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Trajectory Tracking of Mobile Robot Based on Improved Hierarchical Sliding Mode Control

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Abstract: A control algorithm based on PD control and non-singular terminal hierarchical sliding mode control is proposed for tracking control of mobile robot systems. First establish a dynamic model of mobile robot under cartesian coordinate system, the system can be divided into subsystems of posture and position, attitude subsystem by using PD control, location subsystem using non-singular terminal hierarchical sliding mode control, the introduction of the system state higher order term, improve the system in the convergence speed and stability of the sliding surface, the system reaches a steady state in a limited time; Finally, the stability of the system is proved theoretically, and the effectiveness of the control method is verified by simulation.

Keywords: Mobile Robot; Hierarchical Sliding Mode Control; Terminal Sliding Mode Control; PD Control.

1. Introduction

In underactuated systems, the number of control variables is less than the number of degrees of freedom, so that the partial degrees of freedom of the system cannot be directly controlled; common underactuated systems include manipulators, mobile robots, quadrotor UAVs, etc.; among them, the mobile robot is a multi-input and multi-output system, and the research is relatively complex. The trajectory tracking technology is an important part of the mobile robot control field. The error between the trajectory of the mobile robot and the reference trajectory is an important indicator of the trajectory tracking performance [2,3].

Sliding mode control method is a simple and effective robust control method, and the response is fast. The control method designs the error-related sliding surface of the system error. When the system control quantity reaches the sliding surface, the system tends to be stable, so as to realize the trajectory tracking control of the mobile robot. Lee et al. proposed to apply sliding mode control to wheeled mobile robot, so that the mobile robot can run steadily on the target trajectory. However, the above method is only to construct a single-layer sliding mode. For multi-input multi-output system, it is difficult to control all variables of the whole system by constructing a sliding surface. Wang et al. [5, 6, 7] proposed a robust nonlinear controller based on hierarchical sliding mode control for a class of underactuated systems to achieve the balance and motion of the underactuated system; the stability of the closed-loop system is obtained by using Lyapunov stability criterion and Barbalat lemma. Pham et al. proposed a new trajectory tracking control algorithm for wheeled mobile robots, which combines hierarchical sliding mode and backstepping control methods. The tracking control guarantees the closed-loop stability and zero tracking error. However, the hierarchical sliding mode control in the above literature adopts ordinary linear sliding surface, which can only guarantee the asymptotic convergence of the system but cannot guarantee the convergence time. Man [9, 10] et al. when designing the sliding surface; the nonlinear sliding surface is used to replace the traditional linear sliding surface, and the terminal sliding mode control is proposed, which makes the convergence time of the system limited. However, singular problems will occur when the parameters are not set. Then a nonsingular terminal sliding mode control is proposed to solve the singular problem of general terminal sliding mode control.

The control objective of this paper is to control the mobile robot system to complete the tracking control with the actual control input torque τ_1, τ_2 . The mobile robot can keep moving along the ideal trajectory. Linear sliding surface is used in the hierarchical sliding mode control method in the above literature. Although the convergence problem of the system is solved, the sliding surface does not converge in finite time, which makes the system chattering larger. In this paper, on the basis of Reference, the terminal sliding mode control instead of the ordinary sliding mode control can effectively improve the time of the system convergence to the stable state and the effect of the system to suppress the chattering problem that often occurs in the sliding film controller. Secondly, the non-singular terminal sliding mode control can avoid the singular point problem in the hierarchical sliding surface. At the same time, the convergence time of the system can be obtained. The fast power reaching law can improve the speed of the system convergence, effectively eliminate the chattering phenomenon and verify the asymptotic stability of the system.

2. Dynamic modeling of mobile robots

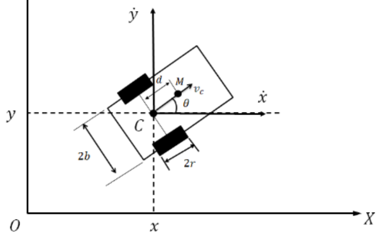

Figure 1. Mobile robots

According to the structure of the nonholonomic mobile robot, point C is the geometric center between two wheels, point M is the focus of the robot, and $q = [x \quad y \quad \theta]^T$ represents the position coordinates of point M in Cartesian coordinate system. d is the distance between two points, and θ is the direction angle of the robot. Assuming that the mobile robot can only move along the direction of the vertical drive wheel, it must meet the condition of pure rolling without sliding

