PR's demand information, the retail platform can estimate the 3PR's lot-sizing decision. Previous literature has studied the well-funded 3PR's lot-sizing policy when facing uncertain demand; however, the lot-sizing policy for the capital-constrained 3PR when facing uncertain demand is still Unknown. In addition, how to design a trade credit contract for the capital-constrained 3PR is also deserved to explore. This leads to our second research question: What are the optimal lot-sizing decision and the trade credit contract for the 3PR and the retail platform, respectively?

Since the retail platform is willing to provide trade credit to the 3PR, the 3PR may conceal their initial budget information, and obtain as many loans as possible from the retail platform, which undoubtedly amplifies the risk of trade credit for retail platforms; hence, how to control the risk of trade credit under asymmetric initial budget information is very important for the retail platform. This lead to our third research question: How does 3PR's asymmetric initial budget information influence the trade credit contract?

To answer these questions, we consider a two-echelon supply chain consisting of a retail platform and a capital-constrained 3PR. The 3PR applies for trade credit from the retail platform and makes a lot-sizing decision. The retail platform estimates the 3PR's demand and provides a reasonable trade credit contract. In addition, the 3PR's lot-sizing decision is influenced by two important factors (i.e., wholesale price and production cost) which are not considered by retail platform; hence, to facilitate our discussion, we assume the retail platform is able to observe the wholesale price and the production cost, and the retail platform will take the wholesale price and the production cost into consideration when it decides the optimal amount of trade credit to the 3PR. To cope with the first research question, we design a survey to obtain the experimental data from the domain experts, and an experimental uncertainty distribution (Liu [16]) is established through the experimental data, and the parameters of the uncertainty distribution are estimated by the method of least squares. Since the future demand is unknown, the 3PR can be seen as a newsvendor, and an uncertain newsvendor model with trade credit is formulated to answer the second research question. The initial budget information is further considered as asymmetric information, and the third research question is answered by solving the an uncertian newsvendor model with asymmetric initial budget information.

The main findings are as follows.

(1) The 3PR will order more units of goods when it has a slight initial budget. This seems counter-intuitive, and one might think no matter how much initial budget the retailer has, the 3PR may claim it has no initial budget and place a huge order. This is because the trade credit is a kind of non-resource loan, hence the less initial budget it has, the cheaper it is to default. If the retail platform does not take any action, then it will severely damage the retail platform's profit.

(2) The retail platform will not always satisfy 3PR's trade credit application. As we have mentioned, when the 3PR has a slight initial budget, the 3PR will behave aggressively when setting purchasing orders, which results in ir- rational trade credit amount; the 3PR's irrational behavior will be regulated and avoided through our model, and the trade credit contract will then be a win-win strategy for both the 3PR and the retail platform.

(3) The 3PR will overstate its initial budget and ask for a larger mount of trade credit under certain conditions. The rationale behind is that when the 3PR overstates its initial budget, the retail platform may mistakenly believe that the 3PR has a good financial position, and then approve a large amount of loan, which is very risky. By inviting experts to estimate the uncertainty distribution of 3PR's initial budget, the negative effect can be mitigated to a certain extent.

The rest of the paper is organized as follows. Section 2 reviews related literature on newsvendor problem with demand forecast, newsvendor problem with uncertain demand, and newsvendor problem involving trade credit. Section 3 shows the basic notations and assumptions that are used i this paper. Section 4 constructs the model and solves this problem through uncertain optimization. Section 5 presents a numerical example for the reader and shows how to use the method we gave in this paper in a practical issue.

2. Literature Review

Our work is related to three streams of literature: (i) newsvendor problems with demand forecast, (ii) newsvendor problems with un- certain demand, (iii) trade credit under newsvendor framework.

The first stream is related to newsvendor problems with demand forecast. Ar- row et al. [1] established distribution-free method, in which only expected value and variance were deemed to be known. Graves [12] noted that the demand was usually time-dependent, and it was not proper to assume demand was independent and identically distributed. Therefore, Avyv [2] applied AR model to forecast de- mand, Dong and Lee [11] made use of the martingale method and Avyv [3] used the Kalman filter method. Most of previous literature was under the assumption of sufficient data, however, there were a few clues toward circumstance when little data is available. To address this limitation, we adopt a new method which estimating the distribution of the demand based on experts' experimental data. This method derives from uncertainty theory and proposed by Liu [16]. This method is differ- ent from preceding probability-based method, and can be used for data-deficient circumstances.

The second stream is related to the newsvendor problem with uncertain demand. Newsvendor problem with uncertain demand is proposed by Qin and Kar [18]. Ding [10] thought that the newsvendor usually sold multi-product simultaneously and faced storage-constrained problem. Therefore, they proposed multi-product newsvendor problem with chance constraint model. In his model, the demand is also described by an uncertain variable. Wang et al. [19] noticed that both random- ness and uncertainty may simultaneously appear in a newsvendor problem, hence, they employed uncertain random variable to describe newsvendor's demand. This paper assists in this stream of research in two ways. First, all previous research im- plicitly assumed that when there are little historical data, and the demand should be described by an uncertain variable with a known uncertainty distribution, but none of them mentioned how to obtain the uncertainty distribution of the uncer- tain demand, so in this paper, we propose a new method which explicitly enables the decision maker to obtain the uncertainty distribution. Second, all previous re- search implicitly assumed that the newsvendor is well-funded, however, in reality, the newsvendor is usually capital-constrained, hence, we introduce trade credit to previous research and analyse how trade credit influences newsvendor's decision.

The third stream is related to trade credit under newsvendor framework. Arrow et al. [1] initiated stochastic newsvendor problem in which supplier produces a single product to the retailer and the retailer faces a stochastic demand. Chen and Wang [9] studied a stackelberg game between a supplier and a capital constrained retailer. They discussed the optimal ordering quantity for the retailer when it received trade credit from the supplier and showed that wholesale price contract can coordinate the supply chain. Gupta and Wang [13] studied a multi-period stochastic inventory model with trade credit. Konvelis and Zhao [14] considered the newsvendor is financed by bank credit and trade credit simultaneously, and the retailer will always prefer optimally-structured trade credit to bank credit. Bing et al. [5] discussed how production cost influenced retailer's choice between bank credit and trade credit. The previous research discussed retailer's optimal operational decision under the assumption that the demand is characterised by a stochastic variable with known distribution. Carrizosa et al. [8] showed that this assumption may be unrealistic in many cases, for example, due to small sample size, if the demand is inferred from sample data, then some desirable statistical properties, e.g. consistency, may not hold any more. This paper is free from this assumption by assuming the demand is an uncertain variable whose distribution function is estimated by experts' experimental data instead of historical data, and we also derive similar and reasonable conclusions as they did.

