Critical length parameter of HDPE and its use in fatigue lifetime predictions

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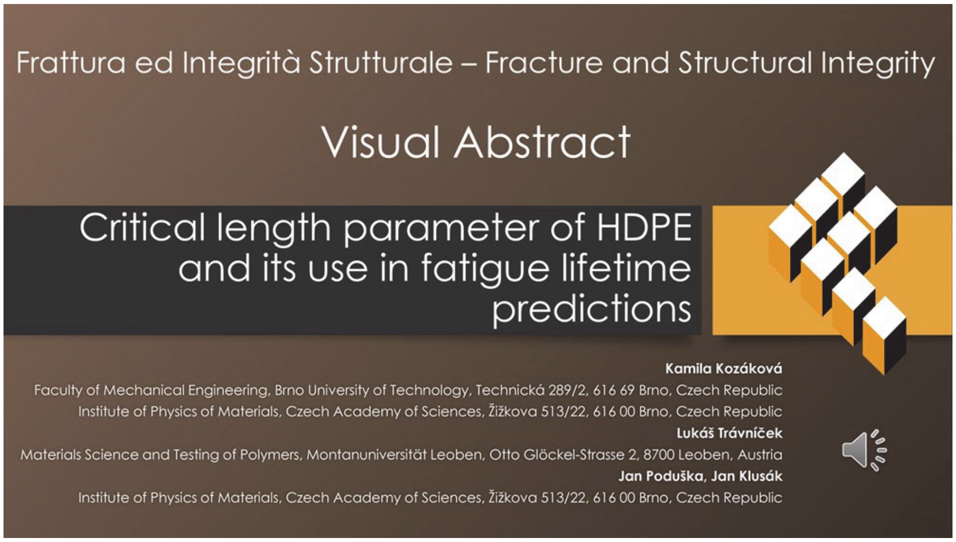

Frattura ed Integrità Strutturale – Fracture and Structural Integrity

Visual Abstract

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INTRODUCTION

The theory of critical distance (TCD) is a bundle of methods for predicting notch fatigue effects in metallic materials predominantly [1]. Its basis was defined well over 60 years ago, but it started receiving attention only in the last couple of decades with greater availability of finite element analysis methods, which are necessary for its successful application. It was shown in [2,3] that the prediction of fatigue lifetime of notched specimens can be done by the TCD. It assumes that the critical distance value, used to calculate effective equivalent stress ahead of the notch, is a material property whose value increases with the decreasing number of cycles to failure. The applicability of the TCD principles to metals proven in [4–6] suggests there may be a possibility to successfully apply the theory to other materials. Especially those, to which other fatigue and fracture theories were successfully applied – e.g. the high-density polyethylene (HDPE) for piping applications.

Resistance to crack propagation is a quality closely watched in polymers used for piping applications. Pressure pipes made of polymer materials are used frequently for water or gas distribution. They are designed to withstand long operating times (longer than 50 years). The main mechanism causing failures after a very long operating time is usually the so-called slow crack growth (SCG) [7] – a process where cracks initiate on the inner surface of the pipe and propagate under the stable pressure load in a creep-like manner towards the outside until the pipe starts to leak. Therefore, various tests were designed to measure the resistance to this mechanism.

It was concluded in several works that the principles of linear-elastic fracture mechanics can be applied to the slow crack growth mechanism in HDPE [8]. Although the mechanism is different from similar processes in metals, the small-scale yielding condition is generally met. The plastic deformation the material achieves during the process is relatively large, but also very localized to a wedge-shaped process zone ahead of the crack tip. Then, parameters like the stress intensity factor can be used to describe and simulate the crack propagation using FEA [8–11].

The Cracked Round Bar (CRB), standardized by ISO 18489 [12] is a test procedure similar in some features to the classic fatigue testing to obtain *S-N* curves. The specimens are cylindrical bodies with razor-sharp, circumferential notches in the middle (hence cracked round bars). They are loaded by cyclic loading and the number of cycles to failure is measured. The resultant dependency of the loading stress range (or stress intensity factor range) on the cycles to failure is used as a measure of the material's crack propagation resistance [13–15]. However, no prediction of the lifetime of a HDPE pipe can be made of such data, because the pipes are not loaded cyclically. On the other hand, fatigue loading, and notches may play an important role in other applications of HDPE in which case the prediction can be useful.

Critical distance theory has already been used to predict the fatigue life of notched specimens based on stress distributions and fatigue S-N curves of model notched and smooth specimens, see [5,6]. This paper deals with the application of the critical distance theory to predict the fatigue life of notched components made of two types of HDPE. In the approach presented here, fatigue life predictions are performed based on data (stress distribution and fatigue curves) corresponding to model notch and cracked specimens (CRB). This novel approach leads to the determination of critical distances and introduces a new type of modification. The modification uses the ratio of the stress concentration factors of the predicted notch and the model notch. Unlike the modification in [5], here the ratio of stress concentration factors is expressed as a reciprocal of that in [5]. The reason for using the reciprocal value is that in the case of HDPE materials, the critical distance is calculated from the sharp stress concentrator (CRB) and model notches, whereas in the case of metals the distance is determined from smooth specimens and model notches.

The main goal was to try the method on materials other than metals and to see if the crack resistance curves measured on the sharp-notched specimens can be used to predict the fatigue lifetime of other notched parts made of HDPE. Preliminary studies of HDPE were carried out in [16]. Fatigue tests on CRB specimens as well as on other notched specimens with notch radii $r = 0.1$ mm, 0.2 mm, and 0.4 mm were carried out in this study. For the sake of comparison, the CRB specimens can also be considered notched specimens with a notch radius of 0.01 mm [12]. The critical distance theory applied here uses the line method to calculate the critical distance. The effective notch stress is the average stress along the critical distance – see [1,3,4,17,18] for a detailed description.

