

Trajectory Tracking of Open-pit Mining Trucks based on MPC and Pure Pursuit Control

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Abstract: Regarding the trajectory tracking problem of mine trucks in intelligent mining environments, This paper presents a Integrated trajectory tracking approach based on Model Predictive Control (MPC) and Pure Pursuit control. This approach is primarily driven by the Model Predictive Control (MPC) controller, The lateral trajectory error is derived through the Pure Pursuit (PP) control algorithm, The vehicle steering angle control input is generated using a weighted hybrid control approach, To address the trajectory tracking challenge of open-pit mine trucks in dynamic and complex environments. Finally, through simulations using CarSim and Matlab/Simulink, the performance of the mine truck in tracking a dual-shift trajectory was evaluated. The results indicate that, compared to the standalone MPC controller, the integrated control method demonstrates a significant improvement in tracking accuracy.

Keywords: Trajectory Tracking; Model Predictive Control; Weighted Hybrid Control.

1. Introduction

The development of green mines serves as a critical measure for promoting high-quality growth in the mining industry, an important approach to advancing ecological progress within the sector, and an inevitable choice for achieving harmonious coexistence between humans and nature [1]. Open-pit mining operations constitute a significant component of modern mineral production, where haul trucks, as the primary means of transport, play a vital role. With advancements in modern science and technology, research into unmanned driving for these trucks has entered a new phase. The main function of haul trucks is to transport extracted ore, waste material, and other supplies from the mining pit to processing plants, storage yards, or other designated locations. Given the harsh conditions of open-pit mines, which require drivers to remain in a constant state of high alert, coupled with the highly repetitive nature of the task, research on path tracking for these trucks is of considerable necessity. The objective of intelligent truck trajectory tracking is to control the vehicle's speed and steering angle, enabling it to follow a planned path with minimal deviation while ensuring stability and safety[2][3].

The origins of unmanned driving technology for mining trucks can be traced back to the 1970s. However, progress was relatively slow due to limitations in technology and other factors at the time[4]. Current research predominantly focuses on control algorithm design based on preview theory and model predictive control theory. In terms of control methodologies, path tracking algorithms can be categorized into three types: The first category includes the Pure Pursuit (PP) algorithm and the Stanley algorithm, which are based on geometric models for path tracking control. The second category is the Model Predictive Control (MPC) algorithm, which utilizes a kinematic model for path tracking. The third category comprises the PID and LQR algorithms, which rely on both kinematic and dynamic models for path tracking control[5]. The PP algorithm operates by identifying a preview point on the reference path, then calculating the

front-wheel steering angle based on the vehicle's position and the preview point, using the rear axle as a reference and the preview distance[6]. The Stanley algorithm is a classic method in the field of unmanned vehicle path tracking, known for its computational simplicity and effective tracking performance. However, it requires the desired path to be sufficiently smooth; suboptimal smoothness in path curvature can lead to issues such as excessive overshoot in vehicle response [7]. Fuzzy PID control enhances traditional PID control by leveraging fuzzy logic to dynamically adjust PID parameters, thereby improving adaptability and optimization when dealing with uncertain, nonlinear, and complex systems. Regarding the MPC algorithm, numerous studies have been conducted by scholars both domestically and internationally. For instance, in 2019, Guo et al. [8] proposed an MPC path tracking strategy that accounts for constraints including road drivable areas and vehicle geometry. This approach addressed model mismatch caused by varying road conditions by treating them as measurable disturbances, thereby enhancing the applicability of the MPC algorithm for path tracking under routine operating conditions.

Building upon the aforementioned research, this paper designs a hybrid weighted control approach that integrates MPC and PP. This method demonstrates effective performance for path tracking in open-pit mine haul trucks under low-speed operating conditions.

