

# Asset allocation strategy based on CRITIC method and BP neural network

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**Abstract:** Investment portfolio optimization has become more and more mature after more than 30 years of development in China's capital market. For ordinary investors, how to reasonably match holding assets to ensure a certain level of returns and reduce investment risks is an important issue. Using 100 securities in the China Securities Dividend Index as index samples, firstly, a BP neural network prediction model is established for prediction, then the CRITIC method is used to evaluate and score the 100 securities, and finally a multi-objective planning model is established for investment portfolio. The results show that: solving the optimal solution of the multi-objective programming model to obtain the investment portfolio and obtaining the investment portfolio strategy; using the appropriate prediction model, evaluation model and establishing the multi-objective programming model can not only reduce investment risk for ordinary investors, but also reduce investment risk. received a certain level of income.

**Keywords:** Portfolio optimization; Multi-objective programming; BP neural network; CRITIC method.

## 1. Introduction

The investment portfolio optimization problem has always been one of the hot research issues that has attracted much attention, and it is also a core topic of modern finance. After more than 30 years of development, China's capital market has developed from a single market structure in the early stage to a diversified market, forming a hierarchical capital market and gradually mature [1]. Markowitz (1952) [2] first proposed the Portfolio Theory, which includes the mean-variance analysis method and the portfolio efficient frontier model. The theory quantifies risk and return, but the requirement to calculate the covariance matrix of all assets severely restricts its application in practice. William (1963) [3] proposed the "single index model", which assumes that asset returns are only related to the overall return of the market, which greatly simplifies Markowitz's theory and promotes the practical application of portfolio theory. Hiroshi and Ken (1995) [4] studied the mean-variance-skewness model in the case of asymmetric returns, which has value in the case of asymmetric return distributions.

Although the domestic capital market started late, it has developed rapidly, and relevant academic research has also achieved great results. Guo et al. (2004) [5] discussed the establishment of the mean-VaR portfolio model under investment opportunity constraints. Sun and Wei (2005) [6] studied the portfolio strategy under changing market conditions. Zhang et al. (2011) [7] proposed a new investment portfolio optimization model by adding the minimum transaction unit and transaction cost constraints, and proposed an intelligent algorithm based on genetic algorithm to solve it. Fang et al. (2015) [8] established an improved CvaR-based portfolio model of the typical transaction cost function through the nonlinear increase of the transaction cost function. Huang (2017) [9] comprehensively evaluate and score stocks based on the entropy weight-TOPSIS method, and use the mean-CVaR portfolio model to make portfolio strategies. Zhou (2022) [1] uses the principal component analysis method to comprehensively score the financial indicators of each company and makes portfolio strategies based on the

mean-variance model.

To sum up, the academia has optimized and improved the investment portfolio optimization problem with many evaluation angles and models, and provided the investment portfolio. Based on Markowitz's classic mean-variance model, many new solutions have been given. This article selects 100 securities of listed companies with high cash dividend rate, stable dividend distribution and a certain scale of CSI Dividend Index (000922.CSI) in Shanghai and Shenzhen markets as index specimens, and uses neural network prediction model to predict the closing price of these 100 securities, then use the CRITIC method as the evaluation method to comprehensively evaluate 100 securities, and finally establish a multi-objective programming mathematical model based on the principle of the largest return and the highest comprehensive score of the investment portfolio. The reliability, versatility and reliability provide references for investors to make investment portfolios.

## 2. Related Theories and Methods

### 2.1. Choose a prediction model

#### 2.1.1. Comparison and selection of prediction models

There are many predictive models, ranging from traditional single consumption methods to more modern techniques such as fuzzy mathematics, neural networks, preferred combinatorial methods, and expert systems. By comparing the advantages and disadvantages of forecasting models such as exponential smoothing method, grey forecasting model, Markov forecasting, BP neural network, etc. Considering that the selection of investment portfolio requires long-term accurate forecasting, and its internal mechanism is more complex, compared with Markov prediction and grey prediction that are not suitable for long-term prediction, BP neural network is suitable for long-term prediction, and compared with the exponential smoothing method which is suitable for short-term prediction and has low accuracy, BP neural network has higher accuracy. Therefore, BP neural network is finally selected for prediction.

### 2.1.2. Build predictive models

The BP neural network is one of the more mature neural networks currently in development. Its main feature is that by adjusting the connection strength between the input node and the hidden layer node, the connection strength between the hidden layer node and the output node, and the threshold, the error can be adjusted along the gradient. After repeated learning and training, the weight and threshold corresponding to the minimum error are determined, and the training is stopped.

