

# Optimization Model of Survey Line Layout for Multibeam Bathymetric Systems

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**Abstract:** Multibeam bathymetric system is an underwater terrain measurement technology with high efficiency, high accuracy and high resolution. Based on the multibeam bathymetric system, this paper establishes an ideal geometric model and an isobath iterative model. Through the physical method of ideal model, the actual physical problem of multibeam sounding is abstracted into a geometric model about a conic curve, and under the constraint of overlap rate, the coordinates of the next survey line are iterated according to the current depth to solve the optimal set of survey lines. First, the seawater depths, strip coverage widths, and overlap rates with the previous line are calculated for lines at different distances from the center point. Then, the angle between the direction of the survey line and the projection of the normal direction of the seafloor slope on the horizontal plane is introduced to solve for the current coverage width of the survey line. Again, the optimal set of survey lines is solved under the constraint of the overlap rate. Finally, Sobel edge detection is used to divide the surface of the seafloor terrain into multiple planes with defined slope angles to solve the optimal line set for the entire seafloor surface. The structure of the proposed mathematical model is clear and simple, which is in line with the actual situation.

**Keywords:** Ideal Geometric Model; Conic Curves; Isometric Iterative Model; Sobel Edge Detection; Surface Fitting.

## 1. Introduction

The multibeam bathymetric system has a wide range of application prospects in China's ocean exploration and marine resource management, maintains China's marine environment and homeland security, and provides a powerful tool for scientific research and ocean management [1-3].

Multibeam bathymetric systems utilize multiple acoustic beams to simultaneously transmit and receive acoustic signals to measure the depth and topography of the water column [4-5]. Compared with single-beam bathymetry, multibeam systems can realize full-coverage bathymetry in flat areas of the seafloor, which improves data density and topographic information acquisition.

In order to solve the above problems, this paper establishes an ideal geometric model and an isobath iterative model based on the multibeam bathymetry system. Through the physical method of ideal model, the actual physical problem of multibeam sounding is abstracted into a geometrical model about a conic curve, and under the constraint of overlap rate, the coordinates of the next survey line are iterated according to the current depth, thus solving the optimal set of survey lines. First, the seawater depths, strip coverage widths, and overlap rates with the previous line are calculated for lines at different distances from the center point. Then, the angle between the direction of the survey line and the projection of the normal direction of the seafloor slope on the horizontal plane is introduced to solve for the current coverage width of the survey line. Again, the optimal set of survey lines is solved under the constraint of the overlap rate. Finally, the surface of the seafloor terrain is segmented into multiple planes with defined slope angles using Sobel edge detection to solve for the optimal set of survey lines for the entire seafloor surface.

## 2. Model Formulation and Solving

### 2.1. Modeling and Solving When the Survey Line is Parallel to the Seafloor Slope

A geometric analysis of the multibeam bathymetry scene is shown in Figure 1.

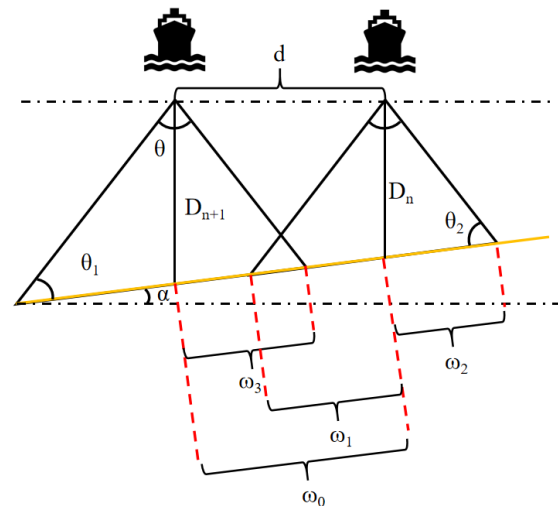


Figure 1. Geometric analysis of the multibeam bathymetry scene

When the survey lines are parallel to each other and the seafloor topography is flat, then the overlap rate  $\eta$  between neighboring strips is defined as:

$$\eta = 1 - \frac{d}{W} = \frac{\frac{W}{2} + \frac{W}{2} - d}{W} \quad (1)$$

Thus defining the overlap rate  $\eta$  between neighboring strips on a slope with a gradient  $\alpha$  is defined as:

$$\eta = \frac{W_1 + W_3 - W_0}{W_1 + W_2} \quad (2)$$

That is, the width of the overlap as a percentage of the width of the coverage of the line at the shallower depths of the sea.

Let the angle between the left beam and the seafloor slope in Fig. 1 be  $\theta_1$ , and the angle between the right beam and the seafloor slope be  $\theta_2$ .