2. Vehicle Error Model

Considering that we are addressing the low-speed motion of haul trucks in open-pit mines, a kinematic vehicle model is established here. The vehicle's motion process is typically described using two coordinate systems, the inertial coordinate system (XOY) and the vehicle body-fixed coordinate system (xoy). The vehicle is modeled based on the Ackermann steering geometry. The coordinates (X_r ; Y_r) and (X_f ; Y_f) represent the centers of the rear and front axles, respectively, V_r denotes the velocity at the rear axle center, L is the wheelbase, δ_f represents the steering angle of the front wheels, and θ indicates the vehicle's heading angle. As

illustrated in Figure 1, the kinematic equations of the vehicle are established accordingly.

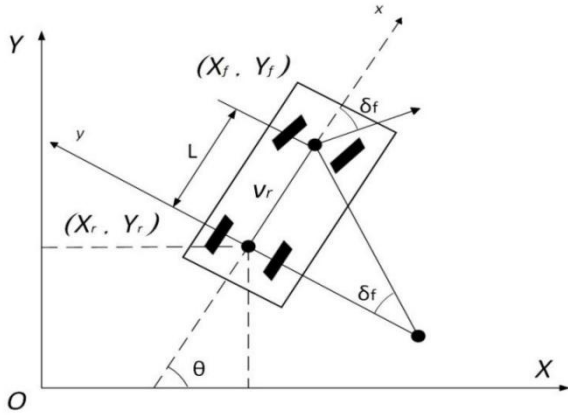


Figure 1. Vehicle Kinematic Model

The vehicle can thus be regarded as a control system with the motion state $\chi[x, y, \theta]^T$ and output $u[v, \delta]^T$, its state can be represented by Equation (1).

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos \theta \\ \sin \theta \\ \tan \delta_f / L \end{bmatrix} V_r \quad (1)$$

Since the focus of this research is path tracking, it can be inferred from the above vehicle model that we need to obtain the motion state variables of the reference path. The given reference trajectory is also considered to satisfy this vehicle model, and Equation (2) can be expressed as

$$\dot{\chi} = f(\chi_r, u_r) \quad (2)$$

A linear error model for vehicle trajectory tracking is derived by applying a Taylor series expansion to Equation (1) at the reference trajectory point, neglecting higher-order terms during the expansion to simplify the model expression. Subtracting Equation (2) from the resulting Taylor polynomial yields the linear error model, which can be expressed as Equation (3).

$$\dot{\tilde{\chi}} = \begin{bmatrix} \dot{x} - \dot{x}_r \\ \dot{y} - \dot{y}_r \\ \dot{\theta} - \dot{\theta}_r \end{bmatrix} = \begin{bmatrix} 0 & 0 & -v_r \sin \theta_r \\ 0 & 0 & v_r \cos \theta_r \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x - x_r \\ y - y_r \\ \theta - \theta_r \end{bmatrix} + \begin{bmatrix} \cos \theta_r & 0 \\ \sin \theta_r & 0 \\ \frac{\tan \delta_{r,f}}{L} & \frac{v_r}{L \cos^2 \delta_{r,f}} \end{bmatrix} \begin{bmatrix} v - v_r \\ \delta - \delta_r \end{bmatrix} \quad (3)$$

To facilitate the application of this model in the design of a model predictive controller, Equation (3) must be discretized. Discretization transforms a continuous-time control system into a discrete-time model suitable for processing by digital computers. This enables the controller to perform prediction and optimization control at each sampling instant based on the current system state, thereby improving computational efficiency and practical implementability in real control systems. The discretized form is given by Equation (4)

$$Y(t) = \begin{bmatrix} \eta(t+1|t) \\ \eta(t+2|t) \\ \vdots \\ \eta(t+N_c|t) \\ \vdots \\ \eta(t+N_p|t) \end{bmatrix}; \Psi = \begin{bmatrix} \tilde{C}_{t,t} \tilde{A}_{t,t} \\ \tilde{C}_{t,t} \tilde{A}_{t,t}^2 \\ \vdots \\ \tilde{C}_{t,t} \tilde{A}_{t,t}^{N_c} \\ \vdots \\ \tilde{C}_{t,t} \tilde{A}_{t,t}^{N_p} \end{bmatrix}; \Delta U(t) = \begin{bmatrix} \Delta u(t|t) \\ \Delta u(t+1|t) \\ \vdots \\ \Delta u(t+N_c|t) \end{bmatrix}$$