The training steps are as follows:

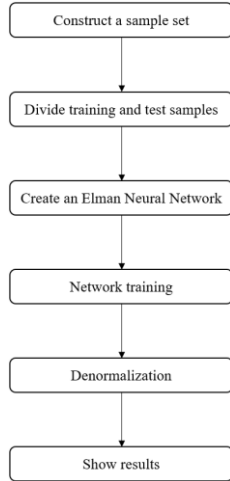


Fig. 1 BP neural network training steps

## 2.2. Determine the evaluation model

### 2.2.1. Selection of evaluation model

There are many kinds of evaluation models, the core of which is to determine the weights and evaluate and score with a suitable system. Among them, the determination weight generally includes AHP, standard deviation method, entropy weight method, CRITIC method and so on. The AHP method has a short decision-making time, but its subjectivity is strong; the standard deviation method will reduce the accuracy when the amount of data is large; the entropy weight method has high accuracy and strong objectivity, but it ignores the importance of the indicators themselves; The CRITIC method is more objective than the AHP method, and is a better objective weighting method than the entropy weight method and the standard deviation method. Considering that the evaluation model of securities needs to be objective, and the evaluation indicators are related to each other, the CRITIC method is used for evaluation.

### 2.2.2. Analysis of CRITIC principle

The CRITIC method is an objective weighting method of evaluation indicators proposed by Diakoulaki. The method revolves around two aspects when calculating the weight of the indicators: contrast and contradiction. The contrast represents the size of the value gap between the evaluation schemes of the same index, which is expressed in the form of standard deviation. The greater the standard deviation, the greater the difference between the values of each scheme. The contradiction is based on the correlation between the two indicators, such as a strong positive correlation between the two indicators, indicating that the two indicators have low conflict. Supposing that there are  $m$  objects to be evaluated and  $n$  evaluation index. The steps of the CRITIC method are as follows:

Step1: Screen positive and negative indicators. The larger the positive indicators are, the more favorable they are, and

the negative indicators are the opposite. The positive and negative indicators are standardized using different rules respectively. Let the standardized elements be  $x_{ij}$ , then

$$x_{ij} = \frac{\max(x_{ij}) - x_{ij}}{\max(x_{ij}) - \min(x_{ij})}, \quad x_{ij} \text{ is a positive indicator}$$

$$x_{ij} = \frac{x_{ij} - \min(x_{ij})}{\max(x_{ij}) - \min(x_{ij})}, \quad x_{ij} \text{ is a negative indicator}$$

Step2: Use the standard deviation to express the contrast of indicator  $j$ :

$$\sigma_j = \sqrt{\frac{\sum_{i=1}^m (x_{ij} - \bar{x}_j)^2}{m - 1}}$$

Step 3 Calculate the contradiction: The contradiction reflects the degree of correlation between different indicators. If there is a significant positive correlation, the smaller the contradiction is, and vice versa. Let the magnitude of the contradiction between the indicator  $j$  and the rest of the indicator be  $f_j$ :

$$f_j = \sum_{i=1}^m (1 - r_{ij})$$

Among them,  $r_{ij}$  represents the correlation coefficient between indicator  $i$  and indicator  $j$ , and it is necessary to consider the type of indicators and select an appropriate correlation coefficient.

Step 4 Calculate the information carrying capacity  $C_j$ :

$$C_j = \sigma_j f_j$$

Step 5 Calculate weights and scores:

Let the weight of the indicator  $j$  be  $w_j$ , calculate the weight:

$$w_j = \frac{C_j}{\sum_{j=1}^n C_j}$$

The weight has a positive correlation with the information carrying capacity, that is, the greater the information carrying capacity is, the greater the weight is.

Let the score of the indicator  $i$  be  $S_i$ , calculate the score:

$$S_i = \sum_{j=1}^n w_j x_{ij}$$

## 2.3. Build investment strategy formulation models

### 2.3.1. Symbol Description

Table 1. Multi-objective programming notation description

symbol	Symbol Description	symbol	Symbol Description
$i$	Number of securities ( $1 \leq i \leq n$ )	$P_1$	Funds invested in securities
$C_i$	0-1 decision variable, 0 means do not invest in the security $i$ , 1 means invest in the first $i$ security	$P_2$	stable investment
$z_i$	Composite indicator for each security	$X_i$	The biggest gain possible in the coming week
$x_i$	Amount invested in each security	$S$	Total revenue
$P$	total investment amount	$h$	monthly return on investment
$a$	risk-free annualized rate of return	$b$	number of securities selected
$w$	Positions per security	$t$	monthly rate of return