To summarize, our research contributes literature by analysing a capital-constrained retailer who is financed by its upstream supplier through trade credit when there is no historical data of sales are available. As far as we know, this is the first paper that focusses on capital-constrained retailer's lot-sizing decision when no or only a little historical data of sales are available. This paper fills two research gaps. First, this paper use a new method, which is based on experts experimental data ([16]), to forecast newsvendor's demand. This method has never been used in uncertain newsvendor problem. In addition, this paper introduces trade credit into uncertain newsvendor model.

3. Notations and Assumptions

The following notations and assumptions are used in this paper.

Parameter	Description
D	3PR's demand, an uncertain variable
QR	3PR's ordering quantity
QS	The optimal upper limit of 3PR's ordering quantity set by the retail platform
QU	3PR's ordering quantity in a standard uncertain newsvendor problem.
p	Retail price
w	Wholesale price
c	Production cost
B	Retailer's initial capital
$\Phi(x)$	The uncertainty distribution of D
$\emptyset(x)$	The derivative of $\Phi(x)$
ΠR	3PR's expected payoff
ΠS	Retail platform's expected payoff
ΠUR	Well-funded 3PR's expected payoff
ΠUS	Retail platform's expected payoff when the 3PR is well-funded

3.1. Notations

Assumptions. Assumption 1: The 3PR has little historical information about uncertain demand.

As we have discussed in the introduction section, most of 3PRs that apply for trade credit are start-up enterprises or individuals. The law of large number may be untenable in this situation [17]. As in [10], [18] and [19], we assume that the demand is described by an uncertain variable.

Assumption 2: The salvage value of the product is zero.

To simplify the model, just as in [9], we assume the salvage value of the product is zero.

Assumption 3: When the 3PR's initial capital B is not enough to pay off the ordering cost, the retail platform is ready to offer trade credit.

As we have discussed in the introduction section, many retail platforms have strong willpower to offer trade credit to 3PRs.

4. Model

4.1 Obtain uncertainty distribution. First and foremost, we need to derive the uncertainty distribution of retailer's demand. We reiterate that the retailer is a start-up enterprise or individual, so that the 3PR does not have any historical data, therefore, the 3PR is not able to estimate probability distribution of its demand. At this time, the 3PR is in a position to invite some domain experts and collect their experimental data and obtain the uncertainty distribution of its demand based on uncertainty theory. Hence, we design a questionnaire to obtain the experimental data from the experts. The questionnaire includes two questions. First, the 3PR should ask the domain expert

'How many products may I sell?'

The domain expert will choose a feasible value x, e.g., 100, according to his or her knowledge, and x is the value that the uncertain variable D will take. After that, the retailer should ask the domain expert

'How likely is D less than or equal x?'

The domain expert will choose a number between 0 and 1, e.g., 0.5. This number is called belief degree in uncertainty theory [16]. In this way, we derive an expert's experimental data $(100, 0.5)$. Repeating the above steps, we can get the experts'

experimental data $(x1, \alpha1), (x2, \alpha2), \cdots, (xn, \alpha n)$.

Next, we use the method of least squares to obtain parameters of the uncertainty distribution. That is to solve

$$\min_{\theta} \sum_{i=1}^{n} (\Phi(x_i|\theta) - \alpha_i)^2$$

where $\Phi(\cdot)$ represents the uncertainty distribution and θ represents the unknown pa- rameter (vector) of the uncertainty distribution. Normal uncertainty distribution and linear uncertainty distribution are two commonly used uncertainty distribu-tions. Their distribution functions are

$$\Phi(x) = \left(1 + \exp\left(\frac{\pi(e-x)}{\sqrt{3}\sigma}\right)\right)^{-1}$$

and

$$\Phi(x) = \begin{cases} 0, & \text{if } x \le a \\ (x-a)/(b-a), & \text{if } a \le x \le b \\ 1, & \text{if } x \ge b, \end{cases}$$

respectively, where the parameters a, b, e and σ are real numbers with $\sigma > 0$.

Based on the discussion above, we have the following algorithm to obtain an uncertain distribution of retailer's uncertain demand.

Table 1. An algorithm to obtain uncertainty distribution of 3PR's demand through experts's empirical data

Algorithm
Step1. Collect empirical data from domain experts through questionnaire.
Step2. Choose an uncertainty distribution.
Step3. Estimate parameters of the distribution with the principle of least square.

Example 4.1. Assuming that the retailer wants to estimate its uncertainty distribu- tion via least squares method, and the 3PR has obtained the experimental data from domain experts, that is, $(10, 0.1), (30, 0.2), (50, 0.6), (70, 0.7), (80, 0.8), (90, 0.9)$. As

to estimate the parameters a and b, the retailer need to solve

$$\min_{a,b} \sum_{i=1}^{6} \left(\frac{x-a}{b-a} - \alpha_i\right)^2$$

which yields $\hat{a}$=0 and $\hat{b}$= 104.6.

4.2. 3PR's decision. We consider a two-echelon supply chain consisting of a re- tail platform and a capital-constrained 3PR. The 3PR faces unknown demand, D. Because the 3PR has no historical data, the 3PR employs uncertainty theory and the unknown demand D follows an uncertainty distribution, instead of a proba- bility distribution. The target of the 3PR is to find optimal ordering quantity Q. When the 3PR is well-funded, then this is a standard uncertain newsvendor problem discussed by Qin and Kar [18]. The optimal ordering quantity Q_U^* satisfies

$$\Phi(Q_U^*) = 1 - \frac{w}{p},$$

and 3PR's optimal expected payoff is

$$\Pi_{UR}^* = (p-w)Q_U^* - p\int_0^{Q_U^*} \mathcal{M}\{D \le x\}\mathrm{d}x$$

where $\mathcal{M}$ is uncertain measure, the definition of uncertain measure can be seen in Liu [16].

