A simplified nonlinear model for bamboo-reinforced concrete beams based on lumped damage mechanics

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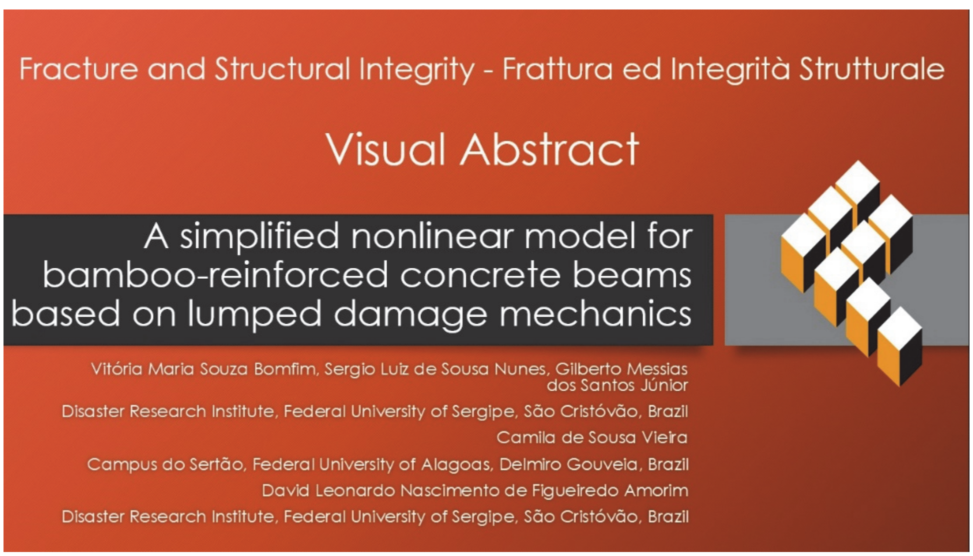

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INTRODUCTION

Bamboo is a renewable and sustainable material with high tensile strength that has been used as an alternative to steel as reinforcement of concrete elements [1]. The study of flexural performance of bamboo-reinforced concrete (BRC) beams in terms of load capacity, deflection and failure showed its feasibility [2-3]. However, to design BRC beams, it is necessary to consider the intrinsic characteristics of bamboo, it such as its reduced bond with concrete [4]. The behaviour and strength of BRC beams depend on the bond strength between bamboo and cement, the reinforcement ratio, the application of confinement, the presence of admixtures, and the strength of concrete [5]. Ibrahim et al. [6] tested experimentally bamboo-reinforced concrete beams subjected to flexural loads. The study verified the influence of bamboo's cross-sectional area on the beam's load-carrying capacity and the influence of its ultimate tensile strength on deflection.

Besides experiments, the behaviour of BRC elements is also analysed through numerical methods. Awoyera et al. [7] validated the experimental evaluation of flexural behaviour of large-scale BRC beams with finite element modelling performed using ABAQUS® software. They demonstrated that members reinforced with 50% bamboo, although with about 14% lesser strength but with minimal deformation and crack propagation, can also be a sustainable alternative for construction. Besides real-life experiments, Mondal et al. [8] utilised finite element numerical experiments to develop a load and resistance factor design framework for BRC beams. They showed that a strength reduction factor to consider the slippage of bamboo inside the concrete could be utilised in the design equation. These papers usually apply two- or three-dimensional finite element analysis, which might require considerable computational effort.

Alternatively, lumped damage mechanics (LDM) can be an interesting tool for approaching bamboo-reinforced concrete structures since it is based on key concepts of classic fracture [9-11] and damage mechanics [12]. LDM was originally developed for seismic analysis of conventional reinforced concrete frames [13]. Later, it was developed for different materials and load conditions [14-21]. Recently, LDM was extended to two-dimensional continuum media [22-23] and reinforced concrete slabs [24].