$$\dot{y}\cos\theta - \dot{x}\sin\theta - d\dot{\theta} = 0 \quad (1)$$

By nonholonomic constraints

$$A^T(q)\dot{q} = 0 \quad (2)$$

The constraint matrix is

$$A^T(q) = [-\sin\theta \quad \cos\theta \quad -d]^T \quad (3)$$

According to Euler-Lagrange principle, the dynamic model of the robot is[11,12]

where $M(q) \in R^{3\times3}$ is the inertia matrix, $C(q,\dot{q}) \in R^{3\times3}$ is a matrix of centripetal and Gosnad forces, $G(q) \in R^3$ is the gravity matrix, $F(\dot{q}) \in R^3$ is friction, $\tau_d \in R^3$ For external disturbances, $A^T \in R^{3\times1}$ is the constraint matrix; $\lambda \in R$ is a Lagrange multiplier, $B(q) \in R^{3\times2}$ is the input transformation matrix, $\tau \in R^2$ is the control input torque, where $M(q), C(q,\dot{q}), B(q), G(q), \tau, \lambda$ is defined as

$$M(q) = \begin{bmatrix} m & 0 & md\sin\theta \\ 0 & m & -md\cos\theta \\ md\sin\theta & -md\cos\theta & I \end{bmatrix},$$

$$C(q,\dot{q}) = \begin{bmatrix} 0 & 0 & -md\dot{\theta}\sin\theta \\ 0 & 0 & md\dot{\theta}\cos\theta \\ 0 & 0 & 0 \end{bmatrix}, G(q) = 0,$$

$$B(q) = \frac{1}{r}\begin{bmatrix} \cos\theta & \cos\theta \\ \sin\theta & \sin\theta \\ R & -R \end{bmatrix}, \tau = \begin{bmatrix} \tau_r \\ \tau_l \end{bmatrix},$$

$$F(\dot{q}) = 0, \lambda = -m(\dot{y}\cos\theta + \dot{x}\sin\theta)\dot{\theta}.$$

where m is the quality of the robot; r is the radius of the left and right wheels; I is the moment of inertia; τ_r, τ_l are the control torque of the left and right wheels; Since the control input is the torque of the robot's left and right wheels, it is ordered

$$\begin{cases} u_1 = \tau_r + \tau_l \\ u_2 = \tau_r - \tau_l \end{cases} \quad (4)$$

In this paper, considering the case where the geometric center C and the center of gravity M coincide, that is d=0, the dynamic model of the robot can be obtained:

$$\begin{cases} \ddot{x} = \frac{\lambda}{m}\sin\theta + \frac{1}{mr}u_1\cos\theta \\ \ddot{y} = -\frac{\lambda}{m}\cos\theta + \frac{1}{mr}u_1\sin\theta \\ \ddot{\theta} = \frac{R}{Ir}u_2 \end{cases} \quad (5)$$

3. Controller design

Since the mobile robot is a typical underdrive system, the input control amount of the system is (u_1, u_2), and the controlled output amount is (x, y, θ), In order to ensure that the system can be effectively controlled, the system is divided into a posture subsystem and a position subsystem, The attitude subsystem utilizes PD control, and the position subsystem utilizes improved hierarchical sliding mode control to ensure that the system reaches a stable state.

3.1. Position subsystem controller

In this paper, hierarchical sliding mode control method is chosen to make a single control input to control the two subsystems ideal stable state, so that the entire position control system to achieve the ideal state.

3.1.1. Sub-section Headings

Nonsingular terminal sliding mode control

In the design of the sliding surface, the nonlinear function is used instead of the linear sliding surface, so that the tracking error of the system can converge to zero in a finite time. Thus, improving the dynamic performance of the system. The general form of nonsingular sliding surface is:

$$s = x + \frac{1}{c}\dot{x}^{\frac{p}{q}} \quad (6)$$

where $c > 0$, p, q are positive odd, and $p > q$.

3.1.2. Hierarchical singular terminal sliding mode controller design

By using hierarchical sliding mode control, the conversion from multi-objective control to single-objective control is realized, and the control design of the system is simplified. The above dynamic model can be represented by a general second-order underactuated system model:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f_1(x) + g_1(x)u(t) \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = f_2(x) + g_2(x)u(t) \end{cases} \quad (7)$$

where

$$f_1(x) = \frac{\lambda}{m}\sin\theta, g_1(x) = \frac{\lambda}{mr}\cos\theta,$$

$$f_2(x) = -\frac{\lambda}{m}\cos\theta, g_1(x) = \frac{\lambda}{mr}\sin\theta,$$