METHODOLOGY

Cylindrical CRB specimens with the dimensions according to the standard (ISO 18489)[12]: outer diameter $D = 14$ mm, length $l = 100$ mm, and circumferential razor-sharp notch $a_{\mathrm{ini}} = 1.5$ mm; were turned from compression molded sheets. Two types of HDPE were tested, they are denoted as PE1 and PE2. The standard defines the maximal radius of the notch tip as 0.01 mm, so this value was taken as the crack tip radius for calculations performed further. Besides the CRB specimens, notched specimens were machined and tested. The notch radius was varied to 0.1, 0.2, and 0.4 mm, see the detail in Fig. 1.

The uniaxial fatigue tests were performed at room temperature using the computer-controlled testing machine INSTRON E3000, which loads the samples with a frequency of 10 Hz. The cyclic loading was controlled by force, and it had the sine form defined by the maximum force $F_{\max}$ and cyclic load ratio $R = 0.1$ ($R = F_{\min}/F_{\max}$). The corresponding loading stress range $\Delta\sigma$ related to the ligament area was chosen within 8 and 14 MPa for CRB specimens. For the notched specimens, the loading was increased to reach the fatigue failure within a reasonable time.

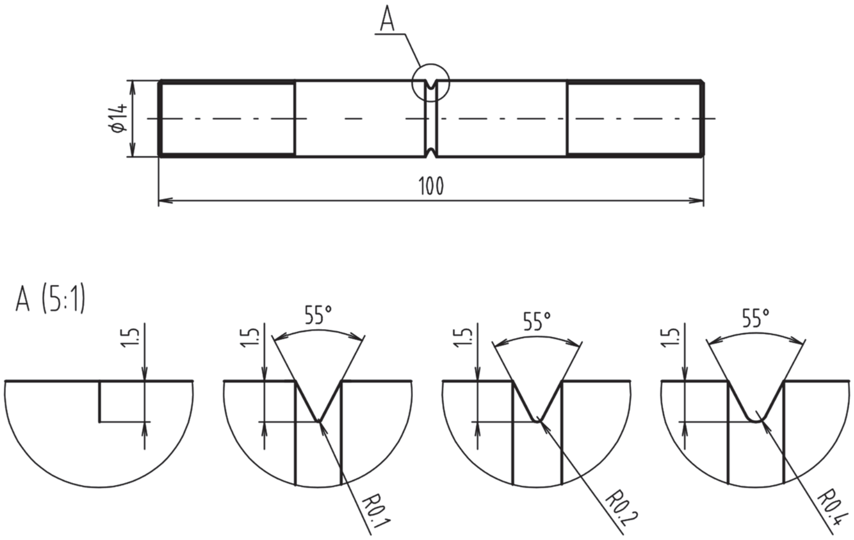

Figure 1: Design of CRB and notched specimens Figure.

In this paper, the methodology used for fatigue lifetime prediction is based on knowledge of the stress distribution ahead of the notches. The respective stress was determined using FEA. The axial stress distributions, together with fatigue data, were used to calculate the critical distance. Critical distances calculated from each pair of curves (describing CRB specimens and specimens with larger notch radii) were compared. Then these critical distances were used for fatigue lifetime predictions of tested notches, so the predictions could be verified easily with the experimental data.

Numerical simulations

Knowledge of stress distribution around a crack and notch tip is necessary to calculate the critical parameter. Stress distribution is determined using Finite element analysis (FEA), software ANSYS [19]. Cracks in CRB specimens are modeled as a notch with a notch radius of r = 0.01 mm, which reflects the radius of the razor blade used to manufacture the circumferential crack. Material parameters of both tested HDPE types are listed in Tab. 1. The type denoted as PE1 is a material from polyethylene (PE) grades used for blow-molding, which is comparable to pipe grades with lower crack resistance, it generally results in shorter lifetimes in fatigue tests. The type PE2 is a genuine pipe grade from the PE100 category. These materials have a very high crack resistance. They are designed to be used in polymer pressure pipes with very long lifetimes.

	density, ϱ [g/cm^3]	Young's modulus, E [MPa]	Poisson's ratio, μ [-]
PE1	0.94169	1412	0.35
PE2	0.95921	1869	0.35

Table 1: Material parameters of tested polyethylenes.

2D models of the specimens were created using the Plane183 elements with axisymmetric conditions. The shape of the notch is shown in Fig. 1. The mesh was refined in the notch area, and the size of the smallest element was 0.04 mm.

The specimens are loaded by the normalized stress so that the nominal stress is equal to 1 MPa. The stress in y direction along the path, which leads from the notch tip to the center of the specimen is plotted in Fig. 2. The highest stress is at the notch tip. The stress concentration factors $K_t = \sigma_{max}/\sigma_{nom}$ of tested of tested notches are listed in Tab. 2.

notch radius, r [mm]	0.4	0.2	0.1	0.01 (crack)
stress concentration factor, K_t [-]	4.13	5.51	7.27	22.23

Table 2: Stress concentration factors.