### 2.3.2. Objective function

The formulation of investment strategy is actually a multi-obj linear programming problem, which should satisfy the two objectives of maximizing returns and maximizing the comprehensive evaluation score of securities investment portfolio at the same time. First consider the goal of maximizing the comprehensive evaluation score of the securities portfolio. For this purpose, set the comprehensive index of each security  $z_i(1 \leq i \leq n)$  and a decision coefficient  $C_i(1 \leq i \leq n)$ . Assuming that the maximization of the comprehensive evaluation score of the portfolio is  $max z$ , the first objective function can be determined:

$$max z = \sum_{i=1}^n C_i z_i (1 \leq i \leq n)$$

Next is the goal of maximizing revenue. First, set the investment amount of each security as  $x_i$ , the initial total investment amount as  $P$ , this  $P$  value is updated as the investment progresses. Set the amount invested in securities as  $P_1$ , the other funds for stable investment as  $P_2$ , then there exists:

$$P_1 = \sum_{i=1}^n C_i x_i (1 \leq i \leq n)$$

$$P_2 = P - P_1$$

$X_i$  indicates the maximum possible increase of each security in the next week, that is, the rate of return of investing in the stock, set the total return as  $S$ , and the risk-free annualized rate as  $a$ , then  $S$  consists of two parts. The first part is  $\sum_{i=1}^n C_i x_i X_i (1 \leq i \leq n)$ , the second part is  $(P - \sum_{i=1}^n C_i x_i) \times \frac{a}{360} \times 7$ , then the total benefit  $S$  is:

$$S = \sum_{i=1}^n C_i x_i X_i + \left( P - \sum_{i=1}^n C_i x_i \right) \times \frac{a}{360} \times 7 (1 \leq i \leq n)$$

In summary, the objective function of the multi-objective linear programming model is:

$$max z = \sum_{i=1}^n C_i z_i (1 \leq i \leq n)$$

$$max S = \sum_{i=1}^n C_i x_i X_i + \left( P - \sum_{i=1}^n C_i x_i \right) \times \frac{a}{360} \times 7 (1 \leq i \leq n)$$

### 2.3.3. Model Construction

$$max z = \sum_{i=1}^n C_i z_i (1 \leq i \leq n) \quad (1)$$

$$max S = \sum_{i=1}^n C_i x_i X_i + \left( P - \sum_{i=1}^n C_i x_i \right) \times \frac{a}{360} \times 7 (1 \leq i \leq n) \quad (2)$$

$$0 \leq \sum_{i=1}^n C_i \leq b (1 \leq i \leq n) \quad (3)$$

$$x_i \leq P \times w (1 \leq i \leq n) \quad (4)$$

$$\frac{(h-t) \times 7}{30} P \leq S \leq P \frac{(h+t) \times 7}{30} \quad (5)$$

$$C_i \in 0,1 (1 \leq i \leq n) \quad (6)$$

$$x_i \geq 0 (1 \leq i \leq n) \quad (7)$$

Equations (1) ~ (2) indicate that the multi-objective linear programming model has the highest total comprehensive evaluation score and the highest total return. Equation (3) indicates that at most  $b$  securities are selected from 100 sample securities to construct asset portfolio allocation. Equation (4) indicates that the position of each security cannot exceed the  $w$  of the total assets. Equation (5) indicates that the monthly dividend income can float up and down  $t$ . Equation (6) is a binary variable constraint. Equation (7) is a non-negative constraint on the investment amount.

## 3. Examples of investment strategy formulation

This article uses the actual data of CSI Dividend Index (000922.CSI) to select 100 securities of listed companies with high cash dividend yield, stable dividend distribution, certain scale and liquidity from Shanghai and Shenzhen markets for analysis. Section 3.1 analyzes the data selection and the specific settings of the example, Section 3.2 analyzes the multi-objective linear programming model corresponding to the example, Section 3.3 gives the evaluation indicators and securities evaluation scores, and Section 3.4 gives specific investment strategies.

### 3.1. Data Selection and Instance Conditions

In this part, the historical data of stock closing prices of 100 securities in the CSI Dividend Index from 2006/01 to 2022/04 are selected. This instance is in the following markets:

The initial total capital is RMB 100 million;

No fees for buying and selling stocks;

The warehouse conversion is allowed once a week;

Stocks with daily limit up or down limit cannot be traded;

The transaction price is based on the closing price;

No other derivatives can be used;

No short-selling or securities lending;

The risk-free annualized rate of return is 2.4%;

The position of each stock shall not exceed 15% of the total assets;

This market is an ideal market, that is, the financial crisis, epidemic and other factors are not considered.