$x = [x \quad y \quad \theta]^T, u(t) = u_2(t)$ is the system input, the tracking error of two state variables of the position subsystem is defined as?

$$\begin{cases} e_1(t) = x(t) - x_d(t) \\ e_2(t) = \dot{x}(t) - \dot{x}_d(t) \\ e_3(t) = y(t) - y_d(t) \\ e_4(t) = \dot{y}(t) - \dot{y}_d(t) \end{cases} \quad (8)$$

The parameter trajectory of. Design the first sliding surface for (8) :

$$s_1 = e_1 + \frac{1}{c_1}sig(e2)^{\gamma_1}, s_2 = e_3 + \frac{1}{c_2}sig(e4)^{\gamma_2} \quad (9)$$

where $c_1, c_2, \gamma_1, \gamma_2$ are positive constant, and $0 < \gamma_1 < 1, 0 < \gamma_2 < 1$. According to the equivalent control method, each equivalent control quantity of the position subsystem can be obtained as follows:

$$u_{eq1} = -\frac{1}{g_1(x)}(f_1(x) + c_1\frac{q_1}{p_1}e_2^{2-\frac{p_1}{q_1}}) \quad (10)$$

$$u_{eq2} = -\frac{1}{g_2(x)}(f_2(x) + c_2\frac{q_2}{p_2}e_4^{2-\frac{p_2}{q_2}}) \quad (11)$$

Construct the second sliding surface:

$$S = \alpha s_1 + \gamma s_2 \quad (12)$$

w α, γ here is a normal number, and $\alpha g_1(x) + \gamma g_2(x) \neq 0$. In order to ensure that the two state variables of the position subsystem can slide stably along the respective sliding surface, the control input of the position subsystem needs to contain the equivalent control quantity on each sub-sliding surface. Therefore, the total control input of the position subsystem is:

$$u_1 = u_{eq1} + u_{eq2} + u_{sw} \quad (13)$$

where u_{sw} is the switching control quantity of the position subsystem in the approach stage? In order to obtain the control input u_1 of the position subsystem, the switching control part u_{sw} of the controller needs to be further determined.

Where $f_1(x), f_2(x), g_1(x), g_2(x)$ will be respectively denoted by f_1, f_2, g_1, g_2. The exponential approach law is adopted:

$$\dot{S} = -\eta sat(S) - kS \quad (14)$$

Where η, k are the normal number, $sat(S)$ is the saturation function, $\Delta$is the boundary layer, $k = \frac{1}{\Delta}$, and its specific definition is as follows:

$$sat(S) = \begin{cases} 1 & S > \Delta \\ ks & |S| \le \Delta \\ -1 & S < -\Delta \end{cases} \tag{15}$$

The switching control quantity of the position subsystem in the approach stage is:

$$u_{sw} = -\frac{\gamma g_2(x) u_{eq1}}{\alpha g_1(x) + \gamma g_2(x)} - \frac{\alpha g_1(x) u_{eq2}}{\alpha g_1(x) + \gamma g_2(x)} - \frac{\eta sat(S) + kS}{\alpha g_1(x) + \gamma g_2(x)} \tag{16}$$

Then the control quantity of the position subsystem is:

$$\begin{aligned} u_1 &= u_{eq1} + u_{eq2} + u_{sw} \\ &= -\frac{\alpha(f_1(x) + c_1 \frac{q_1}{p_1} e_2^{2-\frac{p_1}{q_1}})}{\alpha g_1(x) + \gamma g_2(x)} - \frac{\gamma(f_2(x) + c_2 \frac{q_2}{p_2} e_4^{2-\frac{p_2}{q_2}})}{\alpha g_1(x) + \gamma g_2(x)} - \frac{\eta sat(S) + kS}{\alpha g_1(x) + \gamma g_2(x)} \end{aligned} \tag{17}$$

The Lyapunov theorem is used to find the switching control quantity of the position subsystem, and the Lyapunov function is constructed as follows