By geometric analysis  $\theta_1$  and  $\theta_2$  are satisfied:

$$\begin{cases} \theta_1 = \frac{\pi}{2} - \frac{\theta}{2} - \alpha \\ \theta_2 = \frac{\pi}{2} - \frac{\theta}{2} + \alpha \end{cases} \quad (3)$$

From the sine theorem, the relationship between the opening angle  $\theta$  of the multibeam transducer, the depth  $D$  of the slope, the angle  $\theta$  between the right beam and the seafloor slope  $\theta_2$  and  $W_1$  can be expressed as follows:

$$W_1 = \frac{D_n \sin \frac{\theta}{2}}{\sin \theta_1} \quad (4)$$

Likewise:

$$\begin{cases} W_0 = \frac{d}{\cos \alpha} \\ W_2 = \frac{D_n \sin \frac{\theta}{2}}{\sin \theta_2} \\ W_3 = \frac{D_{n+1} \sin \frac{\theta}{2}}{\sin \theta_2} \end{cases} \quad (5)$$

From the geometric relationship

$$D_{n+1} = D_n + d \tan \alpha \quad (6)$$

$x$  represents the distance of the survey line from the center point, and the horizontal direction pointing to the rise of the seabed slope is the positive direction of  $x$ . From the geometric relationship the coverage width  $W$  can be expressed as:

$$W = (W_1 + W_2) \cos \alpha \quad (7)$$

From the geometric model the depth of seawater  $D$  can be expressed as:

$$D = h_c + d \tan \alpha \quad (8)$$

The opening angle  $\theta$  of the multibeam transducer is  $120^\circ$ , the slope  $\alpha$  is  $1.5^\circ$ , the depth of seawater  $h_c$  at the center point of the sea is 70 m, and the spacing of the survey lines  $d$  is 200 m, i.e.:

$$\begin{cases} \theta = 120^\circ \\ \alpha = 1.5^\circ \\ h_c = 70m \\ d = 200m \end{cases} \quad (9)$$

The spacing of the survey lines  $d$  is 200 m, and the distance from the survey line to the center point is -800 m. The seawater depths, coverage widths, and overlap rates with the previous survey line corresponding to different distances of the survey lines are obtained as shown in Figure 2.

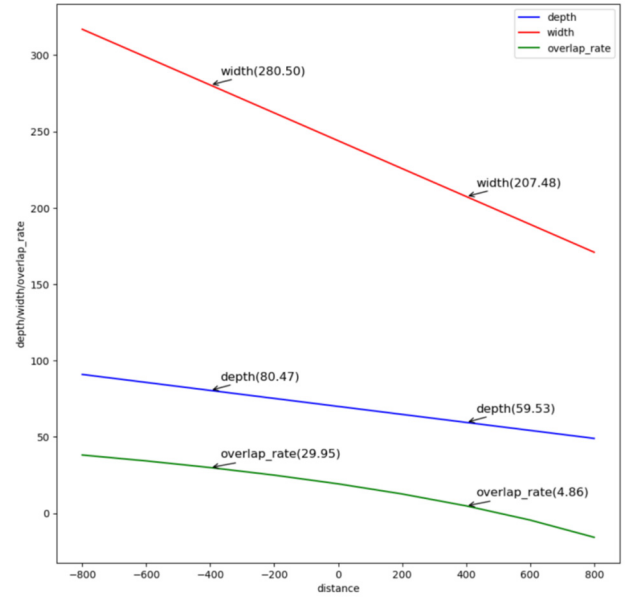


Figure 2. Plot of data corresponding to measuring lines of different distances

From Figure 2, it can be seen that the depth of the sea water, the coverage width and the overlap rate with the previous measurement line decrease with the increase of the distance from the center point, and the overlap rate  $\eta$  is negative at  $x=600m$  and  $x=800m$ , which means that the "leakage phenomenon" has occurred.

## 2.2. Modeling and Solving Multibeam Bathymetric Coverage Width at Different Points

The angle  $\beta$  between the direction of the survey line and the projection of the normal direction of the seafloor slope on the horizontal plane, the direction of the survey line is perpendicular to the beam sector, and the range of  $\beta$  is  $[0, 2\pi]$ , as shown in Figure 3.