$$\tilde{\chi}(k+1) = A \tilde{\chi}(k) + B \tilde{u}(k) \quad (4)$$

$$A = \begin{bmatrix} 1 & 0 & -v_r \sin \theta_r T \\ 0 & 1 & v_r \cos \theta_r T \\ 0 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} T \cos \theta_r & 0 \\ T \sin \theta_r & 0 \\ \frac{\tan \delta_{r,f}}{L} T & \frac{v_r}{L \cos^2 \delta_{r,f}} T \end{bmatrix}, T$$

denotes the sampling time.

3. Design of the MPC Controller

3.1. State Prediction Model

The model achieves desired control objectives by predicting future states and optimizing control inputs. During this process, the controller utilizes the current state and the system model to forecast system behavior over a future time horizon and computes the optimal control sequence. Furthermore, the system model is capable of handling various constraints, such as limitations on inputs, outputs, or states, thereby enhancing the feasibility and efficiency of the control strategy in practical applications. Model predictive control excels in delivering superior performance in complex and constrained environments. To enable direct constraints on control increments during execution, Equation (4) is transformed as follows

$$\xi(k|t) = \begin{bmatrix} \tilde{\chi}(k|t) \\ \tilde{u}(k-1|t) \end{bmatrix} \quad (5)$$

The transformed state-space representation is obtained as follows

$$\begin{cases} \xi(k+1|t) = \tilde{A}_{k,t} \xi(k|t) + \tilde{B}_{k,t} \Delta U(k|t) \\ \eta(k|t) = \tilde{C}_{k,t} \xi(k|t) \end{cases} \quad (6)$$

In which,

$$\tilde{A}_{k,t} = \begin{bmatrix} A & B \\ 0_{m \times n} & I_m \end{bmatrix}; \tilde{B}_{k,t} = \begin{bmatrix} B \\ I_m \end{bmatrix}; \tilde{C}_{k,t} = \begin{bmatrix} C & 0_{n \times m} \\ 0_{m \times n} & 0_{m \times m} \end{bmatrix}; C = \text{diag}(1 \ 1 \ 1)$$

Here, $n=2$ denotes the dimension of the state vector, $m=3$ denotes the dimension of the control input, $k+1|t$ denotes the prediction horizon, which defines the prediction of system variables from the current time step t to the future time step $k+1$.

Simplified Computation Assumption

$$\begin{cases} \tilde{A}_{k,t} = \tilde{A}_{t,t} \\ \tilde{B}_{k,t} = \tilde{B}_{t,t} \end{cases} k=1, 2, 3, \dots, t+N-1$$

Based on the output of Equation (6), the variable $\eta(k|t)$ can be derived through multiple steps of deduction as follows

$$Y(t) = \Psi \xi(t|t) + \Theta \Delta U(t) \quad (7)$$

In which

$$\Theta = \begin{bmatrix} \tilde{C}_{t,t} \tilde{B}_{t,t} & 0 & 0 & 0 \\ \tilde{C}_{t,t} \tilde{A}_{t,t} \tilde{B}_{t,t} & \tilde{C}_{t,t} \tilde{B}_{t,t} & 0 & 0 \\ \dots & \dots & \ddots & \dots \\ \tilde{C}_{t,t} \tilde{A}_{t,t}^{N_c-1} \tilde{B}_{t,t} & \tilde{C}_{t,t} \tilde{A}_{t,t}^{N_c-2} \tilde{B}_{t,t} & \dots & \tilde{C}_{t,t} \tilde{B}_{t,t} \\ \dots & \dots & \ddots & \dots \\ \tilde{C}_{t,t} \tilde{A}_{t,t}^{N_p-1} \tilde{B}_{t,t} & \tilde{C}_{t,t} \tilde{A}_{t,t}^{N_p-2} \tilde{B}_{t,t} & \dots & \tilde{C}_{t,t} \tilde{A}_{t,t}^{N_p-N_c-1} \tilde{B}_{t,t} \end{bmatrix}$$

N_c denotes the control horizon of the predictive model, while N_p represents its prediction horizon. Based on the system state and error at time t , the model can predict system states over the future N_p steps, thereby achieving accurate vehicle path tracking.

