

Research on Multi-Beam Line Finding Problem and Optimal Line Finding Path based on Exhaustion Method

Yuxuan Lu *

School of Finance, Central University of Finance and Economics, Beijing, China

* Corresponding author Email: lyxcufe2022@126.com

Abstract: In this paper, the multi-beam line measurement problem is studied, solving the relevant data of different seabed slope and line measurement, and establishing a multi-beam line measurement model based on it, simulating the work of multi-beam line measurement, using ergodic and iterative algorithms, to achieve the best line measurement path design and calculation of uneven sea area. First of all, the mathematical model of multi-beam line surveying with variable direction of line surveying is obtained through geometric relation and sine and cosine theorem, etc., and the coverage width W , overlap rate between lines η and sea depth D of measuring ships at different positions are solved. Secondly, the characteristics of sea terrain are analyzed, and the variation law of the coverage width W of the measured line with the angle of the measured line direction β is explored. Finally, whether the beginning and end points of the survey line reach the north and south banks is classified and discussed, and the traversal calculation method is designed to exhaustively explore all possible points and solve the minimum points.

Keywords: Loop Traversal; Calculation Geometry; Multi-Beam Line Measurement.

1. Introduction

In the 1980s, multi-beam sounding technology emerged, and has been developed into the mainstream detection mode of seabed sounding. Multi-beam sounding is based on single beam sounding, which is characterized by full coverage and high precision. The multi-beam surveying line can change the angle of the transducer and send out dozens or even hundreds of beams in the vertical plane of the surveying line direction at a time, so as to enlarge the width of the surveying line and form the purpose of the full coverage of the water depth strip [1-3]. But at the same time, multi-beam sounding also faces the problem of seabed topography fluctuation. Based on the ergodic exhaustive algorithm, this paper studies how to calculate the coverage width and overlap rate of the survey line under different conditions and design a reasonable survey line direction in the face of the rolling seabed topography, which is of great significance for reducing the phenomenon of leakage and data redundancy in the sea area and improving the efficiency and accuracy of multi-beam sounding technology.

2. Mathematical Model of Multi-beam Survey Line

Assuming that the slope α , the opening angle of the multi-beam transducer β and the water depth D of the center point of the sea area are known, this chapter establishes a mathematical model of the coverage width of the multi-beam sounding and the overlap rate of adjacent strips under these conditions, and uses the model to solve the seawater depth, coverage width and overlap rate of the lines with different distances from the center of the sea area.

2.1. The Mathematical Model of Multi-Beam Line Measurement with Constant Line Direction

2.1.1. Coverage Width Calculation Formula

Set the edge and angle and construct the static plane

geometry diagram when the surveying ship is in a certain position, as shown in Figure 1.

As shown in Figure 1, according to the geometric relationship, the angles established meet the following requirements:

$$\begin{cases} \theta_1 = \frac{\pi}{2} + \alpha \\ \theta_2 = \frac{\pi}{2} - \alpha \\ \theta_3 = \frac{\pi}{2} - \frac{\theta}{2} - \alpha \\ \theta_4 = \frac{\pi}{2} - \frac{\theta}{2} + \alpha \end{cases} \quad (1)$$

Therefore, according to the law of sine and the law of cosine, the covering width W can be expressed as:

$$W = D \cos^2 \alpha \sqrt{\frac{1}{\cos^2(\frac{\theta}{2} - \alpha)} + \frac{1}{\cos^2(\frac{\theta}{2} + \alpha)} - \frac{2 \cos \theta}{\cos(\frac{\theta}{2} - \alpha) \cos(\frac{\theta}{2} + \alpha)}} \quad (2)$$

2.1.2. Formula for Calculating Overlap Rate

Assuming that the opening angle of the transducer is unchanged, the schematic diagram of two adjacent multi-beam survey lines on two different positions A and B from the center of the sea area is constructed by the survey ship as shown in Figure 2.