When the 3PR is capital-constrained, the retailer is only able to make an initial payment B, the retail platform will offer trade credit equals $wQR - B$ to the 3PR.

The 3PR realized its demand D at time instant T . If the 3PR sells well, then the retail platform collects $(wQR - B)$. Otherwise, the retail platform can only collect $(wQR - B) - p\ \min\{D, QR\}$. Therefore, the expected profit of the 3PR can be

expressed by

$$\Pi_R(Q_R) = \mathrm{E}[(p\min\{D, Q_R\} - (wQ_R - B))^+]$$

where $(a)^+ = \max\{a, 0\}$, and E[·] is the expected value operator.

The following theorem shows how to obtain 3PR's optimal ordering quantity.

Theorem 4.1. Suppose the 3PR has an initial budget B, and the 3PR's demand follows a normal uncertainty distribution $\mathcal{N}(e,\sigma)$ or a linear uncertainty distribution $\mathcal{L}(a,b)$,

(1) suppose f is a real function, then we have

$$\int_0^{+\infty} \mathcal{M}\{(f(D))^+ \ge r\}dr = \int_0^{+\infty} \mathcal{M}\{f(D) \ge r\}dr.$$

(2) The 3PR's expected payoff is

$$\Pi_R(Q_R) = \int_{(wQ_R - B)/p}^{Q_R} p\mathcal{M}\{D \ge x\}dx.$$

(3) The 3PR's unique optimal ordering quantity, Q_R^*satisfies

$$p\mathcal{M}\{D \ge Q_R^*\} = w\mathcal{M}\{D \ge (wQ_R^* - B)/p\}.$$

Proof. Part (i): According to the set theory, we have

$$\mathcal{M}\{(f(D))^+ \ge r\} = \mathcal{M}\left\{\{r \le 0\} \cup \{f(D) \ge r\}\right\}.$$

It follows from the monotonicity theorem and subadditivity axiom [15], we have

$$\mathcal{M}\{f(D) \ge r\} \le \mathcal{M}\{\{r \le 0\} \cup \{f(D) \ge r\}\}$$

and

$$\mathcal{M}\{\{r \le 0\} \cup \{f(D) \ge r\}\} \le \mathcal{M}\{r \le 0\} + \mathcal{M}\{f(D) \ge r\}.$$

Moreover, we have

$$\int_0^{+\infty} \mathcal{M}\{f(D) \ge r\}\mathrm{d}r \le \int_0^{+\infty} \mathcal{M}\{(f(D))^+ \ge r\}\mathrm{d}r$$

and

$$\begin{aligned} &\int_0^{+\infty} \mathcal{M}\{(f(D))^+ \ge r\}\mathrm{d}r \\ &\le \int_0^{+\infty} \mathcal{M}\{f(D) \ge r\}\mathrm{d}r + \int_0^{+\infty} \mathcal{M}\{r \le 0\}\mathrm{d}r \\ &\le \int_0^{+\infty} \mathcal{M}\{f(D) \ge r\}\mathrm{d}r. \end{aligned}$$

Therefore, we have

$$\int_0^{+\infty} \mathcal{M}\{(f(D))^+ \geq r\}\mathrm{d}r = \int_0^{+\infty} \mathcal{M}\{f(D) \geq r\}\mathrm{d}r$$

Part (ii): Based on the definition of expected value [15], we have

$$\begin{aligned}
&\mathrm{E}[(p\min\{D,Q_R\} - wQ_R - B)^+] \\
&= \int_0^{+\infty} \mathcal{M}\{(p\min\{D,Q_R\} - (wQ_R - B))^+ \geq r\}\mathrm{d}r \\
&\quad - \int_{-\infty}^0 \mathcal{M}\{(p\min\{D,Q_R\} - (wQ_R - B))^+ \leq r\}\mathrm{d}r \\
&= \int_0^{+\infty} \mathcal{M}\{p\min\{D,Q_R\} \geq (wQ_R - B) + r\}\mathrm{d}r \\
&= \int_0^{+\infty} \mathcal{M}\{pQ_R - p(Q_R - D)^+ \geq (wQ_R - B) + r\}\mathrm{d}r \\
&= \int_0^{+\infty} \mathcal{M}\{(Q_R - D)^+ \leq ((p-w)Q_R + B - r)/p\}\mathrm{d}r
\end{aligned}$$

Let $((p-w)QR + B - r)/p = u$, then we have

$$\begin{aligned}
&\mathrm{E}[(p\min\{D,Q_R\} - wQ_R - B)^+] \\
&= -\int_{((p-w)Q_R+B)/p}^{-\infty} p\mathcal{M}\{(Q_R - D)^+ \leq u\}\mathrm{d}u \\
&= \int_0^{((p-w)Q_R+B)/p} p\mathcal{M}\{(Q_R - D)^+ \leq u\}\mathrm{d}u \\
&= \int_0^{((p-w)Q_R+B)/p} p\mathcal{M}\{D \geq Q_R - u\}\mathrm{d}u
\end{aligned}$$

Let $x = QR - u$, then we have

$$\begin{aligned}
&\mathrm{E}\left[(p\min\{D,Q_R\} - wQ_R - B)^+\right] \\
&= -\int_{Q_R}^{(wQ_R-B)/p} p\mathcal{M}\{D \geq x\}\mathrm{d}x \\
&= \int_{(wQ_R-B)/p}^{Q} p\mathcal{M}\{D \geq x\}\mathrm{d}x \\
&= \int_{(wQ_R-B)/p}^{Q} p\mathcal{M}\{D \geq x\}\mathrm{d}x
\end{aligned}$$

Part (iii): The uncertainty distribution of retailer's demand is denoted by $\Phi(x)$, and $\Phi(x) = \mathrm{d}\Phi(x)/\mathrm{d}x$. Taking the derivative of 3PR's expected payoff w.r.t. QR, and we have

$$\begin{aligned}
&\frac{\mathrm{d}\Pi_R(Q_R)}{\mathrm{d}Q_R} \\
&= \mathcal{M}\{D \geq Q_R\} - \frac{w}{p}\mathcal{M}\{D \geq (wQ_R - B)/p\} \\
&= (1 - \Phi(Q_R)) - \frac{w}{p}(1 - \Phi((wQ_R - B)/p)).
\end{aligned}$$