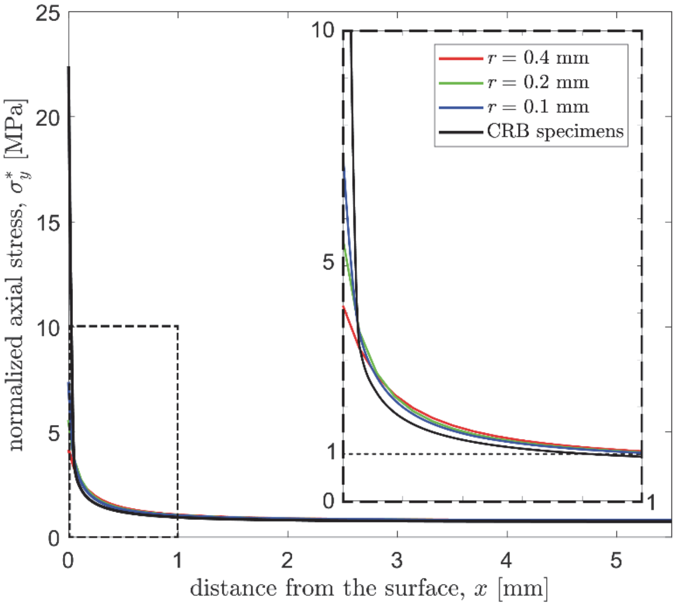

Figure 2: Normalized axial stress along a notch tip to the center of the specimen.

Determination of critical distance

In the next step, the critical distance is calculated. The critical distance is determined from two approximated *S-N* curves, one describes broken CRB specimens, and the other approximates notched specimens. The principle is shown in Fig. 3, where a determination of critical distance for a specific number of cycles to failure N_f is described. The procedure is repeated for all numbers of cycles N_f within the tested range, and the dependency of the critical distance on the number of cycles is obtained. In each step, the exact axial stress σ_y is plotted for the CRB and notched specimens with a model notch. Average stress $\overline{\sigma_y}(x)$ for both of the exact axial stress distributions is calculated using the following Eqn. 1:

$$\overline{\sigma_y}(x) = \frac{1}{x}\int_0^x \sigma_y \mathrm{d}x \tag{1}$$

where x is the distance from the surface. The intersection of the average stress functions is the point of the critical distance for the respective N_f. Hence, when S_1 equals S_2 at the same number of cycles to fracture, the critical distance is found.

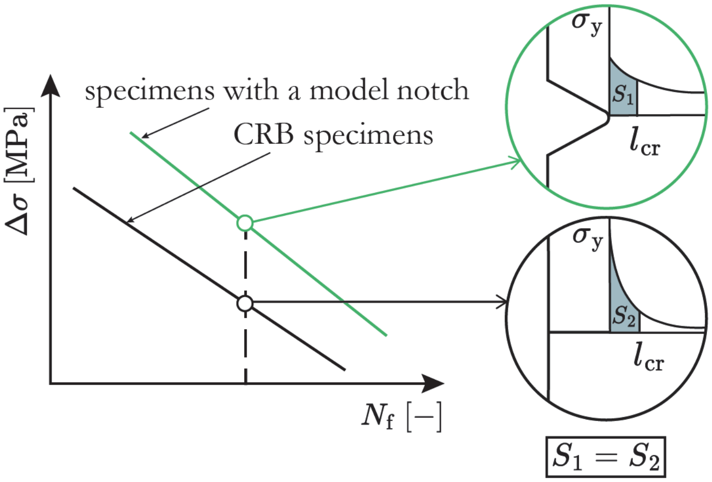

Figure 3: Determination of the critical distance.

The notch radius of the model notch can be chosen. In our study, the critical distance dependency on the number of cycles is determined for all pairs of approximations of CRB specimens and notched specimens with notch radii r = 0.1 mm, 0.2 mm, and 0.4 mm. The curves of critical distance dependencies are obtained. For fatigue lifetime predictions, just one curve of critical distance is needed. To demonstrate the reliability of the method, predictions are made from each model notch separately.

Fatigue lifetime predictions of notched specimens

The critical distances depend on the notch radius; therefore, it is necessary to modify the critical distance. The critical distance used for fatigue lifetime predictions of specimens with the specific notch radii is denoted as l_p. The ratio K_{tp}/K_{tm} represents a relation between the model notch (l_{cr} is determined from a model notch) and the notch whose fatigue lifetime we want to predict. This critical distance used for predictions can be calculated from Eqn. 2, where K_{tp} is the stress concentration factor of the predicted notch and K_{tm} is the stress concentration factor of the model notch. Model notch is the notch, which is used for critical distance l_{cr} calculation.

$$l_{\mathrm{p}} = l_{\mathrm{cr}} \cdot \frac{K_{\mathrm{tp}}}{K_{\mathrm{tm}}} \tag{2}$$

The predictions are performed as an inverse task to the determination of the critical distance, see Fig. 4. Predictions are based on the dependency of the critical distance of the predicted notch l_p and S-N data of the CRB specimens. When the S-N curve of CRB specimens is known, the axial stress distribution of CRB specimens at a specific number of cycles can be calculated. Then the average stress $\overline{\sigma_{\mathrm{y,CRB}}\left(l_{\mathrm{p}}\right)}$ over the critical distance l_p is determined from the axial stress distribution of the CRB specimen. Subsequently, the normalized axial stress distribution of the predicted notch is used. In this axial stress distribution, where the nominal stress is equal to 1 MPa, the average stress over the critical distance l_p is found, $\overline{\sigma^{*}_{\mathrm{y,\,notch}}\left(l_{\mathrm{p}}\right)}$. Note that $\overline{\sigma^{*}_{\mathrm{y,\,notch}}\left(l_{\mathrm{p}}\right)}$ is dimensionless as it represents normalized stress. The predicted fracture stress range of the notched specimen $\Delta\sigma^{\mathrm{n}}$ is calculated from Eqn. 3:

$$\Delta\sigma^{\mathrm{n}} = \frac{\overline{\sigma_{\mathrm{y,CRB}}\left(l_{\mathrm{p}}, N_{\mathrm{f}}\right)}}{\overline{\sigma^{*}_{\mathrm{y,\,notch}}\left(l_{\mathrm{p}}, N_{\mathrm{f}}\right)}} \tag{3}$$

This procedure is repeated for the whole range of cycles to failure N_f and the fatigue curve of the predicted notch is obtained.