Note that other approaches might also be helpful in analysing reinforced (steel or bamboo) concrete structures, such as continuum damage and cohesive fracture approaches. Regarding continuum damage modelling, concrete damage plasticity (CDP) modelling is quite effective in analysing reinforced concrete structures under different load conditions [25-27]. Another option is to analyse complex concrete structures by cohesive fracture models [28-30]. Regardless of the accuracy of such approaches, lumped damage models may present more efficient simulations. According to Bosse et al. [31], when compared to CDP, lumped damage modelling of reinforced concrete structures demands computational resources approximately 10,000 times lower.

Therefore, this paper proposes a novel lumped damage model for bamboo-reinforced concrete beams. The proposed model is easy to implement and feasible for practical applications, especially if several numerical analyses are required, e.g., Monte Carlo simulations on structural reliability.

PROPOSED LUMPED DAMAGE MODEL FOR BAMBOO-REINFORCED CONCRETE BEAMS

Strain equivalence hypothesis and its application in bamboo-reinforced concrete beams

From classic damage mechanics, the first main concept to analyse is the effective stress. For the sake of simplicity, consider a uniaxial case. If the applied Cauchy stress (σ) implies in damaged material, the effective stress can be defined as:

$$\tilde{\sigma} = \frac{\sigma}{1-\omega} \tag{1}$$

where ω is the damage variable.

Then, the strain equivalence hypothesis states that an undamaged material can replace the damaged material submitted to a Cauchy stress state with the same strain state submitted by the effective stress (Fig. 1). Therefore, the elasticity law (Hooke's law) is rewritten using the effective stress, i.e.

$$\tilde{\sigma} = E\varepsilon \quad \Rightarrow \quad \varepsilon = \frac{\sigma}{(1-\omega)E} = \frac{\sigma}{E} + \frac{\omega\sigma}{(1-\omega)E} = \varepsilon^e + \varepsilon^d \tag{2}$$

being E the elasticity modulus and ε the total strain. Note that the total strain can be divided into an elastic (ε^e) and a damaged (ε^d) part.

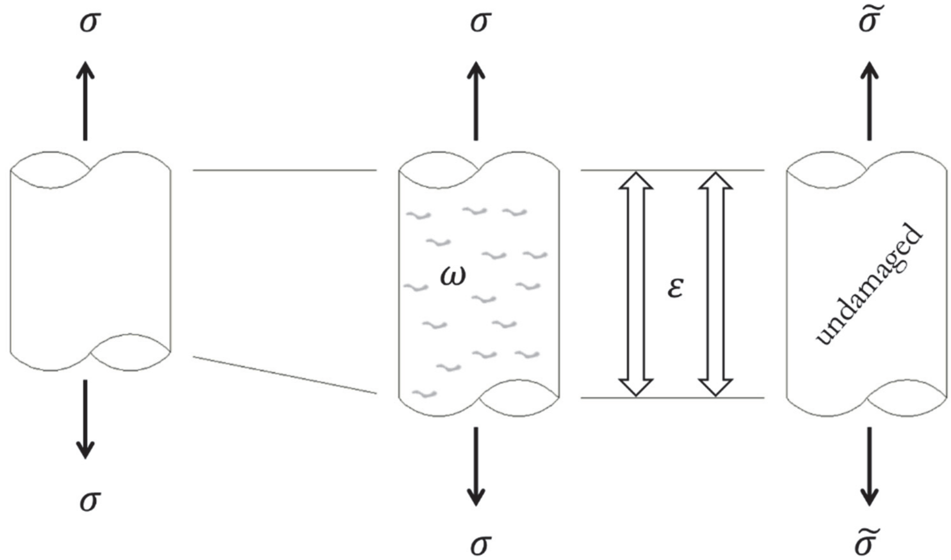

Figure 1: Strain equivalence hypothesis.