3.2. Pure Pursuit (PP) Controller

The core principle of the Pure Pursuit control algorithm is to select a look-ahead point on the target path at a predetermined distance from the vehicle's current state, and then compute the steering angle based on the position of this target point relative to the vehicle's current position. As shown in Figure 2, with the rear axle as the reference point (O), a point (C) is selected on the path at a look-ahead distance Ld . This configuration forms a circular arc centered at point (O), where L represents the wheelbase of the vehicle.

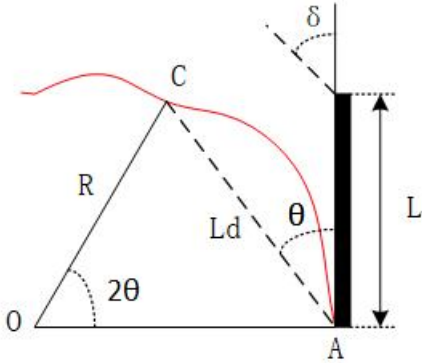


Figure 2. Principle of the Pure Pursuit Control

As illustrated in the figure, the geometric relationship $\frac{Ld}{\sin 2\theta} = \frac{R}{\sin(\frac{\pi}{2} - \theta)}$ can be observed. Based on this,

$R = \frac{Ld}{2\sin\theta}$ can be calculated. To ensure that the center of the rear axle reaches the previewed position after the vehicle's movement, the steering angle of the front wheels must satisfy the kinematic relationship with the wheelbase, as expressed in $\tan\delta = \frac{L}{R}$. From this, $\delta = \arctan(\frac{2L\sin\theta}{Ld})$ can be derived.

3.3. Lateral Error in Pure Pursuit Control

The lateral error of the vehicle at the look-ahead path point is defined as $e_y = Ld \sin\theta$. Under the small-angle assumption, the transformation based on the above equation yields $e_y \approx \frac{Ld^2}{2L} \delta$. Therefore, when the look-ahead

distance and the vehicle wheelbase are fixed, the Pure Pursuit control can essentially be regarded as a proportional (P) controller, whose tracking performance is determined by Ld . Typically, Ld is defined as a speed-dependent expression $Ld = K_v V + Ld_0$. The steering angle computed from the above equation is incorporated as an optimization term in the objective function. Simultaneously, the suggested steering angles from the reference path and the Pure Pursuit control are combined through a weighted fusion strategy, and the resulting output is applied as the final steering command to the vehicle.

4. Design of the MPC Controller

4.1. Objective Function Design

The lateral error derived from the Pure Pursuit (PP) control relative to the reference path is incorporated into the optimal control objective function through proportional weighting. This paper adopts a weighted hybrid approach for reference path tracking, with the specific control procedure detailed in Figure 3.

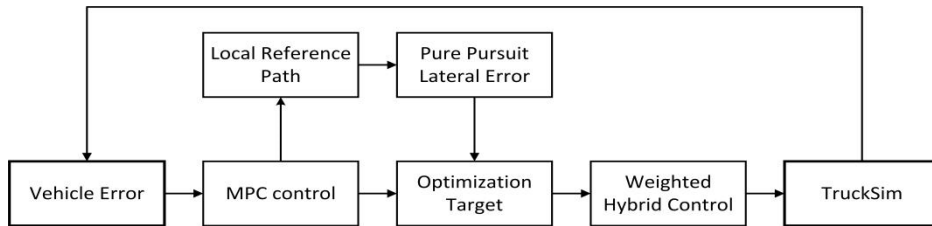


Figure 3. Flowchart of the Path Tracking Control

To minimize both the path tracking deviation and the control input energy, while reducing disturbances during the control process, the following objective function is designed.