$$V = \frac{1}{2} S^2 \tag{18}$$

Take the derivative of the above equation

$$\begin{aligned} \dot{V} &= S\dot{S} = S(\alpha \dot{s}_1 + \gamma \dot{s}_2) \\ &= S(\alpha(e_2 + \frac{p_1}{c_1 q_1} e_2^{2-\frac{p_1}{q_1}} \dot{e}_2) + \gamma(e_4 + \frac{p_2}{c_2 q_2} e_4^{2-\frac{p_2}{q_2}} \dot{e}_4)) \\ &= S(\alpha(e_2 + \frac{p_1}{c_1 q_1} e_2^{2-\frac{p_1}{q_1}} (f_1 + g_1 u_1)) + \gamma(e_4 + \frac{p_2}{c_2 q_2} e_4^{2-\frac{p_2}{q_2}} (f_2 + g_2 u_1))) \\ &= S(\alpha(e_2 + \frac{p_1}{c_1 q_1} e_2^{2-\frac{p_1}{q_1}} (f_1 + g_1(u_{eq1} + u_{eq2} + u_{sw}))) + \gamma(e_4 + \frac{p_2}{c_2 q_2} e_4^{2-\frac{p_2}{q_2}} (f_2 + g_2(u_{eq1} + u_{eq2} + u_{sw})))) \\ &= -\eta |S| - kS^2 \le 0 \end{aligned} \tag{19}$$

3.1.3. Stability analysis

Theorem 1 For the underactuated system shown in Equation (7), if the sub-sliding mode surfaces of each level are designed according to Equations (9) respectively, and the control law shown in Equation (17) is adopted, the sliding mode surface of the second layer of the position subsystem is asymptotically stable.

Proof: According to Equation (19) :

$$\dot{V} = -S * (\eta sat(S) - kS) = -\eta |S| - kS^2 \le 0 \tag{20}$$

Combining equation (18), it can be seen that $V > 0$. According to Lyapunov theorem, the position subsystem is stable.

By integrating t on both sides of Equation (19), we can get:

$$V(0) = V(t) + \int_0^t \eta\, |S| + kS^2 dt \ge \int_0^t \eta\, |S| + kS^2 dt \tag{21}$$

Then according to Barbalat's theorem [13], when $t \to \infty$, $\lim_{t \to \infty} S = 0$. The second sliding mode surface can be proved to be asymptotically stable.

Theorem 2: For an underactuated system shown in Eq. (7), if the sub-surfaces of each level are designed according to Eq.(9), respectively, and the control law shown in Eq. (17) is adopted, the sum of the two first-level sliding surfaces of the position subsystem is asymptotically stable.

From the barbalat theorem and Lyapunov theorem, it can be proved that Theorem 2 holds [7], and the position subsystem is asymptotically stable.

Attitude subsystem controller

For attitude subsystem

$$\ddot{\theta} = \frac{R}{Ir} u_2 \tag{22}$$

Assume that the ideal Angle command is θ_d , and the tracking error is $e_\theta = \theta - \theta_d$, For attitude subsystem, PD control method is adopted to design the control law as follows:

$$u_2 = -K_p e_\theta - K_d \dot{e}_\theta \tag{23}$$

where K_p, K_d is the normal number?

Let's take the Lyapunov function as

$$V = \frac{1}{2} \frac{Ir}{R} \dot{e}^T \dot{e} + \frac{1}{2} K_p e^T e \tag{24}$$

Since I, r, K_p they are all normal numbers, it can be known that V is positive definite. Then the derivative of the above formula can be obtained as follows:

$$\begin{aligned} \dot{V} &= \frac{Ir}{R} \dot{e}\ddot{e} + K_p \dot{e} e = \dot{e}(-K_p e - K_d \dot{e} + K_p e) \\ &= -K_d \dot{e}^2 \le 0 \end{aligned} \tag{25}$$

Since it is semi-negative definite and positive definite, the attitude subsystem is stable.

4. Literature References

In order to verify the effectiveness and robustness of the proposed control method, a trajectory tracking control experiment of wheeled mobile robot was conducted using Matlab/Simulink simulation environment. The parameters of the mobile robot are selected according to the reference [12]. The specific parameters of the mobile robot system are shown in Table 1:

Table 1. Three Scheme comparing

m/kg	R/m	r/m	$I/(kg/m^2)$
4	0.2	0.04	2.5

The parameters of non-singular terminal Layered Sliding mode controller (NTHSMC) are as follows: $\alpha = 1$, $\gamma = 50$, $c_1 = 35$, $c_2 = 35$, $q_1 = 55$, $p_1 = 53$, $q_2 = 55$, $p_2 = 53$, $\eta = 5$, $k = 5$, $\Delta = 0.005$. The parameters of the PD controller are as follows: $K_p = 5$, $K_d = 10$. Set the circle trajectory with the center of the ideal trajectory as the origin and the radius as 2:

$$\begin{cases} x_d(t) = 2\cos\theta_d \\ y_d(t) = 2\sin\theta_d \\ \theta_d = 0.1t \end{cases}$$

Type of; The starting position of the mobile robot is set as $q = [0 \quad 0 \quad \frac{\pi}{2}]$; The mobile robot is simulated and analyzed. The parameters of the layered sliding mode controller (HSMC) are $\alpha = 1$, $\gamma = 50$, $c_1 = 35$, $c_2 = 35$, $\eta = 5$, $k = 5$, $\Delta = 0.005$.