Now, considering a BRC beam element (Fig. 2), its deformed shape can be described by two relative rotations (ϕ_i and ϕ_j) at the elements' ends. Such relative rotations are assembled in the generalised deformations matrix as follows:

$$\{\mathbf{\Phi}\} = \{\varphi_i \quad \varphi_j\}^T \tag{3}$$

where the superscript T means 'transpose of'.
To consider the aforementioned concepts from classic damage mechanics, the generalised deformations matrix is described as a sum of two parts: elastic $\{\mathbf{\Phi}^{\mathbf{e}}\}$ and damaged one $\{\mathbf{\Phi}^{\mathbf{d}}\}$, i.e.

$$\{\mathbf{\Phi}\} = \{\mathbf{\Phi}^{\mathbf{e}}\} + \{\mathbf{\Phi}^{\mathbf{d}}\} \tag{4}$$

Note that the elastic part of the generalised deformations $\{\mathbf{\Phi}^{\mathbf{e}}\}$ represents the elastic behaviour of the beam element. On the other hand, the damaged part $\{\mathbf{\Phi}^{\mathbf{d}}\}$ takes into account the concrete cracking of the BRC beam element (Fig. 2), which is considered lumped at the element's ends, mathematically represented by inelastic hinges and calculated by damage variables (d_i and d_j). Note that no plastic deformations were considered since bamboo reinforcement does not yield.

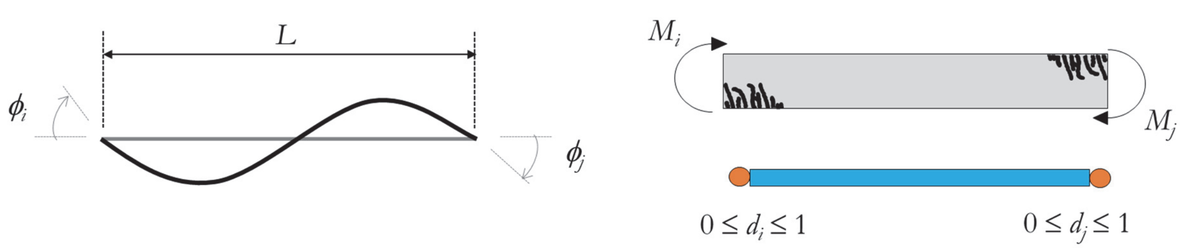

Figure 2: Generalised deformations (left) and stresses (right) of a beam element and the lumped damage of the inelastic hinges.

Therefore, considering an analogous relation as the one presented in (2), the elastic law for a beam element is:

$$\{\mathbf{\Phi}\} = \{\mathbf{\Phi}^{\mathbf{e}}\} + \{\mathbf{\Phi}^{\mathbf{d}}\} = [\mathbf{F}_0]\{\mathbf{M}\} + [\mathbf{C}(\mathbf{D})]\{\mathbf{M}\} = [\mathbf{F}(\mathbf{D})]\{\mathbf{M}\}$$

$$\text{where } [\mathbf{F}_0] = \begin{bmatrix} \frac{L}{3EI} & -\frac{L}{6EI} \\ -\frac{L}{6EI} & \frac{L}{3EI} \end{bmatrix}, \quad [\mathbf{C}(\mathbf{D})] = \begin{bmatrix} \frac{L\,d_i}{3EI(1-d_i)} & 0 \\ 0 & \frac{L\,d_j}{3EI(1-d_j)} \end{bmatrix}, \tag{5}$$

$$\text{and } [\mathbf{F}(\mathbf{D})] = \begin{bmatrix} \frac{L}{3EI(1-d_i)} & -\frac{L}{6EI} \\ -\frac{L}{6EI} & \frac{L}{3EI(1-d_j)} \end{bmatrix}$$

being $[\mathbf{F}_0]$ the elastic flexibility matrix, $[\mathbf{C}(\mathbf{D})]$ the compliance matrix due to concrete cracking in the inelastic hinges, $[\mathbf{F}(\mathbf{D})]$ the flexibility matrix, L the element's length, EI the element's flexure stiffness, and $\{\mathbf{M}\}$ the generalised stress matrix, which contains two bending moments located at the elements' ends (M_i and M_j):

$$\{\mathbf{M}\} = \{M_i \quad M_j\}^T \tag{6}$$

Note that $[\mathbf{C}(\mathbf{D})]$ is null if both damage values are zero, and its main diagonal tends to infinity if both damage values tend to one, which reproduces perfect hinges. Both terms in the main diagonal are obtained directly from the strain equivalence hypothesis (2).