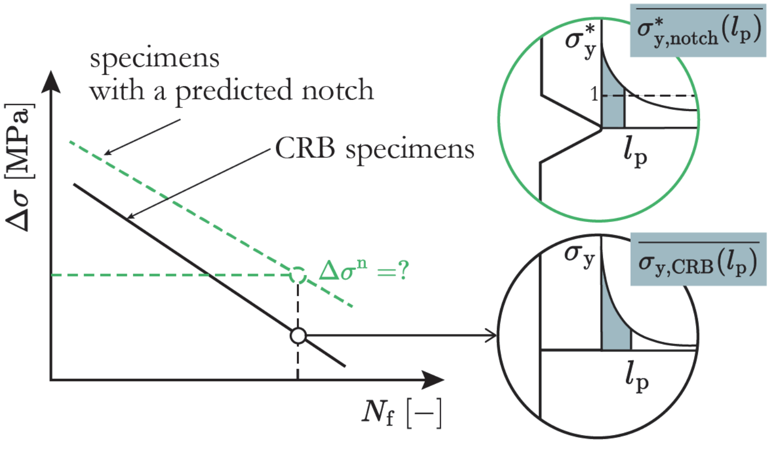

Figure 4: Determination of the predicted fatigue curve of notched specimens.

RESULTS

According to the methodology described, the critical distances were calculated based on experimental fatigue data. Two types of HDPE were tested. The fatigue lifetime predictions of notched specimens were calculated and verified. All results are described in the following chapters.

Experimental fatigue data

Fatigue tests of CRB and all notched specimens were performed, so the fatigue lifetime predictions could be compared to experimental data. Specimens were tested with the stress ratio $R = 0.1$, which ensures tension of the component. Fatigue data of PE1 range between 2×10^5 and 3×10^6 cycles. Fatigue data of PE2 range between 8×10^5 and 3×10^6 cycles. As expected, fatigue lifetime decreases with notch sharpness, see Fig. 5, 6. Fatigue data are approximated by an exponential Eqn. 4, the coefficients of approximations are listed in Tab. 3, 4.

$$\Delta\sigma = A \cdot N_{\mathrm{f}}^{B} \tag{4}$$

The confidence intervals are wider with increasing notch radius. This is due to the nature of the damage evolution process. The very sharp notch in CRB specimens causes the crack to initiate early and propagate fast, resulting in a smaller scatter. In specimens with blunter notches, the crack initiation process is longer and depends also more on the quality of the surface or the presence of defects. That leaves more space for scatter. This effect is much more pronounced in the more durable PE2.

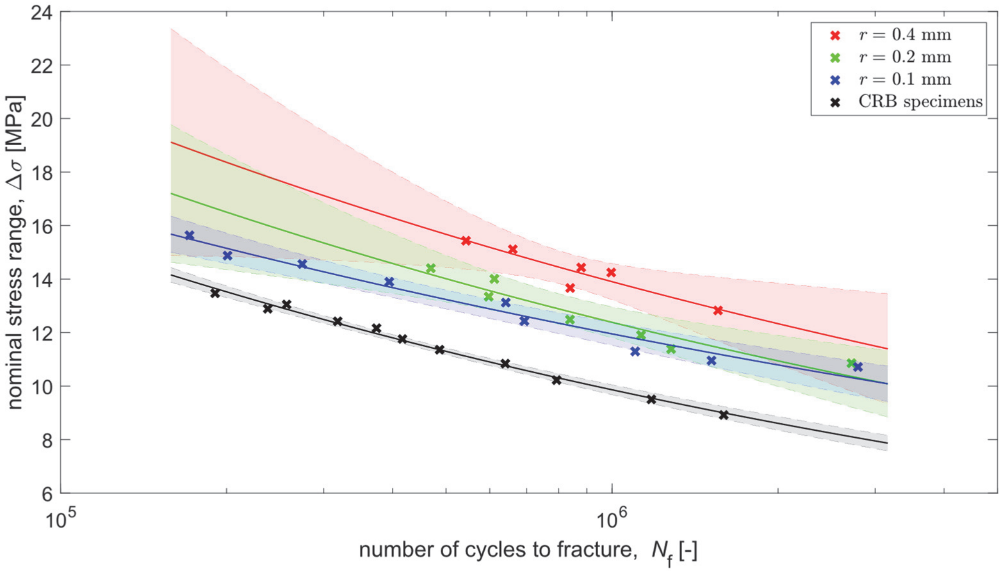

Figure 5: PE1: Experimental fatigue data of CRB and notched specimens.

notch radius, r [mm]	0.4	0.2	0.1	0.01 (crack)
A [MPa]	151.5	145.6	91.51	148.2
B [-]	-0.173	-0.178	-0.147	-0.196

Table 3: PE1: Coefficients of approximations A, B.

Dependency of the critical distance on the number of cycles

The dependency of the critical length parameter on the number of cycles is calculated from the pair of the following S-N curves: approximation of CRB specimens, and approximation of notched specimens with a model notch. The critical distance decreases with the increasing number of cycles to failure in the case of PE1. On the other hand, the critical distance increases with the increasing number of cycles in the case of PE2, see Fig. 7. Generally, the sharper the notch, the greater the critical distance. The critical distance is a parameter that describes a ratio between the fatigue characteristics of two sets of samples. Its dependency on the number of cycles to failure is not indicative of the general fatigue performance of the material but rather of the mutual position of the curves.