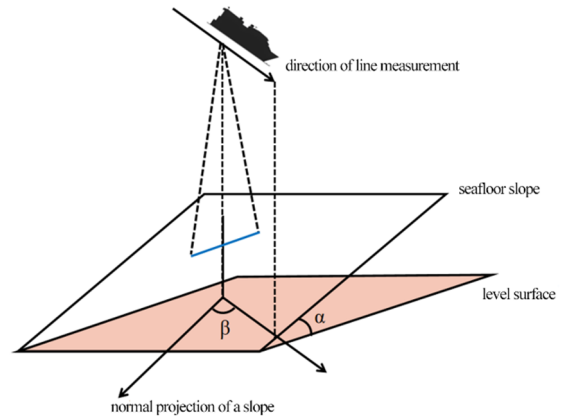


Figure 3. Schematic diagram of the angle  $\beta$  between the direction of the survey line and the projection of the normal direction of the seabed slope on the horizontal plane

The angle  $\beta$  between the direction of the survey line and the normal direction of the seafloor slope projected on the horizontal plane changes from small to large in the range of  $[0, 2\pi]$ , and the corresponding beam sector rotates to form a cone with the ship as the vertex, which intersects with the seafloor slope to form an ellipse. The different diameters  $W\beta$  of this ellipse are the detection widths corresponding to

different pinch angles  $\beta$ . When  $\beta$  is  $0^\circ, 45^\circ, 90^\circ, 135^\circ, 180^\circ, 225^\circ, 270^\circ, 315^\circ$ , the corresponding  $W\beta$  is:

$$\begin{cases} W_{0^\circ} = W_{180^\circ} = 415.55 \\ W_{45^\circ} = W_{135^\circ} = W_{225^\circ} = W_{315^\circ} = 416.05 \\ W_{90^\circ} = W_{270^\circ} = 416.55 \end{cases} \quad (10)$$

The distance  $dX$  traveled by the boat in the direction of the survey line is decomposed into the horizontal distance  $dZ$  pointing down the slope and the direction  $dY$  parallel to the slope, i.e:

$$dz = dx \cos \beta \quad (11)$$

The orthogonal decomposition of the distance  $dz$  in the horizontal direction pointing to the slope decline is decomposed into the distance  $dP$  along the direction of the slope normal vector and the distance  $dC$  perpendicular to the slope normal vector, i.e:

$$dP = dx \cos \beta \cos \theta \quad (12)$$

From geometric modeling:

$$\theta = \frac{\pi}{2} - \alpha \quad (13)$$

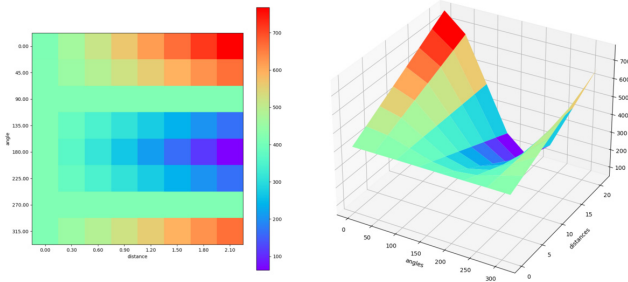
This is obtained from the properties of similar triangles:

$$\frac{H}{H + x \cos \beta \cos \theta} = \frac{W_\beta}{W'} \quad (14)$$

The opening angle  $\theta$  of the multibeam transducer is  $120^\circ$ , the slope  $\alpha$  is  $1.5^\circ$ , the depth of the sea water  $D_0$  at the center point of the sea area is 120 m, and the spacing of the survey lines  $d$  is 0.3 nautical miles =  $0.3 * 1852m = 555.6$  m, i. e:

$$\begin{cases} \theta = 120^\circ \\ \alpha = 1.5^\circ \\ D_0 = 120m \\ d = 555.6m \end{cases} \quad (15)$$

The distance  $d$  between survey lines is 0.3 nautical miles (555.6 m), and the distance from the survey line to the center point is 0 m. The coverage widths corresponding to different distances from the survey line and the angle  $\beta$  in the direction of the survey line are obtained as shown in Figure 4.



**Figure 4.** Plot of data corresponding to measuring lines of different distances

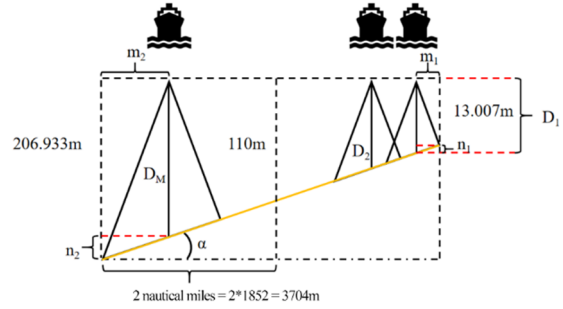
From Figure 4, when the angle  $\beta \in \left(0, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{2}, 2\pi\right)$ , i.e., the ship's has a downward velocity component along the slope, the coverage width increases with the distance of the measuring ship from the center point of the sea area; When the angle  $\beta \in \left(\frac{\pi}{2}, \frac{3}{2}\pi\right)$ , i.e., the boat has a

velocity component that is upward along the slope, the width of coverage decreases as the distance of the measuring boat from the center point of the sea area increases.