This means the 3PR's optimal ordering quantity Q_R^* must satisfy the following first order condition, i.e.,

$$w(1 - \Phi((wQ_R - B)/p)) = p(1 - \Phi(Q_R)). \quad (1)$$

Moreover, we have

$$\begin{aligned}
&\left.\frac{\mathrm{d}^2\Pi_R(Q_R)}{\mathrm{d}Q_R^2}\right|_{Q_R} \\
&= -\phi(Q_R) + \frac{w^2}{p^2}\phi((wQ_R - B)/p) \\
&= -(1 - \Phi(Q_R))\frac{\phi(Q_R)}{1 - \Phi(Q_R)} \\
&\quad + \frac{w^2(1 - \Phi((wQ_R - B)/p))}{p^2}\frac{\phi((wQ_R - B)/p)}{1 - \Phi((wQ_R - B)/p)} \\
&= -(1 - \Phi(Q_R))\frac{\phi(Q_R)}{1 - \Phi(Q_R)} \\
&\quad + \frac{w(1 - \Phi(Q_R))}{p}\frac{\phi((wQ_R - B)/p)}{1 - \Phi((wQ_R - B)/p)} \\
&= -(1 - \Phi(Q_R))\left(\frac{\phi(Q_R)}{1 - \Phi(Q_R)} - \frac{w}{p}\frac{\phi((wQ_R - B)/p)}{1 - \Phi((wQ_R - B)/p)}\right)
\end{aligned}$$

Suppose D follows uncertain normal distribution, then

$$\Phi(x) = \frac{1}{1 + \exp\left(\frac{\pi(e - x)}{\sqrt{3}\sigma}\right)}$$

and

$$\phi(x) = \frac{\frac{\pi}{\sqrt{3}\sigma}\exp\left(\frac{\pi(e - x)}{\sqrt{3}\sigma}\right)}{\left(1 + \exp\left(\frac{\pi(e - x)}{\sqrt{3}\sigma}\right)\right)^2},$$

Setting

$$\Psi(x) = \frac{\phi(x)}{1 - \Phi(x)},$$

then we have

$$\Psi(x) = \frac{\pi}{\sqrt{3}\sigma\left(1 + \exp\left(\frac{\pi(e-x)}{\sqrt{3}\sigma}\right)\right)}.$$

It is not hard to see $\Psi(x)$ is an increasing function w.r.t. x. Similarly, we can get the same conclusion when D is an uncertain linear variable.

Furthermore, it is not hard to prove $QR > (wQR - B)/p$, then we have

$$\frac{\mathrm{d}^2\Pi_R(Q_R)}{\mathrm{d}Q_R^2} < 0.$$

So Q_R^* is 3PR's unique optimal ordering quantity.

Theorem 4.1 discusses 3PR's optimal ordering quantity when it receives trade credit from its upstream retail platform. Theorem 4.1 part (i) proves a key step that is used for simplifying and calculating retailer's expected payoff, and the simplified result is shown in Theorem 4.1 part (ii). Theorem 4.1 part (iii) finds 3PR's unique optimal ordering quantity.

Corollary 1. If the demand is described by a normal uncertain variable $\mathcal{N}(e,\sigma)$, then Q_R^* solves from

$$w\left(1 - \left(1 + exp\left(\frac{\pi(pe - wQ_R + B)}{\sqrt{3}\sigma p}\right)\right)^{-1}\right) = p\left(1 - \left(1 + exp\left(\frac{\pi(e - Q_R)}{\sqrt{3}\sigma}\right)\right)^{-1}\right)$$

Corollary 2. If the demand is described by a linear uncertain variable $\mathcal{L}(a,b)$, then

$$Q_R^* = \frac{(p-w)pb - wB}{p^2 - w^2}.$$

Corollary 3. Retailer's optimal ordering quantity with trade credit, Q_R^*, is greater than that without trade credit, Q_U^*.

The implication behind corollary 3 is that the 3PR will order more when the retail platform offers trade credit. This is because the 3PR can leverage more initial capital through trade credit, i.e., with trade credit, the 3PR can use extra $(wQR - B)^+$.

The insight behind corollary 3 is that the retail platform can stimulate the 3PR through trade credit. However, this benefit is at the cost of sharing 3PR's marketing risk. If the 3PR overestimates the demand, then the retail platform will face a maximum loss that equals $(wQR - B)^+$.

Corollary 4. 3PR's optimal ordering quantity, Q_R^* , is a decreasing function w.r.t.B.

Proof. Setting

$$G(Q_R, B) = w(1 - \Phi((wQ_R - B)/p)) - p(1 - \Phi(Q_R)),$$

then

$$\frac{\mathrm{d}Q_R}{\mathrm{d}B}$$

$$= -\frac{\mathrm{d}G/\mathrm{d}B}{\mathrm{d}G/\mathrm{d}Q_R} = \frac{\frac{w}{p}\phi\left(\frac{wQ_R - B}{p}\right)}{\frac{w^2}{p}\phi\left(\frac{wQ_R - B}{p}\right) - p\phi(Q_R)}$$

$$= \frac{\frac{w}{p}\phi\left(\frac{wQ_R - B}{p}\right)}{(1 - \Phi(Q_R))\left(w\Psi\left(\frac{wQ_R - B}{p}\right) - p\Psi(Q_R)\right)} < 0.$$

The implication behind corollary 4 is that the 3PR, who has less initial capital, tries to order more, regardless of the market condition. This is because trade credit is a kind of non-recourse loan, and the 3PR's maximum loss is B. What's more, the cost of 3PR's default decreases with its initial capital. Meanwhile, 3PR's possible future profit increases with its ordering quantity. Therefore, the 3PR orders more when it has less initial capital.

The insight behind corollary 4 is that the retail platform should realize that the 3PR is risk-seeker in this circumstance. In addition, no matter the 3PR has much or little initial capital, it may announce that it has no capital and apply for a huge amount of trade credit. This may damage the retail platform's expected profit. However, according to theorem 4.2, the retail platform will realize the retailer's irrational decision, and will limit the amount of trade credit provided to the 3PR. Corollary 5 further proves that the retail platform's optimal trade credit amount is an increasing function w.r.t. 3PR's initial capital. Therefore, the 3PR's irrational decision will be avoid.