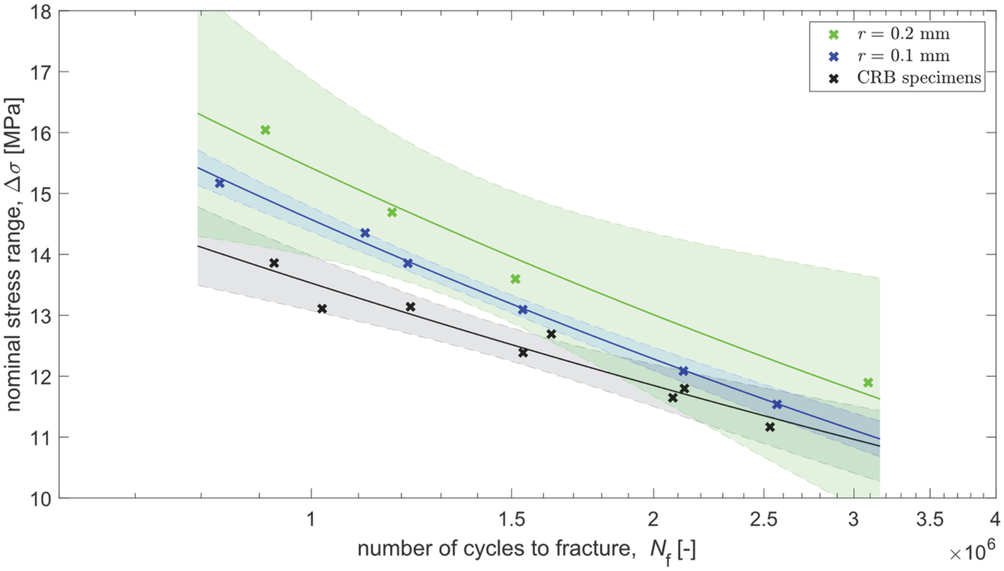

Figure 6: PE2: Experimental fatigue data of CRB and notched specimens.

notch radius, r [mm]	0.2	0.1	0.01 (crack)
A [MPa]	456.3	439.7	190.3
B [-]	-0.245	-0.247	-0.191

Table 4: PE2: Coefficients of approximations A, B.

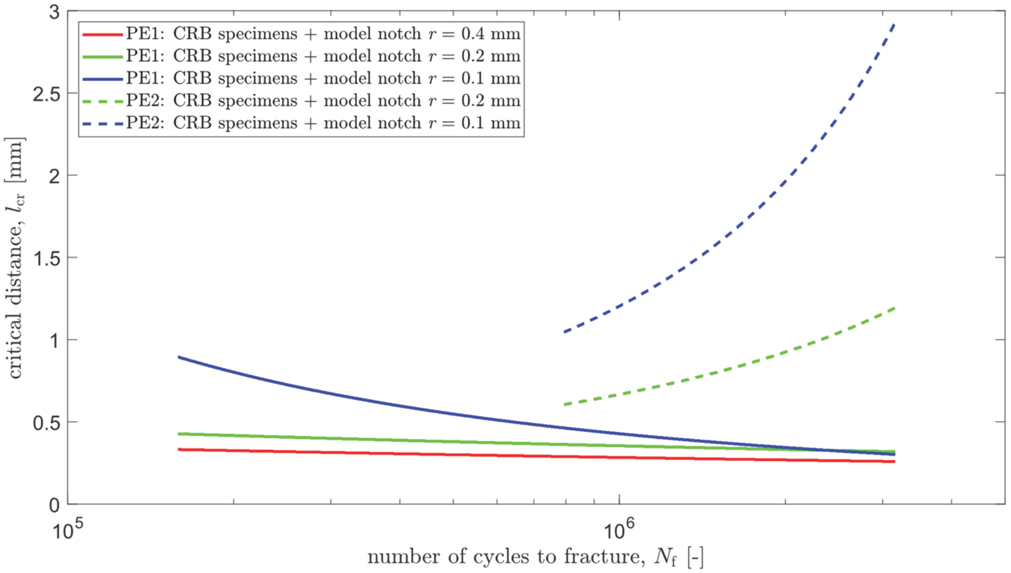

Figure 7: Dependency of the critical distance.

The critical distance of PE1 ranges between 0.25 and 0.89 mm. The mean critical distance calculated from CRB specimens and notched specimens with notch radius $r = 0.4$ mm is 0.29 mm, the mean critical distance calculated from CRB specimens and notched specimens with notch radius $r = 0.2$ mm is 0.37 mm, and the mean critical distance from CRB specimens and notched specimens with notch radius $r = 0.1$ mm is 0.52 mm.

In the case of PE1, the dependency of the critical distance calculated from $r = 0.4$ mm and 0.2 mm have a similar shape and slope, which corresponds to experimental fatigue data. The slope of experimental data from the notched specimens with $r = 0.1$ mm differs from the results of other notched specimens and CRBs, which is manifested in the different values of the exponent B, listed in Tab. 3.

The critical distance of PE2 ranges between 0.61 and 2.93 mm. The mean critical distance calculated from CRB specimens and notched specimens with notch radius $r = 0.2$ mm is 0.85 mm. The mean critical distance from CRB specimens and notched specimens with notch radius $r = 0.1$ mm is 1.75 mm.

In the case of PE2, the critical distances grow rapidly. This phenomenon comes from the experimental data, the approximation curves converge in the right part of the graph, see Fig. 6. The closer the notched sample approximation is to the CRB samples approximation curve, the larger the value of the critical distance.

The values of critical distances are slightly higher but still comparable to critical distances reported for amorphous polymers by D. Taylor in [18], although the PE1 and PE2 are semicrystalline polymers.