Here, Q represents the weighting on the error terms, which, as indicated in the aforementioned flowchart, comprise the

sum of the vehicle state error and the steering angle error derived from Pure Pursuit (PP) control. The R matrix denotes the weighting on the control inputs, while the F matrix focuses on the terminal error. \mathcal{E} represents a slack variable, whose introduction relaxes the constraints on the inputs,

thereby allowing for more aggressive control actions. By substituting Equation (7) into the objective function given in

Equation (8), the objective function can be reformulated as follows

$$J(k) = \sum_{i=1}^{N_p-1} \left\| \eta(k+i|t) - \eta_{ref}(k+i|t) \right\|_Q^2 + \sum_{i=1}^{N_c} \left\| \Delta U(k+i|t) \right\|_R^2 + \eta(k+N_p|t)^T F \eta(k+N_p|t) + \varepsilon^T \rho \varepsilon \quad (8)$$

$$J(k) = \xi(k)^T G \xi(k) + 2\xi(k)^T E \Delta U + \Delta U^T H \Delta U + \varepsilon^T \rho \varepsilon \quad (9)$$

In which $G = \psi^T \bar{Q} \psi$; $E = \psi^T \bar{Q} \Theta$; $H = \Theta^T \bar{Q} \Theta + \bar{R}$, $\bar{R} = \begin{bmatrix} R & & & \\ & R & & \\ & & \ddots & \\ & & & R \end{bmatrix}$; $\bar{Q} = \begin{bmatrix} Q & & & \\ & Q & & \\ & & \ddots & \\ & & & F \end{bmatrix}$

Since G is a known quantity, the first term represents the initial state and does not affect the minimization of the cost function, thus it can be neglected during the optimization process. Following multi-step derivations and omitting

constant terms, the problem can be transformed into a standard quadratic programming form, with the final objective function given by Equation (10).

$$\min J(k) = \frac{1}{2} \left\{ \begin{bmatrix} \Delta U \\ \varepsilon \end{bmatrix}^T \begin{bmatrix} 2H & 0 \\ 0 & 2\rho \end{bmatrix} \begin{bmatrix} \Delta U \\ \varepsilon \end{bmatrix} + \begin{bmatrix} 2\xi(k)^T E & 0 \end{bmatrix} \begin{bmatrix} \Delta U \\ \varepsilon \end{bmatrix} \right\} \quad (10)$$

4.2. Constraint Formulation

Given that haul trucks typically operate at low speeds in localized environments, the design of the control strategy must specifically account for constraints on the velocity as a control variable, as well as limitations on the control

increments. By effectively constraining both the speed and its incremental changes, it is possible to enhance smoothness during control execution, prevent abrupt acceleration or deceleration, and ensure stable vehicle operation with improved control accuracy. The constraints are formulated as follows

$$\begin{cases} u_{\min}(t+k) \leq u(t+k) \leq u_{\max}(t+k) \\ \Delta u_{\min}(t+k) \leq \Delta u(t+k) \leq \Delta u_{\max}(t+k) \end{cases} \quad k = 0, 1, \dots, N_c - 1 \quad (11)$$

5. Simulation and Experimental Analysis

To validate the feasibility and effectiveness of the proposed weighted hybrid control strategy integrating MPC and Pure

Pursuit (PP) control, a co-simulation study was conducted using the CarSim and Matlab/Simulink platform. A double-lane change trajectory was employed as the reference path to evaluate the path-tracking performance of the proposed controller on roads with varying curvature. The mathematical expression of the reference path is given by

$$Y = \frac{dn1}{2} [1 + \tanh(r1)] - \frac{dn2}{2} [1 + \tanh(r2)] \quad (12)$$

$$\theta = \arctan \left[\operatorname{dn1} \left(\frac{1}{\cosh(r1)} \right)^2 \left(\frac{1.2}{dm1} \right) - \operatorname{dn2} \left(\frac{1}{\cosh(r2)} \right)^2 \left(\frac{1.2}{dm2} \right) \right] \quad (13)$$