### 2.3. Optimization Strategy for Line-of-sight Design under Specified Overlap Rates

Since the overlap cannot exceed 20%, the spatial relationship between survey lines should be parallel. Design a set of survey lines with the shortest measuring length that can completely cover the entire sea area to be surveyed, i.e., it is necessary to determine the direction of the survey lines and the spacing of adjacent survey lines. When the vessel stays at a certain point, by rotating the direction of the survey, a cone with the vessel as the apex can be obtained. The intersection of this cone with the seabed slope is an ellipse, and the individual diameters of the ellipse are the detection widths corresponding to different angles  $\beta$ .

The shorter the total length of the lines to be measured, the larger the detection width corresponding to each line should be. The longest diameter in the ellipse is the long axis, so the long axis is used as the detection width, i.e., the line is parallel to the contour line, as shown in Figure 5.



**Figure 5.** Schematic of the profile

From the given information, the shallowest part of the sea (east side) is two nautical miles from the center, then the depth of sea water at the shallowest part is  $D_{min}$ :

$$D_{min} = hc - 2 * 1852 \tan \alpha \quad (16)$$

Similarly, the deepest seawater depth  $D_{max}$  is:

$$D_{max} = hc + 2 * 1852 \tan \alpha \quad (17)$$

To cover the shallowest part of the sea area, the first sounding line sea depth  $D_1$  on the east side is the deepest:

$$D_1 = D_{min} + D_{min} \tan \frac{\theta}{2} \tan \alpha \quad (18)$$

To cover the deepest part of the sea area, the first western sounding line sea depth  $D_S$  is the shallowest:

$$D_S = D_{max} - D_{max} \tan \frac{\theta}{2} \tan \alpha \quad (19)$$

Meanwhile, for the sake of data accuracy and error reparability, the overlap rate should satisfy the requirement of 10%~20%; meanwhile, in order to satisfy the shortest total measurement length, the overlap rate  $\eta=10\%$  is selected.

The opening angle  $\theta$  of the multibeam transducer is  $120^\circ$ , the slope  $\alpha$  is  $1.5^\circ$ , and the depth of seawater  $hc$  at the center point of the sea is 110 m, i.e:

$$\begin{cases} \theta = 120^\circ \\ \alpha = 1.5^\circ \\ hc = 110m \end{cases} \quad (20)$$

The coordinates of the starting point and the depth of the sea water at the starting point are used as initial values, and

then the distribution of the survey lines is derived as shown in Figure 6.

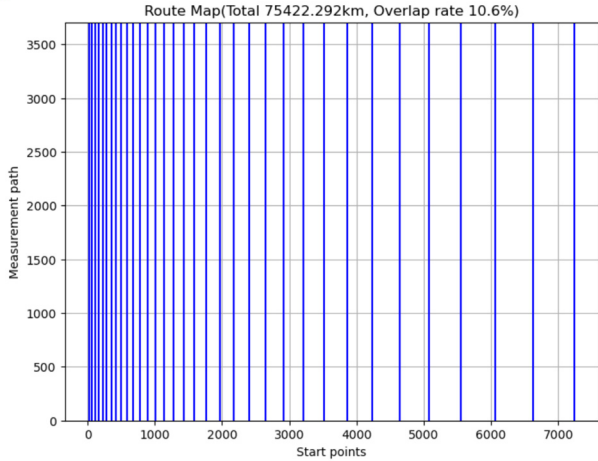


Figure 6. Schematic diagram of the distribution of survey lines

The total length of the survey lines is 75,422.292km, and the overlap rate is 10.6%, which is in line with the condition that the overlap rate is between 10% and 20%, and it is also found that the survey lines are more dense in the "shallow water area", and the arrangement of the survey lines is more sparse in the "deep water area".

#### 2.4. Optimization Strategy for Line-of-sight Design Considering Submarine Surfaces

In order to have a clearer and more intuitive understanding of the seabed topography of the sea area, the given data were visualized.

Since the surface of the seabed terrain should be segmented into a patchwork of multiple planes, the dividing line of each plane needs to be determined. And Sobel filter is a convolutional filter commonly used in image processing to detect edges and contours in an image. It calculates the gradient value of a pixel by applying two 3\*3 convolution kernels to each pixel of the image, which can be used to calculate the horizontal and vertical gradients of the image, which are used to identify the edges in the image, and then to determine how to segment the surface into planes.