Based on the above market, the goal of the investment portfolio strategy is to select up to 10 securities from 100 sample securities for an asset portfolio allocation (short position is allowed), and the warehouse conversion is allowed once a week, and the conversion time is unified every Monday. Up to 10 securities allocation combinations can be re-selected when placing a warehouse, so that the selected asset allocation combination cannot deviate from the monthly return of the CSI Dividend Index  $\pm 1.5\%$ . On this basis, the annual excess return rate is the highest.

### 3.2. Multi-objective Linear Programming Model

According to the market conditions and investment portfolio strategy objectives, the following multi-objective mathematical programming model can be obtained according to Section 2.3:

$$max z = \sum_{i=1}^{100} C_i z_i (1 \leq i \leq 100)$$

$$max S = \sum_{i=1}^{100} C_i x_i X_i + \left( P - \sum_{i=1}^{100} C_i x_i \right) \times \frac{2.4\%}{360} \times 7 (1 \leq i \leq 100)$$

$$0 \leq \sum_{i=1}^{100} C_i \leq 10 (1 \leq i \leq 100)$$

$$x_i \leq P \times 15\% (1 \leq i \leq 100)$$

$$\frac{(h - 1.5\%) \times 7}{30} P \leq S \leq P \frac{(h + 1.5\%) \times 7}{30}$$

$$C_i \in 0,1 (1 \leq i \leq 100)$$

$$x_i \geq 0 (1 \leq i \leq 100)$$

### 3.3. Selection of evaluation indicators

There are many angles to measure the quality of a security. This article is based on the past data of the closing prices of CSI dividend constituents from five indicators:

Indicator  $x_1$ - full distance  $R:R = \max(x) - \min(x)$ ;  
Indicator  $x_2$  - variance  $s^2$  :  $s^2 = \frac{(M-x_1)^2+(M-x_2)^2+\dots+(M-x_n)^2}{n}$ ;

Indicator  $x_3$ - coefficient of variation  $c_v:c_v = \frac{\sigma}{\mu}$ ;

Indicator  $x_4$ - skewness coefficient  $S_k: S_k = \frac{\bar{x}-m_0}{\sigma}$ , where  $m_0$  is the mode;

Indicator  $x_5$ - difference  $\overline{\Delta x}$ : The difference between the predicted five-day mean and the historical group mean.

According to the above five evaluation indicators, 100 securities are evaluated and ranked in descending order, and the following scores are obtained (select the top ten securities):

**Table 2.** Top ten securities

name	score	sort
Huayu Automobile 600741.SH	1	1
Kibing Group 601636.SH	0.7335	2
Weixing Shares 002003.SZ	0.7266	3
Yibai Pharmaceutical 600594.SH	0.7065	4
Jinke Shares 000656.SZ	0.6738	5
Haili De002206.SZ	0.6704	6
Sangang Minguang 002110.SZ	0.6456	7
Longbai Group 002601.SZ	0.6373	8
Shimao Co., Ltd. 600823.SH	0.6365	9
Nangang Co., Ltd. 600282.SH	0.6313	10

### 3.4. Investment strategy formulation

The dynamic process of investment strategy formulation is as follows: first step, use raw data and BP neural network prediction model to predict the closing price of securities in the next week; second step, use CRITIC method to evaluate and score securities; third step, give the portfolio and calculate the return based on the multi-objective linear programming model; in the fourth step, the original data set is updated weekly and the first step is repeated.

Next, start to make a portfolio of securities, first make a portfolio for the first week of May, because the predicted rate of return in the first week is  $1.52\% > 0$ , so it is possible to invest, first use the original data set and BP neural network model to predict the closing price of securities in the first week of the month, and then the CRITIC method is used to evaluate the securities for scoring. Finally, based on the multi-objective linear programming model, the portfolio is given and the returns are calculated to obtain the following results:

**Table 3.** Portfolio for the first week of May

Securities name	score	sort	Investment amount
Jinke Shares 000656.SZ	0.6738	5	1157489
Jizhong Energy 000937.SZ	0.39	62	984831.2
Weixing Shares 002003.SZ	0.7266	3	1351060
C&D Inc. 600153.SH	0.4381	49	923789.1
Yankuang Energy 600188.SH	0.5993	14	1921711
First open shares 600376.SH	0.5272	twenty-three	338674.5
Gemdale Group 600383.SH	0.2155	95	962698.9
Maanshan Iron & Steel Co., Ltd. 600808.SH	0.4724	39	1275520
Pingmei Co., Ltd. 601666.SH	0.3824	64	1282602
China Architecture 601668.SH	0.318	83	1306208