In which

$$r1 = \frac{2.4}{dm1} (x - 60) - 1.2; \quad r2 = \frac{2.4}{dm2} (x - 120) - 1.2$$

$$dm1 = 60; \quad dm2 = 56; \quad dn1 = 5; \quad dn2 = 4.5$$

5.1. Sensitivity Analysis of Weighting Parameters

For the experimental methodology of weighted hybrid tracking, the allocation of tracking weight (α_{track}) and steering weight (β_{steer}) was performed. To systematically analyze the impact of weight distribution between the MPC and Pure Pursuit controllers, a three-level orthogonal array design was adopted in this study to efficiently evaluate each normalized parameter.

α_{track} : Represents the proportion of MPC in the total tracking error weight, with level values {0.2, 0.5, 0.8} corresponding to "PP-dominant," "balanced," and "MPC-dominant" tracking strategies, respectively.

β_{steer} : Denotes the proportion of MPC in the total steering cost weight, with level values {0.3, 0.5, 0.7} used to explore the transition from PP to MPC in steering control.

Based on the above selections, a total of nine experimental orthogonal arrays were derived. Simulations under double-lane change conditions recorded the lateral error results, with experiments conducted at a speed of 15 km/h. The mean absolute error (MAE), error fluctuation (standard deviation, Std), and worst-case deviation (Max) across all time steps and repeated experiments were calculated. The range analysis method of orthogonal experiments was then applied to

determine parameter significance, and the interaction effects were visualized using a heatmap, as shown in Figure 4.

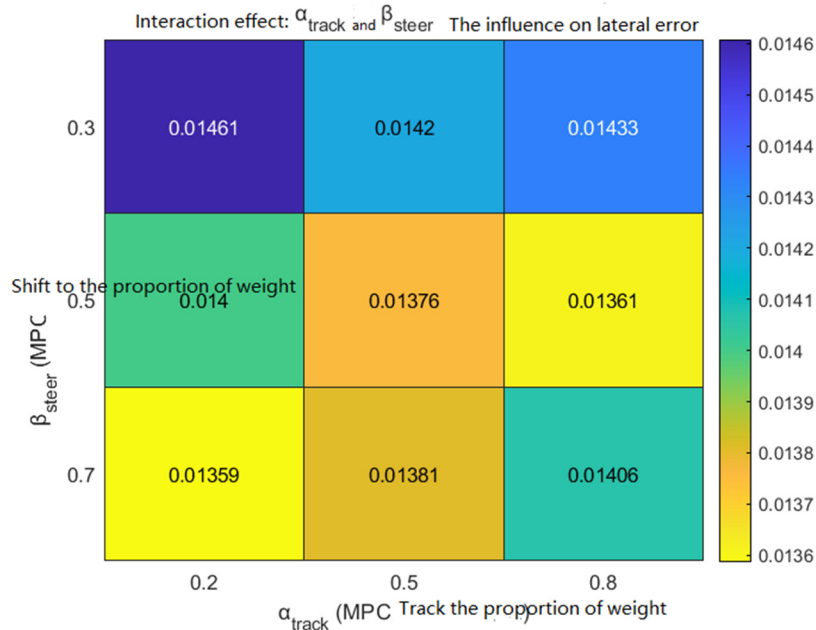


Figure 4. Heatmap of Interaction Effects

Range analysis indicates that the parameter combination ($\alpha=0.2, \beta=0.7$) falls within the low-error region (MAE=0.01359). Consequently, the weight allocation for the weighted hybrid control strategy is determined as shown in Table 1.