Fatigue lifetime predictions of PE1

Having obtained the critical distances from testing data makes it possible to predict failures of notched samples. The predictive ability was tested by calculating the predicted lifetimes and comparing them with the experimentally obtained values. From every pair of CRB + notched specimens, a lifetime prediction was calculated for the remaining notched samples. The fatigue lifetime predictions from each critical distance dependency are shown in the following figures. The full lines represent fatigue lifetime predictions of the relevant notch specimens. Every graph is supplemented with experimental data, their approximation (dotted lines), and confidence intervals. The predictive ability is evaluated by how close the full lines are to the dotted lines and if they fall into the confidence interval of the dotted line.

Fatigue lifetime predictions calculated from CRB specimens and specimens with notch radius $r = 0.4$ mm are shown in Fig. 8. Fatigue lifetime predictions of specimens with notch radius $r = 0.2$ mm are very close to experimental approximation, it corresponds to the fact that the shape and slope of critical distances calculated from specimens with notch radius $r = 0.4$ mm and $r = 0.2$ mm are similar. Fatigue lifetime predictions of notches with a radius $r = 0.1$ mm are less accurate than notches with a radius $r = 0.2$ mm. Although a major part of the prediction is located in the confidence interval of experimental data. Deviations correspond to the fact that the slope of experimental fatigue data of notched specimens with notch radius $r = 0.2$ mm and $r = 0.4$ mm differ. Predictions follow the slope of the approximation of the experimental fatigue data of the model notch, which was used for critical distance calculations.

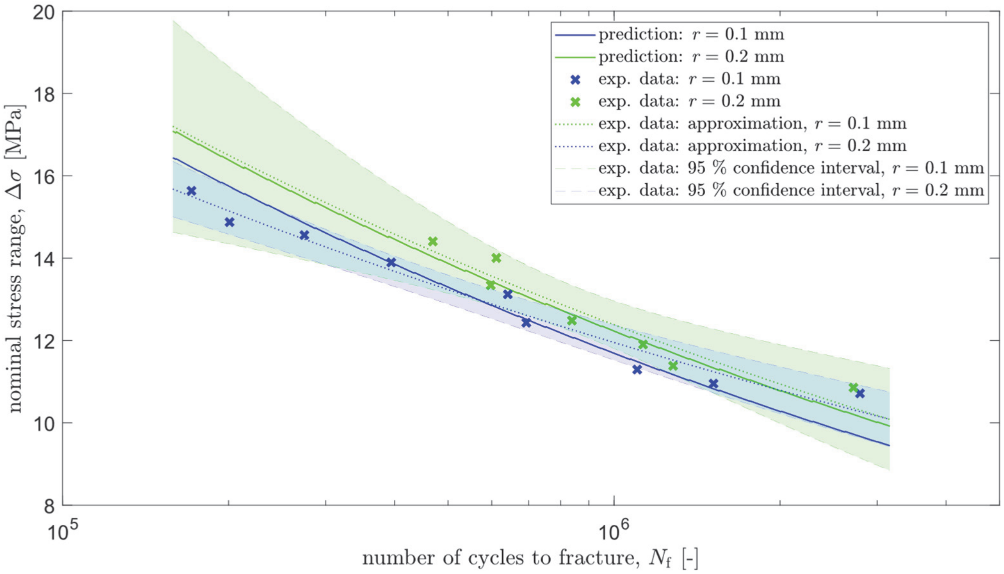

Figure 8: PE1: Fatigue lifetime predictions from the critical distance calculated from $r = 0.4$ mm.

Fatigue lifetime predictions calculated from CRB specimens and specimens with notch radius $r = 0.2$ mm are shown in Fig. 9. Fatigue lifetime predictions of specimens with notch radius $r = 0.4$ mm are very close to experimental approximation. This situation again corresponds to the fact that the shape and slope of critical distances calculated from specimens with notch radius $r = 0.2$ mm and $r = 0.4$ mm are similar. The first part of fatigue lifetime predictions of notched specimens with notch radius $r = 0.1$ mm is out of the confidence interval of experimental data. However, the bigger remaining part of the prediction is located in the confidence interval. The deviation in this case is caused by the different slope of experimental fatigue data of notched specimens with notch radius $r = 0.1$ mm and $r = 0.2$ mm.

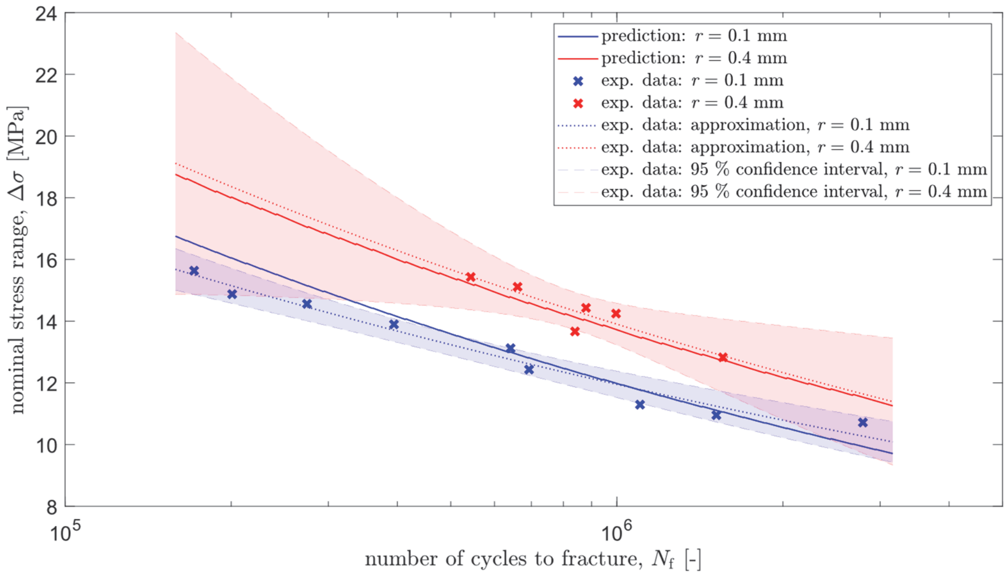

Figure 9: PE1: Fatigue lifetime predictions from the critical distance calculated from $r = 0.2$ mm.