According to the sine theorem:

$$\frac{L_1}{\sin(\frac{\pi}{2} + \frac{\theta}{2})} = \frac{l_1}{\sin \alpha} \quad (3)$$

$$\frac{L_2}{\sin(\frac{\pi}{2} - \alpha)} = \frac{D}{\sin \alpha} \quad (4)$$

According to the proportional relationship between parallel lines, the sea surface height D' at the second line can be measured.

$$\frac{D'}{D} = \frac{L_2 - d}{L_2} \quad (5)$$

Then the overlapping area X is:

$$X = W - x_3 \quad (6)$$

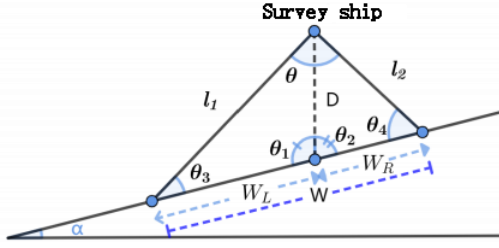


Figure 1. Static plane geometry diagram

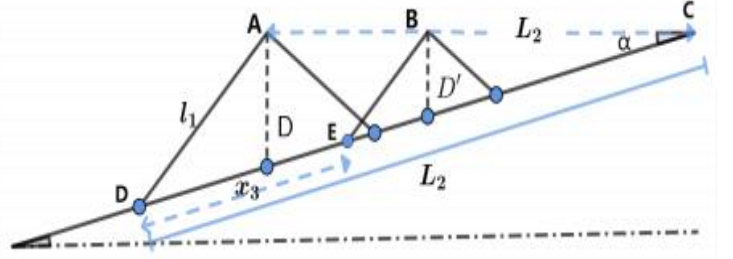


Figure 2. Diagram of adjacent multi-beam line measurement

Get the overlap rate η between the second measurement and the first measurement as:

$$\eta = \frac{x}{W} \quad (7)$$

In summary, the expression obtained is as follows:

$$\eta = \frac{D}{D - dtana} - \frac{d \frac{\theta}{2}}{d \cos(\frac{\theta}{2} + \alpha) \tan \alpha \sqrt{\frac{1}{\cos^2(\frac{\theta}{2} - \alpha)} + \frac{1}{\cos^2(\frac{\theta}{2} + \alpha)} - \frac{2 \cos \theta}{\cos(\frac{\theta}{2} - \alpha) \cos(\frac{\theta}{2} + \alpha)}} \quad (8)$$

Therefore, η is a function related to the depth and the distance between the two measurement lines.

2.2. Multi-beam Line Measurement Mathematical Model with Variable Direction of Line Measurement

2.2.1. Coverage Width Solution

As shown in Figure 3, when the angle between the survey line direction and the direction of ship navigation is β , the inclination angle θ_1' in the cross-sectional view perpendicular to the survey line direction (as shown in Figure 3) differs from θ_1 (as shown in Figure 1) because the slope also changes when viewed from this section. The new slope α' can be calculated from the geometric relationships shown in the figure, satisfying the following equation:

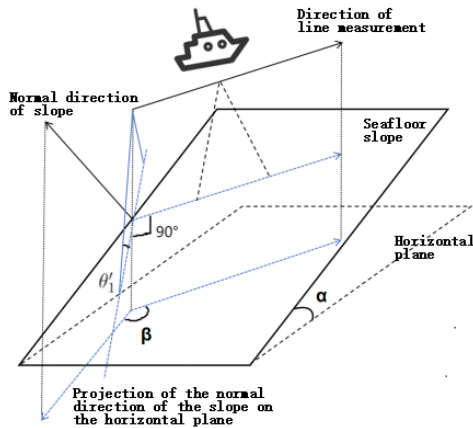


Figure 3. A geometric diagram in the direction of an angular line

In the first scenario, when the angle β between the survey line directions is 0° , and when $\beta_i = 0$, the cross-sectional triangle no longer has a slope, and α' is also 0° , making the equation unable to be solved. When the slope is 0° , the cross-

$$\tan \alpha' = \tan \alpha \sin \beta \quad (9)$$

By replacing the original slope α with the new slope α' , the coverage width of multi-beam sounding can be obtained:

$$D \cos^2 \alpha' \sqrt{\frac{1}{\cos^2(\frac{\theta}{2} - \alpha')} + \frac{1}{\cos^2(\frac{\theta}{2} + \alpha')} - \frac{2 \cos \theta}{\cos(\frac{\theta}{2} - \alpha') \cos(\frac{\theta}{2} + \alpha')}} = W' \quad (10)$$