FIG. 2 shows the motion trajectory of the mobile robot. It can be seen from FIG. 2 that the actual trajectory can quickly track the given desired trajectory and keep the basic coincidence, which highlights the effectiveness of the controller. When the controller parameters are consistent, N TSMC has higher tracking performance than HSMC, but HSMC cannot achieve the tracking purpose. FIG. 3 is the tracking error curve. It can be seen from the curve that around 10s, the system gradually enters the steady state, and the mobile robot starts to run along the circular trajectory, and the tracking error all converges to zero, which has good trajectory tracking performance. FIG. 4 shows the variation trend of each sliding mode surface under non-singular terminal Layered Sliding mode control (NTHSMC) and the total sliding mode surface after linear combination. It can be seen from Figure 4 that the sub-sliding surface and the combined sliding surface under NTHSMC can achieve a fast and stable trend and remain in a stable state, while effectively reducing chattering. Although the convergence speed is different, they all converge to zero in finite time. The convergence speed of each sliding mode surface can be achieved by adjusting the parameter size of the controller, or by improving the reaching law to accelerate the convergence speed.

Figure 5 is the change curve track of the input control sum of the position subsystem and the attitude subsystem. It can be seen that the convergence curve of the controller is gradually stable and finally converges to a stable state.

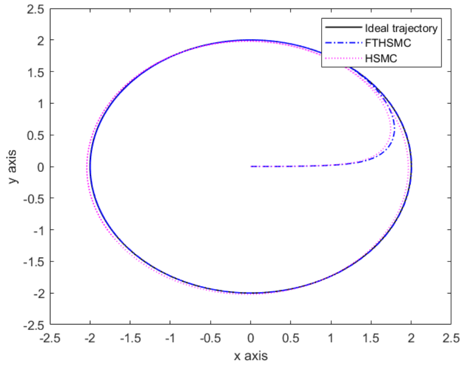

Figure 2. Trajectory of the mobile robot

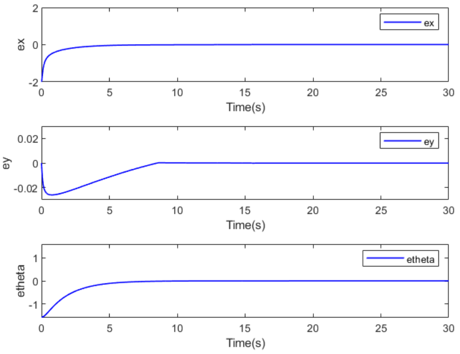

Figure 3. The tracking error curve

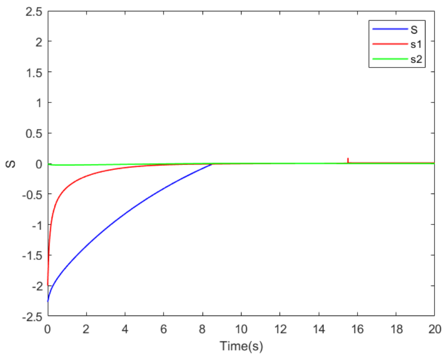

Figure 4. The sliding mode surface

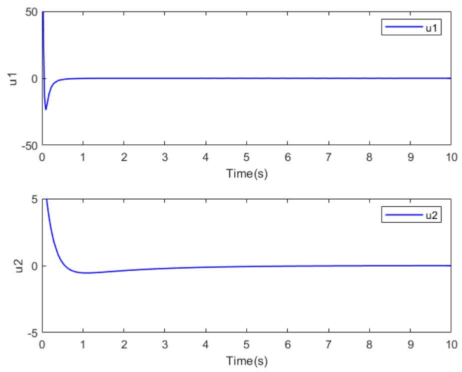

Figure 5. the input control curve

5. Conclusion

This paper presents the design and implementation of a controller based on mobile robot which combines non-singular terminal sliding mode control and PD control. The first layer of the layered sliding mode controller is a non-singular terminal sliding mode control structure for each variable of the position subsystem, and the second layer is a linear combination of the first layer. The advantage of the proposed control method is that it can reach the desired stable state in a given time. The simulation results show that the control method has good stability and robustness.

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