4.3 Retail platform's decision under symmetric initial budget informa- tion. The previous subsection shows that the 3PR will place an aggressive order when it has little initial budget, so in this subsection, we will discuss how the retail platform reacts w.r.t. the 3PR's aggressive order. For a given 3PR's ordering quantity Q_R^* , the retail platform will decide the optimal trade credit amount $wQS - B$.

The retail platform's expected payoff can be written as

$$\Pi_S = \mathrm{E}\left[(w - c)Q_S + \min\{p\min\{D, Q_S\}, (wQ_S - B)\} - (wQ_S - B)\right],$$

where the first term $(\text{i.e.}, (w - c)Q)$ is expect profit from the goods sold. The second term (i.e., min{p min{D, Q}, (w Q − B)} − (wQ − B)) addresses the expected payoff of the trade credit.

Theorem 4.2. Suppose the 3PR has an initial budget B, and the 3PR's demand, D, follows a normal uncertainty distribution $\mathcal{N}(e, \sigma)$ or a linear uncertainty distribution $\mathcal{L}(a, b)$,

(1) the retail platform's expected payoff is

$$\Pi_S = (w - c)Q_S - \int_0^{(wQ_S - B)/p} p\mathcal{M}\{D \le x\}dx$$

where $QS \in [0, Q_R^*]$.

(2) The retail platform will offer limited amount of trade credit w $\min\{Q_R^*, Q_S^*\} - B$ to the 3PR, and Q_S^* solves from

$$\Phi((wQ_S^* - B)/p) = 1 - \frac{c}{w}.$$

Proof. Part (i): At first, we calculate the following expected value

$$\begin{aligned}
&\mathrm{E}\left[((w - p)Q_S - B + p(Q_S - D)^+)^+\right] \\
&= \int_0^{+\infty} \mathcal{M}\{((w - p)Q_S - B + p(Q_S - D)^+)^+ \ge r\}\mathrm{d}r \\
&\quad - \int_{-\infty}^0 \mathcal{M}\{((w - p)Q_S - B + p(Q_S - D)^+)^+ \le r\}\mathrm{d}r \\
&= \int_0^{+\infty} \mathcal{M}\{p(Q_S - D)^+ \ge r + B + (p - w)Q_S\}\mathrm{d}r \\
&= \int_0^{+\infty} \mathcal{M}\{D \le (wQ_S - B - r)/p\}\mathrm{d}r
\end{aligned}$$

Setting $x = (wQ_S - B - r)/p$, and we have

$$\begin{aligned}
&\mathrm{E}\left[((w - p)Q_S - B + p(Q_S - D)^+)^+\right] \\
&= \int_{-\infty}^{(wQ_S - B)/p} p\mathcal{M}\{D \le x\}\mathrm{d}x \\
&= \int_0^{(wQ_S - B)/p} p\mathcal{M}\{D \le x\}\mathrm{d}x.
\end{aligned}$$

Therefore, retail platform's expected payoff equals

$$\begin{aligned}
\Pi_S &= \mathrm{E}\left[\min\{p\min\{D, Q_S\}, (wQ_S - B)\}\right] - cQ_S + B \\
&= \mathrm{E}\left[\min(pQ_S - p(Q_S - D)^+, wQ_S - B)\right] - cQ_S + B \\
&= \mathrm{E}\left[(wQ_S - B) - ((wQ_S - B) - pQ_S + p(Q_S - D)^+)^+\right] - cQ_S + B \\
&= (w - c)Q_S - \mathrm{E}\left[((wQ_S - B) - pQ_S + p(Q_S - D)^+)^+\right] \\
&= (w - c)Q_S - \int_0^{(wQ_S - B)/p} p\mathcal{M}\{D \le x\}\mathrm{d}x.
\end{aligned}$$

Part (ii): The retail platform's optimal trade credit am ount depends on Q_S^* , which maximize its total profit. Therefore, the optimal Q_S^* satisfies the following first order condition, i.e.,

$$\frac{\mathrm{d}\Pi_S}{\mathrm{d}Q_S} = (w - c) - w\Phi((wQ_S - B)/p).$$

When $Q_R^* < Q_S^*$ we find that ΠS is an increasing function w.r.t. Q_S, therefore, the optimal trade credit amount equals $wQ_R^* - B$. When $Q_R^* > Q_S^*$ we find that $\mathrm{dII}_S < \mathrm{d}Q_S$ is a decreasing function w.r.t. Q_S , so II_S is a concave function. At this time, the retail platform will provide a trade credit amount equals $wQ_R^* - B$ to the retailer.

From Theorem 4.2, we find that the retail platform's expected profit is not an increasing function of 3PR's ordering quantity, and as we analysed in Corollary 4, the capital-constrained 3PR may order more when it has slight initial capital, however, the retail platform's expected profit is a concave function w.r.t. Q_S when $Q_R^* > Q_S^*$. Therefore, the retail platform will not provide a trade credit amount $wQ_R^* - B$ to the 3PR, because from the prospective of the retail platform, the optimal trade credit amount should be $wQ_S^* - B$. Therefore, the the 3PR's irrational ordering quantity will be avoided.

Corollary 5. The ordering quantity's upper limits set by the retail platform, Q_S^* is an increasing function w.r.t. B.

Corollary 6. If the demand is described by a normal uncertain variable $\mathcal{N}(e, \sigma)$, then

$$Q_S^* = \frac{1}{w} \times \left(pe + B - \frac{\sqrt{3}\sigma p}{\pi} \ln \frac{w}{w-c} \right).$$

Corollary 7. If the demand is described by a linear uncertain variable $\mathcal{L}(a, b)$, then

$$Q_S^* = \frac{1}{w} \times \left(B + pb - \frac{c}{w}(pb - pa) \right).$$

4.4 Retail platform's decision with asymmetric initial budget informa- tion. In this subsection, we assume 3PR's initial budget is asymmetric information which is not yet known to the retail platform. When the 3PR has slight initial budget, the 3PR will place a huge order Q_R and apply for trade credit of $wQ_R - B$, however the retail platform will only lend $wQ_S - B$ instead of $wQ_R - B$ to the 3PR, and the 3PR can only procure $Q_S (Q_S < Q_R)$ unit of goods. What's more, accord- ing to corollary 5, Q_S is an increasing function w.r.t B. Therefore, the 3PR may overestimate its initial capital to ask for a larger amount of loan. The information about 3PR's initial budget is known to the retail platform through domain experts' experimental knowledge in the form of uncertain variable with an uncertainty dis- tribution $\psi(x)$ and an inverse uncertainty distribution $\psi^{-1}(\alpha)$.