2.2.2. Seawater Depth Solution

According to the proportionality relationship between parallel lines, the sea surface height (D_i) measured at the i -th survey line can be obtained as follows:

$$\frac{D_i}{D} = \frac{L_2 - d}{L_2} \quad (11)$$

Where the positive direction d is the direction from the depth of water to the shallow water, the value L_2 is obtained from the following formula:

$$\frac{L_2}{\sin(\frac{\pi}{2} - \alpha)} = \frac{D}{\sin \alpha} \quad (12)$$

However, it is important to note that different values of the angle β require separate considerations. Figure 4 illustrates a plan view of the measurement of the ship's survey line direction for different values of β .

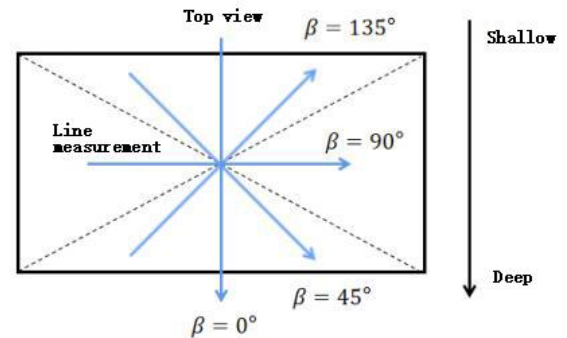


Figure 4. Top view of line direction

section forms an equilateral triangle with a vertex angle θ of 120° . When the depth is D_i' , the width W is $2\sqrt{3}D_i$. For 180° , the reverse situation of 0° , the calculation is similar.

The second case, when the angle is 90° or 270° . When the survey boat is traveling along the survey line, it is parallel to the slope and the depth remains the same.

Other angles can be calculated normally.

3. Build the Optimal Line Path Design Model Based on Exhaustion

Based on the established mathematical model of the coverage width of multi-beam sounding, a set of multi-beam surveying line is designed for the rectangular sea area, so that the total measuring length of the surveying line is shortest, and the whole sea area to be measured can be completely covered.

3.1. Establishment of target model

To explore the relationship between variables, with coverage width W as the dependent variable, and the angle β of the sounding line direction and the distance s of the measurement vessel from the shore as the independent variables. The study found that when the distance s from the shore of the measuring ship is constant, the function image of W is an image of trigonometric function class, its value changes periodically with the change of β , the amplitude A is determined by the corresponding distance s from the shore of the measuring ship, and the period T is constant π . When the angle of the direction of the control line is constant β , W is proportional to the distance from the measuring ship to the shore, and the farther away from the shore, the larger the coverage width. Therefore, it can be inferred that the surveying line will be denser at the shallow water and sparse at the deep water; β_i near 90° , the overall length of the line will be smaller.

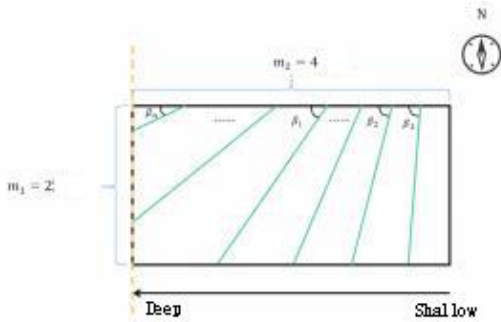


Figure 5. Top view of simulated line process between north and south coasts

$$\sum_{i=1}^n W_N(\beta_i, n) > m_1 \quad (16)$$

$$\sum_{i=1}^n W_S(\beta_i, n) > m_1 \quad (17)$$

By calculating the number of measurements when the direction of the survey line is always 90° , the number of measurements under the direction of other survey lines can be roughly estimated. At this time, the measurement length is only related to the number of measurements, in order to minimize the number of measurements, we should try to ensure the maximum gap, that is, the minimum overlap rate, so the overlap rate is set at 10%. Since the west is deep and the east is shallow, the measurement starts from the West Bank, and the area that the measurement strip cannot cover when it reaches the east coast is much smaller than the area starting