Among them, the income is 693,000, so the total

investment amount to update is 100,693,000 yuan, and it is predicted that the income in the second week is  $1.02\% > 0$ , so it is possible to invest, update the data and start to predict the stock price of the second week of the month with the updated data set and BP neural network prediction model, and then based on the multi-objective programming model, the portfolio is given and the returns are calculated to obtain the following results:

**Table 4.** Portfolio for the second week of May

Securities name	score	sort	Investment amount
Zhongnan Construction 000961.SZ	0.6094	11	466591.4
Weixing Shares 002003.SZ	0.7266	3	1016011
Luyang Energy Saving 002088.SZ	0.5615	18	944199
Yutong Bus 600066.SH	0.4564	44	303584.5
Yankuang Energy 600188.SH	0.5993	14	684017.7
Huayang shares 600348.SH	0.4643	42	785756.9
Yibai Pharmaceutical 600594.SH	0.7065	4	1046788
Sifang Shares 601126.SH	0.5461	20	258392.2
Kibing Group 601636.SH	0.7335	2	821044.2
Buchang Pharmaceutical 603858.SH	0.4185	52	683167.8

Among them, the income is 592074.8, so the updated investment amount is 101285074.8 yuan, and the predicted return rate in the third week is  $1.25\% > 0$ , so it is possible to invest. Similarly, the following results can be obtained:

**Table 5.** Portfolio for the third week of May

Securities name	score	sort	Investment amount
Zhongnan Construction 000961.SZ	0.6094	11	465932.7
Weixing Shares 002003.SZ	0.7266	3	927777.7
Yankuang Energy 600188.SH	0.5993	14	584969.5
Huayang shares 600348.SH	0.4643	42	673381.9
Yibai Pharmaceutical 600594.SH	0.7065	4	758596.2
Greenland Holdings 600606.SH	0.573	17	943337.3
China Shenhua 601088.SH	0.2799	89	722806.8
Sifang Shares 601126.SH	0.5461	20	462658.7
Shaanxi Coal Industry 601225.SH	0.3739	66	353900.1
Kibing Group 601636.SH	0.7335	2	575309.9

Among them, the income is 649912.6, so the updated investment amount is 101934987.4 yuan, and the predicted rate of return for the fourth week is  $-0.0015 < 0$ . Since the predicted profit rate is negative, the market environment is not suitable for investment, and no portfolio selection should be made. Short positions should be chosen to maintain business operations and avoid losses caused by capital flows.

After one month's prediction, evaluation and analysis of the relevant data obtained, according to the objective function and constraints, a portfolio of 100 securities was obtained, and the asset allocation strategy was completed. In this month, the asset allocation strategy included the use of disposal

capital to complete the initial position building, and two positions were adjusted according to the optimal asset portfolio. Since the predicted return rate in the fourth week was negative, the short position was adopted. The four asset allocations reflect the asset allocation process that needs to be completed in the actual situation, so it can be concluded that the investment strategy used in this paper is feasible and effective. By analogy, the above model can be used to effectively complete the asset allocation strategy for subsequent data.

#### 4. Conclusions

Based on the investment portfolio optimization problem in the ideal market environment, this article uses the BP neural network model to predict and uses the CRITIC method to evaluate and score securities. Finally, a multi-objective linear programming model is established with the goal of maximizing returns and evaluation scores.

Compared with the existing investment portfolio optimization methods, the method proposed in this article has the following advantages: the prediction model is relatively accurate and the evaluation model score is objective and effective; the multi-objective linear programming model established in this article has certain applicability and has a high reference Value and flexibility; this article uses MATLAB software for prediction, evaluation and scoring, and uses LINGO software to optimize the multi-objective linear programming model, which greatly saves computing time and is not lacking in accuracy; the investment portfolio optimization method in this article has greater universality, that is, the hierarchy is easy to understand but not lacking in reliability and accuracy; this article also has certain guiding value for ordinary investors, that is, it can be invested in a simpler but accurate method, and it is flexible and can be further optimized according to actual scenarios. .

Subsequent research can be carried out from the three perspectives of adding optimization operators to improve the optimization prediction model, considering factors such as financial crisis and epidemic situation in non-ideal markets, and adding more constraints, such as market scenario constraints and improving multi-objective linear programming models to carry out further research and exploration.

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