Theorem 4.3. Suppose the 3PR has an initial budget B, and it is asymmetric in- formation belonging to 3PR, and it is known to the retail platform through domain experts' experimental knowledge. The retail platform regards it as an uncertain variable with an uncertainty distribution $\psi(x)$ and an inverse uncertainty distribu- tion $\psi^{-1}(\alpha)$, and the 3PR's demand, D, follows a normal uncertainty distribution $\mathcal{N}(e, \sigma)$ or a linear uncertainty distribution $\mathcal{L}(a, b)$,

(1) the retail platform will offer limited amount of trade credit w min $\{Q_R^*, \tilde{Q}_S^*\}$ to 3PR, and $\tilde{Q}_S^*$ solves from

$$(w - c) - \int_0^1 w\Phi((w\widetilde{Q}_S - \Psi^{-1}(\alpha))/p)d\alpha = 0.$$

(2) The 3PR will share the initial budget information when $Q_S^* > \tilde{Q}_S^*$

Proof. (1) According to theorem 4.2, the retail platform's expected payoff equals

$$\begin{aligned}\Pi_S &= \mathrm{E}\left[\min\{p\min\{D, Q_S\}, (wQ_S - B)\}\right] - cQ_S + B \\ &= (w-c)Q_S - \mathrm{E}\left[\int_0^{(wQ_S - B)/p} p\mathcal{M}\{D \le x\}\mathrm{d}x\right] \\ &= (w-c)Q_S - \int_0^1 \int_0^{(wQ_S - \Psi^{-1}(\alpha))/p} p\mathcal{M}\{D \le x\}\mathrm{d}x\mathrm{d}\alpha.\end{aligned}$$

Therefore, the optimal $\tilde{Q}_S$ satisfies the following first order condition, i.e.,

$$\frac{\mathrm{d}\widetilde{\Pi}_S}{\mathrm{d}\widetilde{Q}_S} = (w - c) - \int_0^1 w\Phi((w\widetilde{Q}_S - \Psi^{-1}(\alpha))/p)\mathrm{d}\alpha = 0.$$

(2) When $Q_S^* > \tilde{Q}_S^*$, the 3PR will get more loans if it shares initial budget infor-mation, so at this time, the 3PR will share the initial budget information.

Corollary 8. If the demand is described by a normal uncertain variable $\mathcal{N}(e, \sigma)$ and the initial budget is described by a linear uncertain variable $L(\underline{B}, \overline{B})$, then

$$\widetilde{Q}_S^* = \frac{1}{w} \times \left(pe + \frac{\underline{B} + \overline{B}}{2} - \frac{\sqrt{3}\sigma p}{\pi} \ln \frac{w}{w-c} \right).$$

The 3PR will share the initial budget information when $B > \frac{\underline{B}+\overline{B}}{2}$

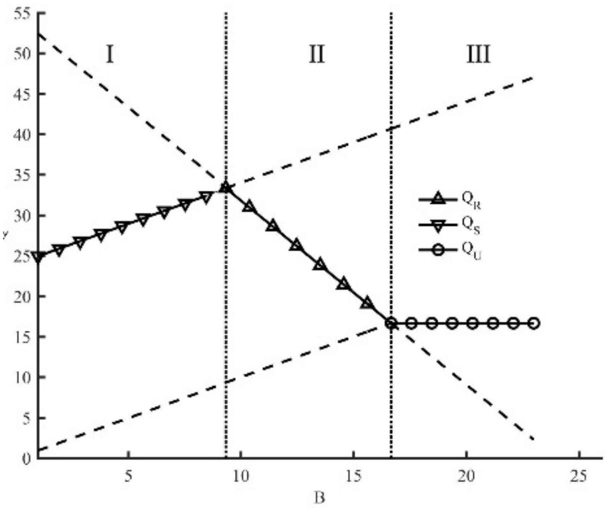

Figure 2. Retailer's optimal ordering quantity with different initial budget B

Corollary 9. If the demand is described by a linear uncertain variable $L(a, b)$ and the initial budget is described by a linear uncertain variable $\mathcal{L}(\underline{B}, \overline{B})$, then

$$\widetilde{Q}_S^* = \frac{1}{w} \times \left(\frac{\underline{B} + \overline{B}}{2} + pb - \frac{c}{w}(pb - pa) \right).$$

The 3PR will share the initial budget information when $B > \frac{\underline{B}+\overline{B}}{2}$

5. Numerical Example

In this section, we provide a numerical example to illus- trate and analyse the effect of trade credit. Consider a two-echelon supply chain with a retail platform and a retailer. The 3PR faces uncertain demand, D, which follows an uncertainty distribution. The 3PR does not have any historical data about the demand, therefore, the 3PR consults experts and collects the experts' experimental data through a questionnaire survey. The collected data are shown in Example 4.1. Based on the least squares method, the retailer derives that the uncertain demand follows a linear uncertainty distribution $\mathcal{L}(0, 104.6)$.

The retail platform's production cost, c, is \$0.8 per unit, and the wholesale price, w, is \$1 per unit. The retailer has an initial budget B, and the retail price, p, is \$1.2 per unit. At the beginning of the sales season, the retailer chooses to order Q units of product. When the 3PR is capital-constrained, that is $B < wQ$, then the 3PR and retail platform will negotiate a trade credit contract. Next, we examine the 3PR's optimal decision, supply chain's profit and supply chain members' profit in different 3PR's initial budget.