Table 1. Weight Allocation

Control Scheme	tracking weight (α_{track})	steering weight (β_{steer})
MPC Control	0.2	0.7
Pure Pursuit Control	0.8	0.3

5.2. Simulation Results Comparison

For the MPC controller, the prediction horizon was set to 50, the control horizon to 20, the simulation time to 40 s, and the vehicle speed to 4 m/s. The tracking performance of both

the MPC and the weighted hybrid control strategies was validated under the same reference path. The specific parameters for the MPC simulation and the weighted hybrid control experiment are listed in Table 2.

Table 2. Experimental Parameters Comparison

Experimental Class	Wheelbase, L (m)	Q	R
MPC	3.75	10	100
Weighted Hybrid Control	3.75	100	10

The path tracking results are shown in Figure 5.

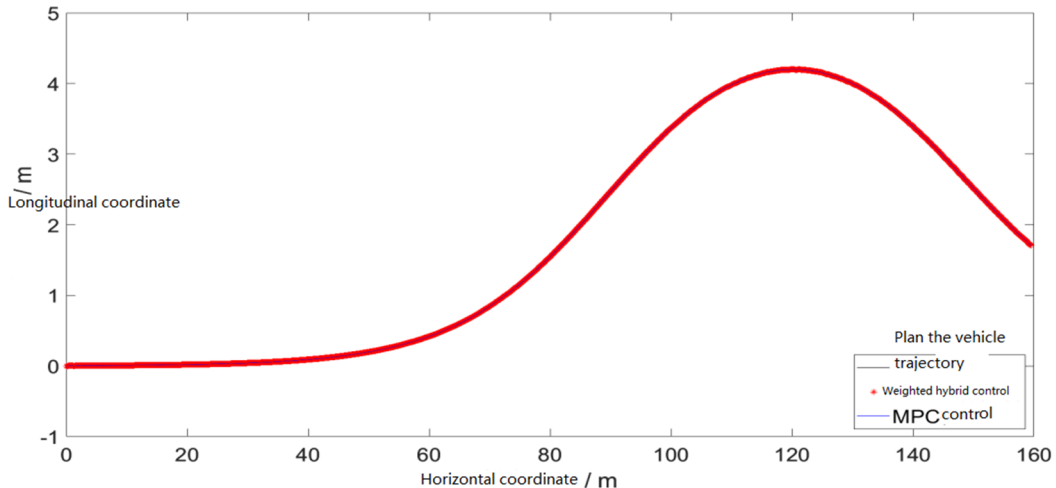


Figure 5. Path Tracking Performance Diagram

Figure 6 illustrates the path-tracking lateral errors under both MPC and the weighted hybrid control strategies,

demonstrating the error variation of each control approach over different time periods. The comparison provides a clear

visualization of the evolving trends in tracking errors for both methods throughout the path-following process.