Proposed lumped damage model

The complementary energy (W) of the BRC beam element is given by:

$$W = \frac{1}{2}\{\mathbf{M}\}^T\{\mathbf{\Phi}\} = \frac{1}{2}\{\mathbf{M}\}^T[\mathbf{F}(\mathbf{D})]\{\mathbf{M}\} \tag{7}$$

The damage driving moments (G_i and G_j) are obtained by differentiating the complementary energy with respect to the damage variables, i.e.

$$\begin{aligned} G_i &= \frac{\partial W}{\partial d_i} = \frac{M_i^2 L}{6EI(1-d_i)^2} \\ G_j &= \frac{\partial W}{\partial d_j} = \frac{M_j^2 L}{6EI(1-d_j)^2} \end{aligned} \tag{8}$$

Since the damage variables represent concrete cracking, the damage evolution laws for each inelastic hinge are defined by a Griffith generalised criterion, such as follows:

$$\begin{aligned} &\begin{cases} \Delta d_i > 0 & \Rightarrow \quad G_i = \mathrm{R}(d_i) \\ G_i < \mathrm{R}(d_i) & \Rightarrow \quad \Delta d_i = 0 \end{cases} \\ &\begin{cases} \Delta d_j > 0 & \Rightarrow \quad G_j = \mathrm{R}(d_j) \\ G_j < \mathrm{R}(d_j) & \Rightarrow \quad \Delta d_j = 0 \end{cases} \end{aligned} \tag{9}$$

where $R(d_i)$ and $R(d_i)$ are the cracking resistance functions for both inelastic hinges.

The cracking resistance function must account for this behaviour since BRC beams tend to be more deformable than conventional ones (steel). Therefore, the proposed cracking resistance function for BRC beams is:

$$R(d) = R_0 + k\frac{\ln(1-d)}{(1-d)}\left[\exp(d)\right]^5 \quad (10)$$

being R_0 and k model parameters.
Note that the model parameters present clear physical meaning since if there is no concrete cracking, i.e. $d = 0$, the parameter k exerts no influence on the cracking resistance; then R_0 is defined as the initial cracking resistance. On the other hand, while concrete cracking propagates, i.e. $d > 0$, the parameter k inserts a second term in the cracking resistance function, which is responsible for increasing the cracking resistance due to the bamboo reinforcement. Finally, the exponential term introduces the necessary deformability for the BRC beams. This characteristic can be observed in a bending moment *vs.* damage curve, as the one presented in Fig. 3. It is worth noting that there is an ultimate damage value (d_u) related to the maximum bending moment (Fig. 3). Numerically the bending moment *vs.* damage curve goes up to $d \rightarrow 1.00$; however, the ultimate damage value (d_u) is considered as the maximum damage value, which is related to the bamboo slippage i.e. collapse. A better explanation about the model parameters (R_0 and k) is presented in the Appendix of this paper.