from the east coast, and through programming verification, the number of measurements starting from the West Bank is less than the east side, so it is assumed that the survey ship starts from the West Bank. Through calculation, at least 34 measurements are obtained, and the measurement line can completely cover the entire sea area. At this time, the total length of the measurement line is 125936m.

3.2. Algorithm Construction

As shown in Figure 6, the total length of the measured distance at this time is [5]:

$$L(\beta) = \sum_{i=1}^n L_i \quad (13)$$

The length of the i -th line L_i satisfies the equation:

$$L_i = \frac{m_1}{\sin\beta_i} \quad (14)$$

Suppose that the overlap rate between adjacent strips is required to be between 10% and 20%, that is:

$$10\% \leq \eta_i \leq 20\%, i = 2, \dots, n \quad (15)$$

Where, η_i refers to the overlap rate between the i -th measurement line and the $i-1$ right measurement line. Since there is overlap between adjacent strips, if you want the survey line to completely cover the entire sea area to be measured, you need the survey line to cover the width of the north and south sides of the bank greater than the width of the shore, that is:

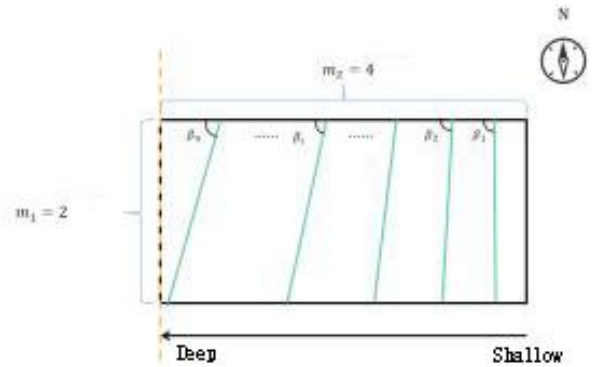


Figure 6. Top view of the survey line reaching the north and south sides of the starting and ending points

from the east coast, and through programming verification, the number of measurements starting from the West Bank is less than the east side, so it is assumed that the survey ship starts from the West Bank. Through calculation, at least 34 measurements are obtained, and the measurement line can completely cover the entire sea area. At this time, the total length of the measurement line is 125936m.

At this time, the length of one measurement is the smallest, so if the measurement length is less than the direction of the measurement line is 90° , the number of measurements should be less than 34 times, so the objective function is as follows:

$$L(\beta) = \sum_{i=1}^n \frac{m_1}{\sin\beta_i}, n < 34 \quad (18)$$

According to the above conditions, use the exhaustive method for all possible traversal, and iteratively find the optimal solution [6].

i represents the cycle of angle β , with the angle value range of (90, 153). For convenience, set the value range of i to (1, 64). j represents the j -th line of the ship moving forward when the angle is i . $\beta_{i,j}$ represents the line angle of the j -th line determined from the previous line after the angle represented by the first ship value is i .

(1) When $j = 1$, the angle of pinch $\beta_{i,j}$ is 91.

In order to ensure that the total length of the measuring line is minimum and the number of measuring lines is small, the distance between each measuring line should be as far as possible. It is necessary to ensure that the distance between the north end of the initial position of the ship and the west bank can be fully covered and the distance from the left side is farthest. Based on the mathematical model of multi-beam measuring line, the farthest d is solved under this fixed angle. At the same time, when the ship is in the initial position, the geometric model is established, as shown in Figure 7, to solve the total coverage length $w_{Si,j}$ and uncovered length x_0 of the south bank of the initial position survey line.