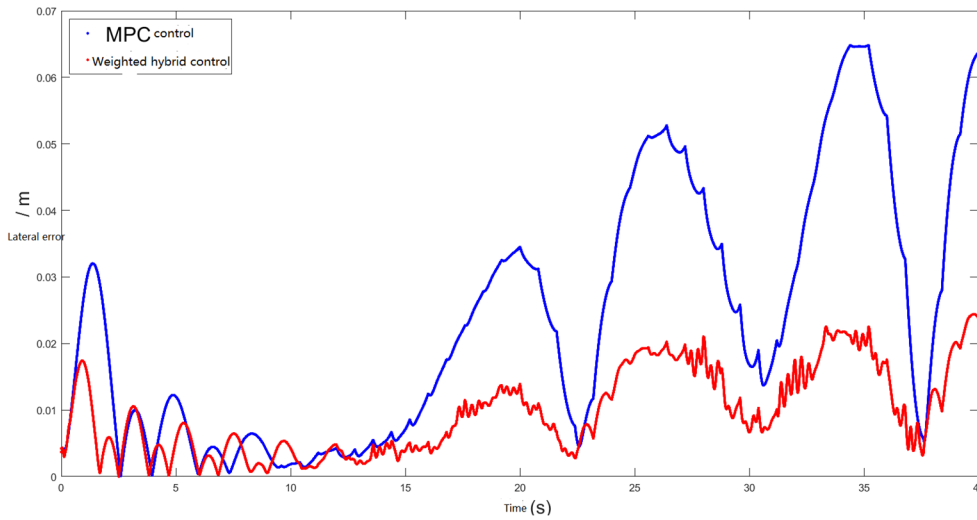


Figure 6. Lateral Error Comparison Diagram

As demonstrated above, the maximum tracking error of the weighted hybrid control remains around 0.02 m, whereas that of the standalone MPC controller reaches approximately 0.07 m. These results indicate that the weighted hybrid control outperforms the standalone MPC in terms of tracking

accuracy, demonstrating a stronger capability to handle system uncertainties and external disturbances, thereby significantly reducing tracking errors.

Figure 7 presents a comparison of lateral errors under the weighted hybrid control strategy at different vehicle speeds.

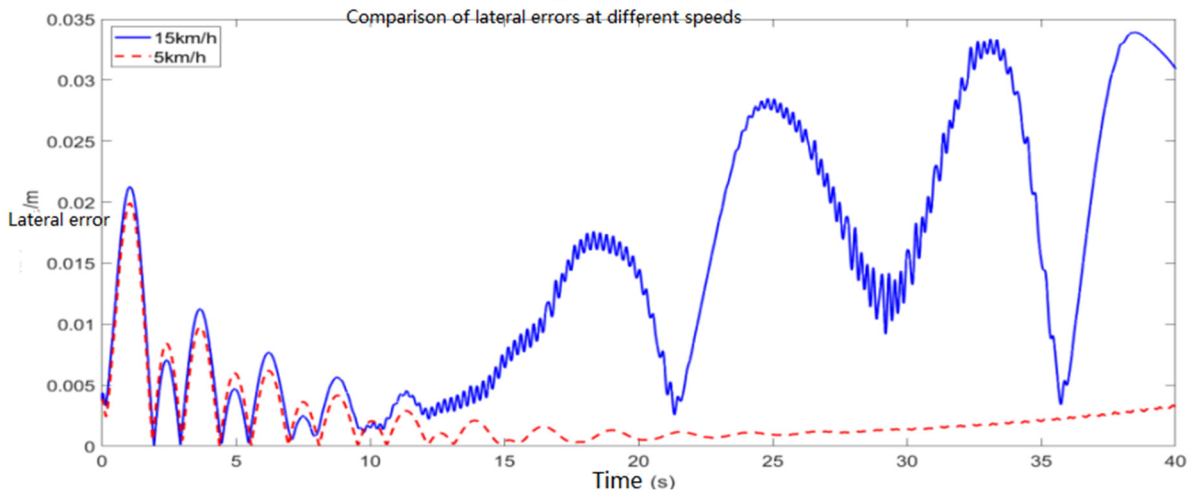


Figure 7. Low-Speed Lateral Error Tracking Comparison

The results indicate that the maximum lateral error at 5 km/h remains around 0.02 m, while even at 15 km/h, it is only approximately 0.035 m. This demonstrates the strong robustness of the proposed method within the low-speed range.

6. Conclusion

This paper presents a coordinated path-tracking method integrating Model Predictive Control (MPC) and the Pure Pursuit algorithm, which innovatively combines the advantages of both control strategies to effectively address the path-tracking challenges faced by mining trucks in complex mining environments. The proposed approach leverages the multi-step prediction capability and constraint-handling strengths of MPC, while incorporating the high tracking accuracy characteristic of the Pure Pursuit algorithm. By implementing a dynamically weighted hybrid control strategy, precise optimization of steering angle control is achieved. In

terms of control architecture design, a parameter sensitivity analysis model was established to determine the optimal weight allocation scheme, enabling the system to autonomously adjust control outputs based on real-time operating conditions. Future research will focus on further optimizing the weight adaptation algorithm to accommodate more complex mining terrains and varying load conditions.

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