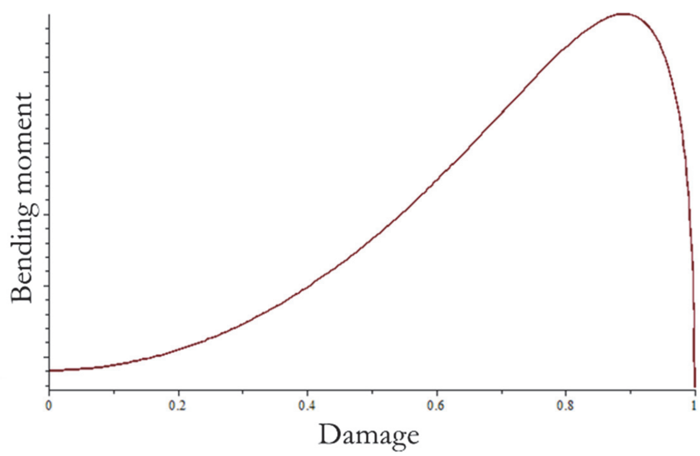

Figure 3: Bending moment vs. damage for a bamboo-reinforced concrete beam.

Notwithstanding, the model parameters are directly related to BRC properties. For any BRC beam, it is possible to calculate the first cracking moment (M_r) and the ultimate bending moment (M_u). First, by equalling (8) and (10), we have the cracking propagation criterion:

$$\frac{M^2 L}{6EI(1-d)^2} = R_0 + k\frac{\ln(1-d)}{(1-d)}\left[\exp(d)\right]^5 \quad (11)$$

Considering the beginning of concrete cracking, $M = M_r$ and $d = 0.0$ in the previous equation. Then, the initial cracking resistance (R_0) is defined as:

$$\frac{M_r^2 L}{6EI} = R_0 \quad (12)$$

The ultimate bending moment (M_u) is reached for the load-bearing capacity, and the damage presents an ultimate value d_u (Fig. 3).

$$\frac{M_u^2 L}{6EI(1-d_u)^2} = \frac{M_r^2 L}{6EI} + k\frac{\ln(1-d_u)}{(1-d_u)}\left[\exp(d_u)\right]^5 \quad (13)$$

Since the previous equation presents two unknown variables (k and d_u), the second equation is:

$$\left.\frac{\partial M}{\partial d}\right|_{d=d_u} = \frac{M_r^2 L}{3EI}\left(1-d_u\right) + k\left\{\left(5d_u - 4\right)\left[\exp\left(d_u\right)\right]^5 \ln\left(1-d_u\right) + \left[\exp\left(d_u\right)\right]^5\right\} = 0 \quad (14)$$

Then, by solving the nonlinear system composed of equations (13-14), the parameters k and d_u are found.

Finite element analysis

Regarding the lumped damage framework, the local analysis in the finite element programming usually splits the classic stiffness-based formulation into three matrix equations, as illustrated in Fig. 4. Note that the generalised displacements $\{\mathbf{U}\}$ are obtained throughout usual finite element analysis for any load step. Then, the generalised deformations matrix is calculated by the kinematic relation, expressed as follows:

$$\{\mathbf{\Phi}\} = [\mathbf{B}]\{\mathbf{U}\}; \quad \text{where } [\mathbf{B}] = \begin{bmatrix} \frac{\sin\alpha}{L} & -\frac{\cos\alpha}{L} & -\frac{\sin\alpha}{L} & \frac{\cos\alpha}{L} \\ \frac{\sin\alpha}{L} & -\frac{\cos\alpha}{L} & -\frac{\sin\alpha}{L} & \frac{\cos\alpha}{L} \end{bmatrix} \quad (15)$$

where α is the beam's inclination. The generalised stresses matrix as well as the damage variables can be calculated by the elasticity law (5), being the damage evolution (9) calculated by a simple prediction-correction algorithm. Finally, the external forces $\{\mathbf{P}\}$ are balanced by the element's internal forces i.e.