5.1 Ordering quantity analysis. Figure 2 shows 3PR's optimal ordering quan- tity with different initial budgets. There are three regions in Figure 2. The full line and its extension dotted line, in region I, depicts the upper limit of the ordering quantity, Q_S^* , set by the retail platform, and Q_S^* increases with B. The full line and its extension dotted line, in region II, depicts capital constrained 3PR's optimal or-dering quantity, Q_R^*, when it receives trade credit from the 3PR, and Q_R^* decreases with B. The full line and its extension dotted line, in region III, depicts 3PR's optimal ordering quantity, Q_U^*, when it receives no trade credit. When the 3PR is capital constrained, the 3PR can only order B/w units of good which deviates from its optimal ordering quantity.

In the region I of Figure 2. The 3PR has only a slight initial capital, just as we analysed, the expected loss of placing a large order is low and limited. The expect profit is an increasing function with the units of good that ordered. Therefore, the 3PR's ordering quantity is a decreasing function w.t.t B. In other words, the 3PR is a risk-seeker. The reason behind this phenomenon is that the market risk is shifted from the 3PR to the retail platform through trade credit. Consequently, the retail platform's expected profit is a concave function w.r.t 3PR's ordering quantity. As to manage its risk, the retail platform will set an upper limit to the 3PR's ordering quantity. And the upper limit works in this region.

In the region II of Figure 2. The 3PR has more initial capital, and the expected loss of placing a large order increases as well. At this time, the 3PR has to make a trade-off between place a reasonable order and place a larger order. The rational 3PR will not do what it did in region I. The retail platform's risk is mitigated due to the increase in 3PR's initial budget. Hence, the retail platform sets a higher upper limit to the 3PR's ordering quantity. And the upper limit does not work anymore. In the region III of Figure 2. The 3PR is not capital constrained anymore. This problem degenerates to classical uncertain newsvendor problem.

5.2 Profit analysis. Figures 3 and 4 show the 3PR's and the retail platform's expected profit with or without trade credit under different initial budget, respec- tively. There are three regions in Figures 3 and 4, the regions are divided under the same principle in Figure 2. Both the 3PR's and the retail platform's expected profit with trade credit is greater than that without trade credit. This is because, with the help of the trade credit, the 3PR will order more units of goods and the retail platform will produce more units of goods, both the 3PR and the retail platform generate extra value through trade credit. The insight behind is that trade credit is a great impetus for supply chain enterprises to increase revenue. What's more, we find that the retail platform's expected profit with trade credit increases first and then decreases. This is because when the 3PR has little initial budget. The 3PR's ordering quantity is decided by the upper limit set by the retail platform, which is derived from the view of the retail platform. At this time, retail platform's expected profit increases w.r.t. B. When the 3PR has more initial capital so that suppler's upper limit is not working anymore, the ordering quantity is derived from the view of the 3PR, not the retail platform. Consequently, the retail platform's expected profit decreases at this time.

The 3PR's expected profit decreases with initial capital in regions I and II. This is because the 3PR's expected profit is a decreasing function w.r.t initial capital.

5.3 Risk analysis. Figure 5 shows retail platform's and 3PR's different expected profit with different demand volatility. The variance of demand measures its volatil- ity which is directly relevant to experts' experimental data. A high volatility means the demand is of high uncertainty, and a low volatility means the demand is of low uncertainty. Note that, Figure 5 shows that the 3PR's expected profit has positive correlation w.r.t. demand uncertainty, while the retail platform's expected profit is a concave function w.r.t. demand uncertainty. We should note that this relationship contrasts with classical newsvendor problem by incorporating trade credit.

It is noticeable that the 3PR's expected profit has a positive correlation with de- mand volatility, but, this conclusion is not consistent with the classical newsvendor problem without trade credit, in which, the newsvendor's expected profit is a con- cave function w.r.t. demand 3PR. This is because the higher demand uncertainty will lead to an increase in 3PR's orders in classical newsvendor problem, and unsold goods will damage 3PR's business. Hence the 3PR's expected profit is a concave function w.r.t. demand uncertainty.

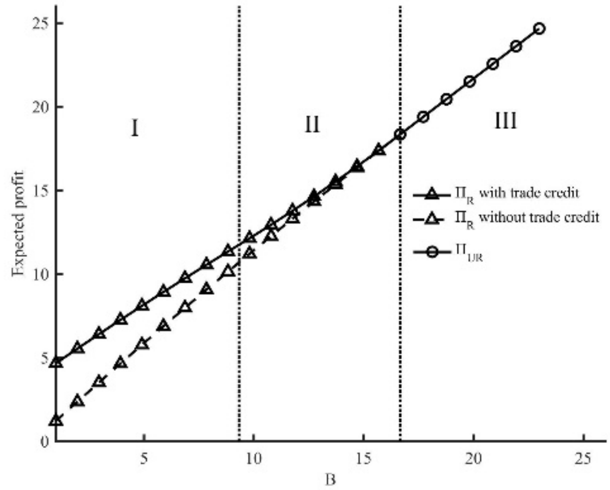

Figure 3. 3PR's expected profit with or without trade credit

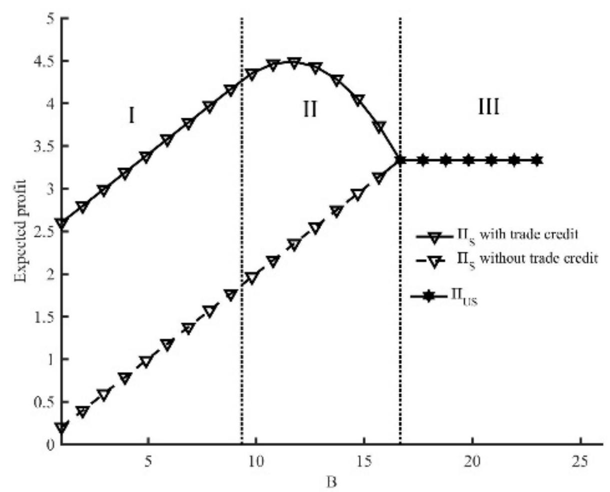

Figure 4. Retail platform's expected profit with or without trade credit

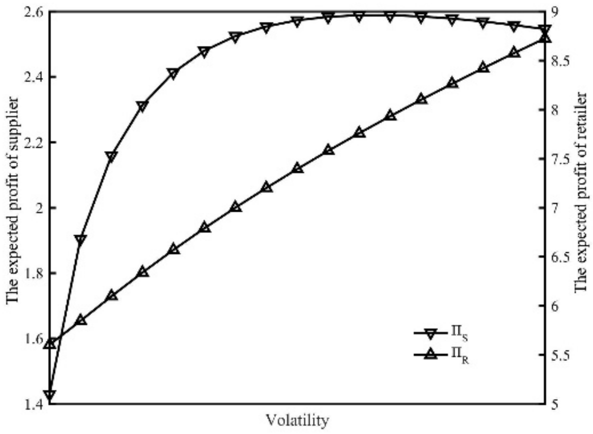

Figure 5. Retail platform's and 3PR's expected profits with different demand volatility

In this paper, the 3PR receives trade credit from the retail platform, and the extra demand uncertainty is shifted to the retail platform through trade credit. Therefore, the 3PR's expected profit has a positive correlation with volatility.