Fatigue lifetime predictions calculated from CRB specimens and specimens with notch radius $r = 0.1$ mm are shown in Fig. 10. Both predictions have a different slope than the experimental approximation. Both predictions have the same slope as an approximation of experimental fatigue data ($r = 0.1$ mm), see Fig. 5. Although the predictions are still situated within the confidence intervals.

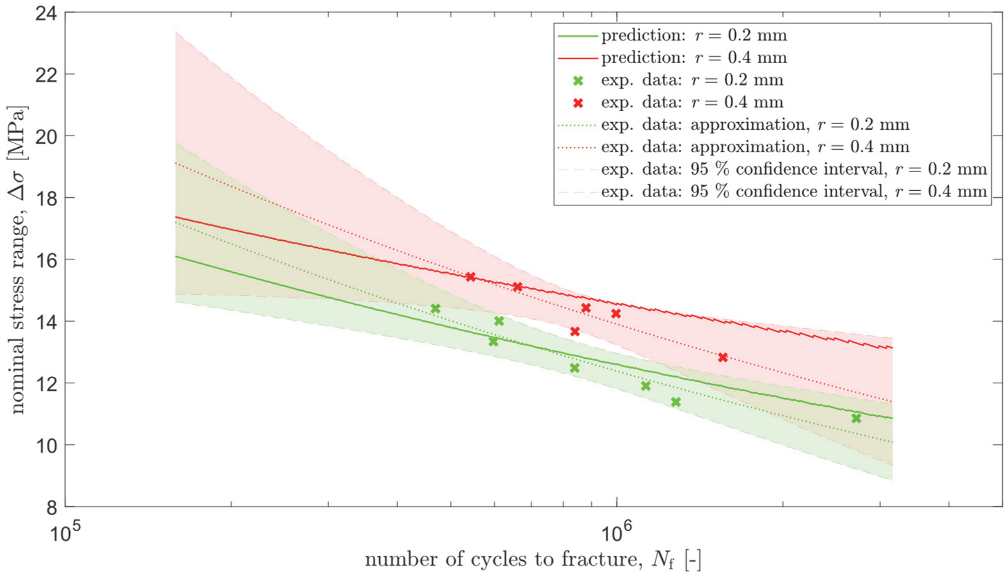

Figure 10: PE1: Fatigue lifetime predictions from the critical distance calculated from $r = 0.1$ mm.

The behavior of the predictions is such that the shape and slope of the predicted curve always depend on the shape of the curve from which it is calculated. Therefore, the predictions are less accurate in cases where the slopes differ more significantly – e.g. in case of predictions from r = 0.1. However, even in such cases, the predictions lie within the confidence intervals of the experimental data.

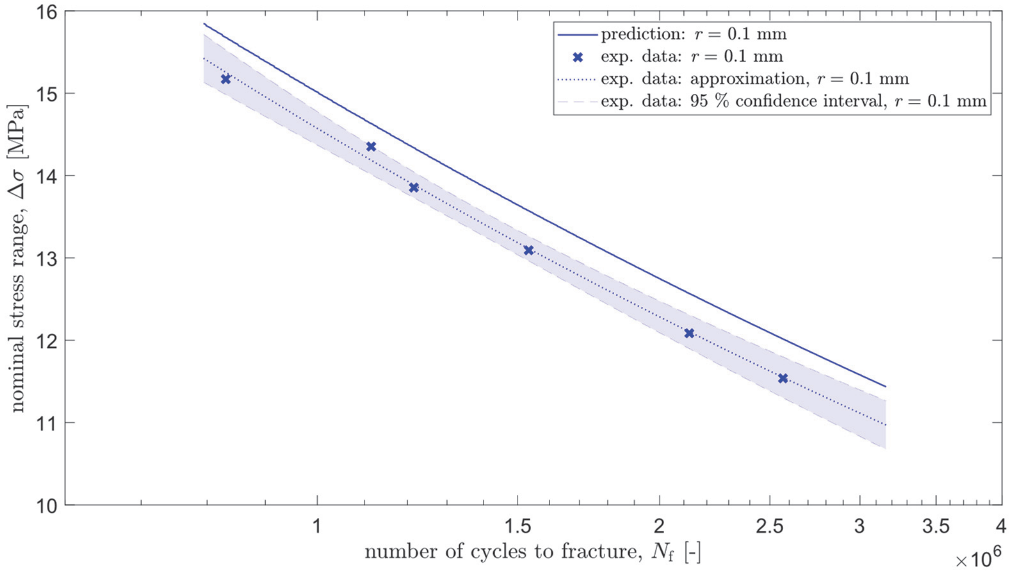

Figure 11: PE2: Fatigue lifetime predictions from the critical distance calculated from $r = 0.2$ mm.