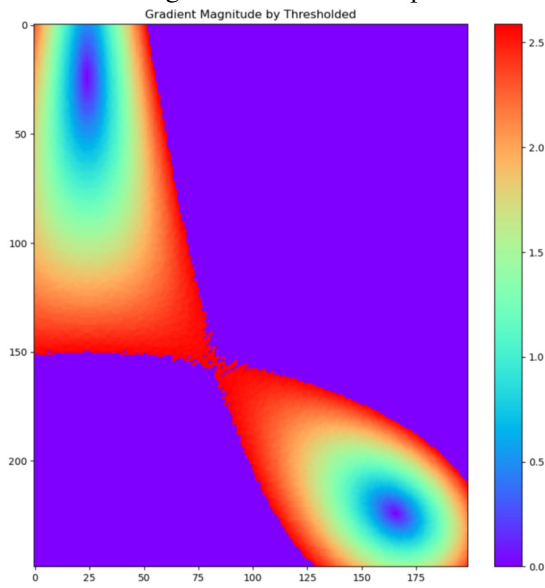


Figure 7. Depth gradient map of the seafloor

The gradient approximation of the seafloor depth data is obtained by two-dimensional convolution of the Sobel filter

with the seafloor depth data, and the gradient threshold is set to 2.588, i.e., the coordinate points with gradient greater than 2.588 are marked in purple, as shown in Figure 7.

Analysis of the gradient diagram shows that there can be two ways of dividing the seabed surface, method one is the square area division and method two is the triangular area division centered on the diagonal. Both method one and method two use the difference between the mean values of the depths of the two ends of the plane to calculate the slope angle of each plane. After calculation, the total length of the measured line of method I is 441044.819m, Leakrate1=5.2%; the total length of the measured line of method II is 325906.828m, Leakrate1=15.7%. By comparing the total line lengths, the total line length of Law II is shorter, the percentage of missed sea area to the total sea area to be measured is larger, and the total length of the part with overlap rate more than 20% is shorter. Method I has a longer total line length, a smaller percentage of the total sea area to be measured, and a longer total length for the portion of the line with an overlap of more than 20%. It was found that fitting a plane to a surface results in a loss of accuracy, and therefore results in an increase in the rate of missed measurements or the total length of the survey line.

### 3. Model Summary

#### 3.1. Model Benefits

1) Based on geometric analysis, this paper establishes an ideal geometric model and an equal-depth iterative model, which explains the problem in an easy-to-understand way, provides a tool for accurate reasoning, and is able to solve the core query effectively.

2) The numerical solutions derived from this model in the application process are more in line with the actual situation and can reasonably describe the bathymetric process of the multibeam sidelobe system.

3) The structure of this model is clear and simple, and its visualization is beautiful and easy to distinguish.

#### 3.2. Model Shortcomings

1) This model needs to use approximation processing when dealing with the seabed surface, which splits the surface into multiple planes, and there is a certain error.

2) There is a certain error in the ideal geometric model, and in real life, because sound waves bend in water, the geometric model can only approximate the composition of the triangular relationship, and thus the calculated numerical solution will have a certain error.

### 4. Conclusion

Based on the multibeam bathymetric system, this paper establishes an ideal geometric model and an isobath iterative model. The actual physical problem of multibeam bathymetry is abstracted into a geometrical model about a conic curve through the physical method of ideal model, and the coordinates of the next line are iterated according to the current depth under the constraints of the overlap rate, thus solving for the optimal set of lines. First, a mathematical model of the coverage width and the overlap rate between adjacent bathymetric strips for multibeam bathymetry is established, and the seawater depths, coverage widths, and overlap rates with the previous survey line corresponding to survey lines at different distances from the centroid are calculated according to this model. Then, the angle between

the direction of the survey line and the normal direction of the seafloor slope projected on the horizontal plane is introduced, and the similar triangle property is utilized to solve the coverage widths corresponding to the survey lines at different distances and the angle  $\beta$ . Again, the optimal survey line is designed, the shortest total survey line is solved by isobath iterative modeling, and the designed combination of survey lines is verified to ensure complete coverage of the entire sea area. Finally, the seafloor terrain is segmented into different planes with slope constraints by Sobel edge detection, and then survey lines are designed for each plane, and the spacing between survey lines is adjusted by the isobath iterative algorithm to meet the overlap rate requirement. The structure of the proposed mathematical model is clear and simple, which is in line with the actual situation.

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