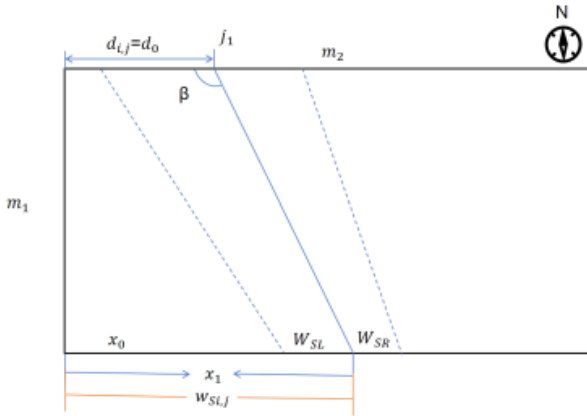


Figure 7. A geometric diagram of the ship's initial position

As shown in the figure, the distance from the measuring ship to the west bank is $X_{W(i,j)}$, the total coverage length of the previous measuring strip on the north bank is $W_{n(i,j-1)}$, the coverage length of the right side of the previous measuring strip is $X_{R(i,j-1)}$, and the distance between the two measuring ships is $d_{i,j}$, then:

$$X_{W(i,j)} = W_{n(i,j-1)} - W_{R(i,j-1)} + d_{i,j} \quad (22)$$

According to the mathematical model of multi-beam measurement line, the depth of the sea water where the measurement ship is at this time and the coverage width of the measured strip here, as well as its left and right coverage width, $d_{i,j}$ can be obtained.

The distance from the measuring ship on the south bank to the west bank can be obtained according to the geometric relationship:

$$X_{S(i,j)} = W_{S(i,j-1)} - W_{SR(i,j-1)} + d_{i,j} + \frac{m_1}{\tan\beta_{i,j}} \quad (23)$$

From this, we can solve $\beta_{i,j}$. According to this, the covering length of the south bank and the north bank can be solved as follows:

$$\omega_S = X_{S(i,j)} + W_{SR(i,j-1)} \quad (24)$$

Suppose the distance from the bow to the west bank $x_{i,j}$, which can be obtained from the knowledge of geometric relations and trigonometric functions:

$$x_{i,j} = d + \frac{m_1}{\tan\beta_{i,j}} \quad (19)$$

Based on the mathematical model of strip coverage width, W_S , W_{SL} and W_{SR} are obtained without coverage:

$$x_0 = x_1 - W_{SL} \quad (20)$$

Then, the total coverage length of the south bank is:

$$W_S = x_1 + W_{SR} \quad (21)$$

(2) Let $j = j + 1$.

The overlap rate $\eta_{i,j}$ between this line and the previous line $j - 1$ is calculated according to the mathematical model of multi-beam line measurement, so that it meets the extreme value $\eta_{i,j} = 0.1$. Subsequently, the unique line angle $\beta_{i,j}$ corresponding to the ship under this line of survey can be calculated. The geometric model of the ship under the survey line was established as shown in Figure 8, and the total coverage length of all the survey lines measured by the ship on the north and south banks was solved respectively.

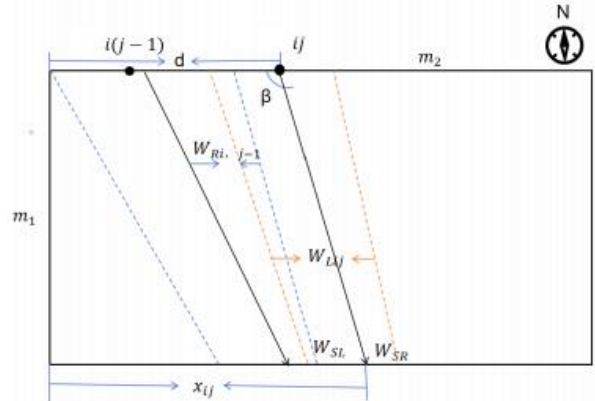


Figure 8. Article j Geometric diagram under the measurement line

$$\omega_N = X_{N(i,j)} + W_{NR(i,j-1)} \quad (25)$$

Determine whether the ship has completed the survey of the sea area at this time. According to the obtained total coverage length $X_{S(i,j)}$ and $X_{N(i,j)}$ of the north and south coasts, if the coverage length of the sea area greater than that of the north and south coasts is satisfied as follows, then the cycle is withdrawn and a set of survey line scheme covering the entire sea area is obtained, and the total length of survey line $L_{i,j}$ at this time is recorded.