Figure 5 show's that the retail platform's expected payoff is a concave function

w.r.t. demand uncertainty, while the retail platform's expected payoff has a posi- tive correlation with demand volatility in classical newsvendor problem. In classical newsvendor problem, the higher demand uncertainty will lead o an increase in 3PR's orders, and the retail platform benefit from the increase in 3PR's orders, therefore, the retail platform's expected profit has a positive correlation with demand uncer- tainty. In this paper, the retail platform provides trade credit to the 3PR. The trade credit on the one hand stimulates the 3PR's ordering quantity, on the other hand shares part of 3PR's demand uncertainty. Note that, the 3PR has limited initial budget which means the 3PR bears only limited demand uncertainty, and the extra demand uncertainty is shifted to the retail platform through trade credit. As we have mentioned in the previous paragraph, the unsold goods will damage 3PR's business, and when the loss is beyond 3PR's initial budget, the extra loss is billed by the retail platform, and this explains why the retail platform's expected profit is a concave function w.r.t. demand uncertainty.

6. Conclusion

This paper studies the optimal operational decision of a capital- constrained third-party retailer (3PR), who gets trade credit from the retail plat- form. We abstract this problem as a newsvendor model. In this model, the 3PR is a start-up enterprise or individual so that only a little historical data is available. Therefore, probability-based methods are not valid for this problem. We use ex- pert's experimental data-based method which is proposed by Liu [16] to obtain the uncertainty distribution of 3PR's uncertain demand. This is the first paper that investigates how to obtain the uncertainty distribution in uncertain newsvendor problem. We use convex optimization method to obtain the 3PR's optimal lot- sizing decision and the retail platform's optimal trade credit amount, and closed- form solutions are derived. We have concluded that a) the method that we use in this paper is applicable when estimating 3PR's uncertainty distribution. b) When the 3PR has a slight initial capital, the 3PR has an incentive to place a huge order, because the trade credit is a non-recourse loan, and in extreme cases, the 3PR may claim that it has no initial capital. However, the retail platform will not provide as much trade credit as the 3PR applies, and the retail platform will offer reasonable amount of trade credit based on the demand information at this time. c) With retail platform's intervention, when the 3PR has slight initial capital, the 3PR may overstate its initial capital and ask for a larger amount of loan. At this time, the retail platform will estimate the 3PR's initial budget information through experts' experimental data and offer reasonable amount of trade credit based on the initial budget information. We also reveal that under certain conditions, the 3PR will share the information.

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References

[1] J. Arrow, T. Harris, J. Marschak, Optimal inventory policy, Econometrica, 19(1951), 250-272.

[2] Y. Avyv, Gaining benefits from joint forecasting and replenishment processes: the case of auto-correlated demand, M&SOM-Manu. Serv. Op., 4(2002), 55-74.

[3] Y. Avyv, A time-series framework for supply-chain inventory management, M&SOM-Manu. Serv. Op., 51(2003), 175-343.

[4] T. Beck, Financing patterns around the world: Are small firms different?, Journal of Financial Economics, 89(2008), 467-487.

[5] J. Bing, X. Chen and G. Cai, Equilibrium financing in a distribution channel with capital constraint, Prod. Oper. Manag., 21(2012), 1090-1101.

[6] J. Buzacott, R. Zhang, Inventory management with asset-based financing, Manage. Sci., 50(2004), 1274-1292.

[7] S. Cao, X. Yang, C. Chen, J. Zhou and Y. Qi, Titant: online real-time transaction fraud detection in ant financial, 2019.

[8] E. Carrizosa, A. Olivares-Nadal and P. Ramrez-Cobo, Robust newsvendor problem with au- toregressive demand, Comput. Oper. Res., 68(2016), 123–133.

[9] X. Chen, A. Wang, Trade credit contract with limited liability in the supply chain with budget constraints, Ann. Oper. Res., 196(2012), 153–165.

[10] S. Ding, Uncertain multi-product newsboy problem with chance constraint, Appl. Math. Com- put., 223(2013), 139–146.

[11] L. Dong, H. Lee, Optimal policies and approximations for a serial multiechelon inventory system with time-correlated demand, Oper. Res., 51(2003), 969–980.

[12] S. Graves, A single-item inventory model for a nonstationary demand process, M&SOM- Manu. Serv. Op., 1(1999), 50-61.

[13] D. Gupta, L. Wang, A stochastic inventory model with trade credit, M&SOM-Manu. Serv. Op., 11(2009), 4-18.

[14] P. Kouvelis, W. Zhao, Financing the newsvendor: supplier vs. bank, and the structure of optimal trade credit contracts, Oper. Res., 60(2012), 566-580.

[15] B. Liu, Uncertainty Theory, Springer-Verleg, 2007

[16] B. Liu, Uncertainty Theory - A Branch of Mathematics for Modeling Human Uncertainty, Springer-Verleg, 2010.

[17] B. Liu, Why is there a need for uncertainty theory, Journal of Uncertain Systems, 6(2012), 3-10.

[18] Z. Qin, S. Kar, Single-period inventory problem under uncertain environment, Appl. Math. Comput., 219(2013), 9630–9638.

[19] D. Wang, Z. Qin, S. Kar, A novel single-period inventory problem with uncertain random demand and its application, Appl. Math. Comput., 269(2015), 133-145.

[20] D. Wu, B. Zhang, O. Baron, A trade credit model with asymmetric competing retailers, Prod. Oper. Manag., 28(2018), 206-231.