$$\{\mathbf{P}\} = [\mathbf{B}]^T \{\mathbf{M}\} \quad (16)$$

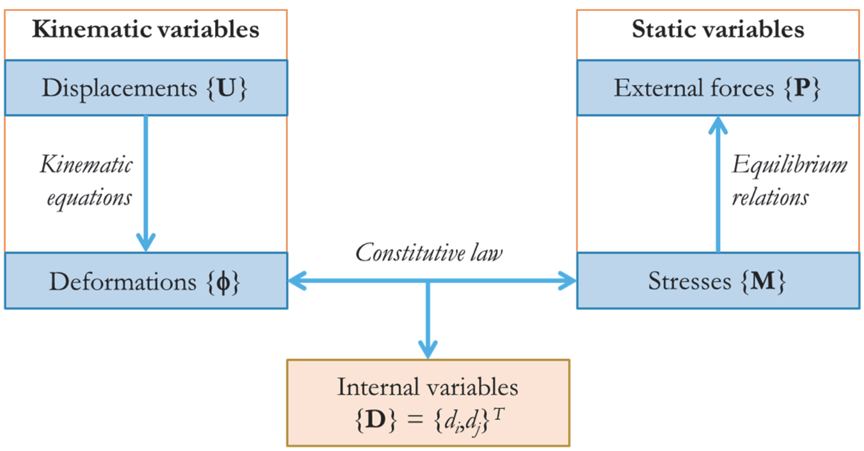

Figure 4: Local finite element analysis.

If convergence is achieved, the global analysis moves to the next load step, where the damage variables penalise the element's stiffness matrix, obtained by substituting (5) and (15) into (16).

RESULTS AND DISCUSSION

This section compares the proposed model to experiments on BRC beams to analyse its accuracy.

BRC beams from Mondal et al. [8]

Mondal et al. [8] tested three sets of BRC beams with different reinforcement ratios: 1.5% (BRCB-1), 2.5% (BRCB-2), and 3.7% (BRCB-3). The characteristic concrete compressive strength was equal to 20MPa for all beams. All beams present a cross-section of 20cm × 25cm and were submitted to a four-point bending test, where the effective span is 2.7m (Fig. 5). The distance between two consecutive load points is the same, i.e. 90cm. The displacements of all beams were measured at mid-span by an LVDT. By taking advantage of the problem's symmetry, only two finite elements were needed (Fig. 5). For the tested beams [8], the first cracking moment (M_r) and the ultimate bending moment (M_u), as well as the model parameters R_0 and k, are given in Tab. 1. Note that the finite element between the support and the applied load is defined as 'element #1' and the finite element between the applied load and the symmetry support is named 'element #2'. Both elements presented the same ultimate damage value for each beam's set: 0.8899 for BRCB-1, 0.8900 for BRCB-2, and 0.8901 for BRCB-3. The comparison between the numerical and experimental results is depicted in Fig. 5. For the three sets, the numerical results present good agreement with the experimental envelope, especially considering the BRC beams' load-bearing capacity. The numerical failure, i.e. when the damage reaches its ultimate value, was quite close to the experiments.