$$X_{S(i,j)} \geq m_2 \parallel X_{N(i,j)} \geq m_2 \quad (26)$$

(3) The end of the cycle $i = 1$.

Let $i = i + 1$, iterate to solve the next set of test lines that meet the conditions.

(4) Loop through until $i = 64$ ends.

According to the above model and code calculation, it is found that there is no direction of the survey line except 90°, so that the survey ship can complete the measurement of the entire sea area when only passing the north and south banks.

3.3. Model Solution Results

After model establishment, constraint conditions were selected, objective function was established, and different cases were classified and discussed, loop traversed, and the

total measurement length at this time was obtained through programming calculation. It was found that the minimum value of the total measurement length appeared in the range of 102° - 104° . Therefore, we adjusted the accuracy of the program to 0.01° , and found that the minimum appeared when the initial $\beta_{i,j}$ was 103.88° , and there were 36 measuring lines with a total length of 124414.50 meters.

4. Conclusion

In this paper, the working principle and measuring process of multi-beam line are analyzed theoretically, and the mathematical model of multi-beam line is constructed. It is found that the coverage width of the line is affected by the depth of the sea water, the slope of the seabed and the angle of the line when the opening angle of the transducer is fixed. Based on the law, the optimal layout of the line is designed. After model establishment and solution, it can be seen that when the angle of the first measuring line is, the optimal situation is obtained, the number of measuring lines is $\beta_{i,1} = 103.88^{\circ}$, and the total length is 124414.5m. At this time, the measurement lines are more densely distributed in the shallow part of the sea bottom, and less distributed in the deep part of the sea water, and the angle of the measurement lines β gradually becomes larger as the sea water goes from shallow to deep. In the same way, the model can be extended to ship surveying, defense and other fields.

References

- [1] Futian Xie, Zehua Li, Chunlei Wang, Jianing Hou. Research on multi-beam line measurement based on differential algorithm [A]. 2023 IEEE International Conference on Electrical, Automation and Computer Engineering (ICEACE) [C], 2023.
- [2] Xiaochen Yan, Lingfeng Lv, Rui Xi. Line Measurement of Multi-Beam Sounding System Based on Annealing Algorithm [A]. 2023 IEEE International Conference on Electrical, Automation and Computer Engineering (ICEACE) [C], 2023.
- [3] Hsi-Tseng Chou, Danai Torrungrueng. Development of 2-D Generalized Tri-Focal Rotman Lens Beamforming Network to Excite Conformal Phased Arrays of Antennas for General Near/Far-Field Multi-Beam Radiations[J]. IEEE Access,2021, Vol.9: 49176-49188.
- [4] Zakir Ullah, Xiaopeng Chen; Siyuan Gou, Yang Xu, Muhammad Salam. FNUG: Imperfect Mazes Traversal Based on Detecting and Following the Nearest-to-Final-Goal and Unvisited Gaps[J]. IEEE Robotics and Automation Letters, 2022, Vol.7(2): 5175-5182.
- [5] Roman Barth, Biswajit Pradhan, Eugene Kim, Iain F Davidson, Jaco van der Torre, Jan-Michael Peters, Cees Dekker. Testing pseudotopological and nontopological models for SMC-driven DNA loop extrusion against roadblock-traversal experiments [J]. Scientific reports,2023, Vol.13(1): 8100.
- [6] Canesche M, Menezes M, Carvalho W, et al. Traversal: A fast and adaptive graph-based placement and routing for cgras[J]. IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems, 2020, Vol.40(8): 1600-1612.