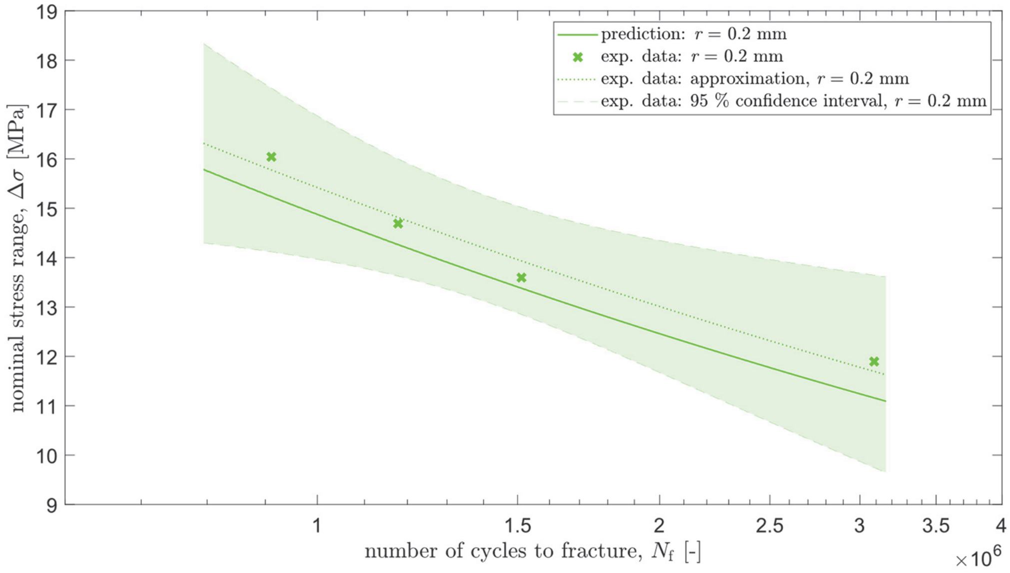

Figure 12: PE2: Fatigue lifetime predictions from the critical distance calculated from $r = 0.1$ mm.

Fatigue lifetime predictions of PE2

In this chapter, fatigue lifetime predictions of the second type of polyethylene are presented. Same as in the previous chapter, the full lines represent fatigue lifetime predictions, cross symbols state for experimental data, and dotted curves state for the approximation of the experimental data. It is assessed whether the prediction curve is in the confidence interval.

Fatigue lifetime predictions calculated from CRB specimens and notched specimens with notch radius $r = 0.2$ mm are shown in Fig. 11. The slope of predicted data is very similar to an approximation of experimental data of notched specimens with notch radius $r = 0.1$ mm, it comes from experimental data. However, the prediction curve is not located in the confidence interval. The predicted values are above the confidence interval, so the predicted life of samples is not conservative.

Fatigue lifetime predictions calculated from CRB specimens and notched specimens with notch radius $r = 0.1$ mm are shown in Fig. 12. The slope of predicted data is almost the same as the approximation of experimental data. The prediction curve is situated in the confidence interval, although not many experiments were run and for that reason, the confidence interval is so big. The prediction curve is below the experimental approximation and leads to conservative results.

In the case of PE2, the slope of prediction agrees with the experimental approximation. On the other hand, there is a problem with offset. The shifts are caused by the ratio of the stress concentration factors. The modification that takes into account the stress concentration factors in the predictions (Eqn. 2) is adapted from the approach designed for metallic materials, for which it leads to quite reliable predictions. Another type of equation may be more suitable for durable semicrystalline polymers like PE2. However, the predicted curves still provide a relatively good idea about the fatigue behavior of the notched specimens.

CONCLUSION

In this paper, a method for fatigue lifetime predictions of notched polyethylene specimens is presented. The method is based on the principles of the theory of critical distances, specifically the Line Method, which averages stress over the (critical) distance from the notch tip. The presented approach is used on two similar types of high-density polyethylene, which differ mainly in their crack resistance. The main findings are listed here:

- The prediction method uses experimental fatigue data of CRB specimens and specimens with a model notch. From these two sets of experimental fatigue data and corresponding axial stress distributions, the critical distance is determined. The critical distance l_{cr} depends on the number of cycles to fracture. The distance ranges between 0.25-0.89 mm in the case of PE1, and between 0.61-1.75 mm in the case of PE2.
- The critical distance decreases with the increasing number of cycles to fracture in the case of PE1. On the other hand, the critical distance of PE2 increases with the increasing number of cycles to fracture which is a completely opposite behavior compared to previously examined carbon steel. The critical distance depends on the notch radius as well. The smaller the notch, the bigger the critical distance. For this reason, modification of the critical distance was introduced here for subsequent fatigue lifetime predictions. The critical distance has to be multiplied by the ratio of stress factors K_{tp}/K_{tm} (the ratio of the stress concentration factor of the predicted notch and the stress concentration factor of the model notch).
- It was found that, even with the introduced modification, fatigue lifetime predictions depend on the approximation of experimental data of model notch. The slope of the experimental approximation indicates the slope of the predictions. The offset of the curves is governed by the ratio of stress concentration factors.
- In the case of PE1, even though the predictions fall within the area of the confidence intervals, the slopes of predicted curves sometimes differ from the experimental data. These discrepancies may have been caused by an insufficient number of tested samples. The larger number of tested samples would improve the *S-N* curve approximations, and therefore, the slope of the predictions. In the case of PE2, the slope of the predicted curves was in line with the experimental curves. However, the position of the predicted curve was a little bit shifted, that was caused by the ratio of the stress concentration factors.

It was concluded that even though the predictions are not always precisely accurate, the approach with the presented modifications can be applied to HDPE. It produces results suitable for preliminary assessment of fatigue behavior of notched specimens and components. This method can lead to the reduction of tested samples and considerable time savings. However, the method still requires further work. Attention should be paid especially to the member that affects the offset of prediction curves and modify it so that it better reflects material properties.

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DATA AVAILABILITY

The data used in this study is available at: https://doi.org/10.5281/zenodo.13981386

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