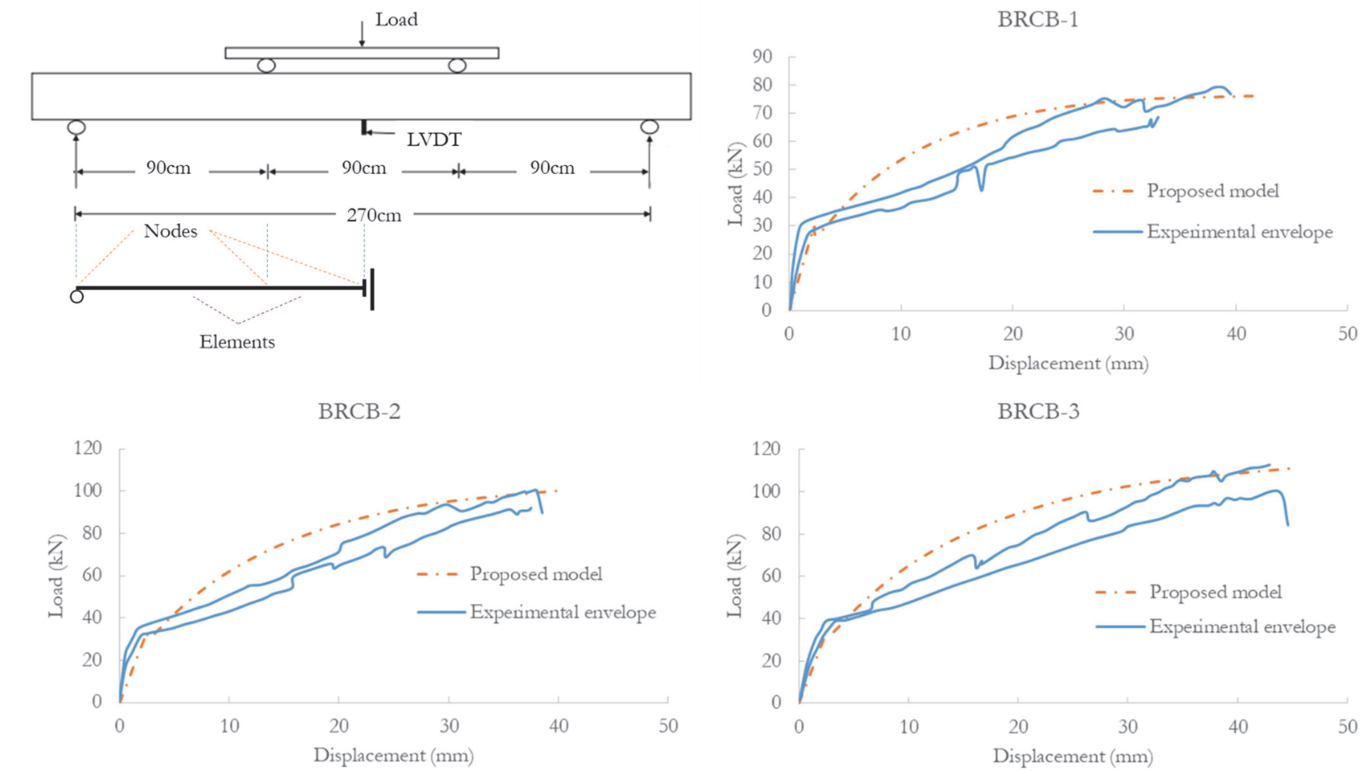

Figure 5: Test set-up [8], mathematical model and numerical results compared to the experimental ones.

Beams	M_r (kN.mm)	M_u (kN.mm)	Element #1		Element #2	
			R_0 (kN.mm)	k (kN.mm)	R_0 (kN.mm)	k (kN.mm)
BRCB-1	13,500.0	34,200.0	5.826	-1.795	2.913	-0.897
BRCB-2	15,007.5	46,107.0	7.199	-3.264	3.600	-1.632
BRCB-3	15,606.0	51,115.5	7.785	-4.013	3.892	-2.006

Table 1: Model parameters for the finite element analysis.

BRC beams from Mali and Datta [32]

Mali and Datta [32] tested several BRC beams under different reinforcement ratios. Only three of those beams showed flexure failure. Such beams have a cross-section of 14cm × 15cm and were submitted to a four-point bending test, where the effective span is L = 1.10m (Fig. 6). By taking advantage of the problem's symmetry, only two finite elements were used (Fig. 6). The longitudinal reinforcement ratio is 3.8%. For the tested beams [32], the first cracking moment (M_r) and the ultimate bending moment (M_u) are 3,300.00kN.mm and 8,121.67kN.mm, respectively. Therefore, the model parameters are R_0 = 0.939kN.mm and k = -0.273kN.mm for the finite element between the support and the applied load, and R_0 = 0.469kN.mm and k = -0.137kN.mm for the finite element between the applied load and the symmetry support. Both elements presented the ultimate damage value of 0.8899. The comparison between the numerical and experimental results is depicted in Fig. 6. Note that the proposed model is well-fitted to the experimental envelope. On the other hand, numerical failure, i.e., when the damage reaches its ultimate value, occurs later than the experimental behaviour.

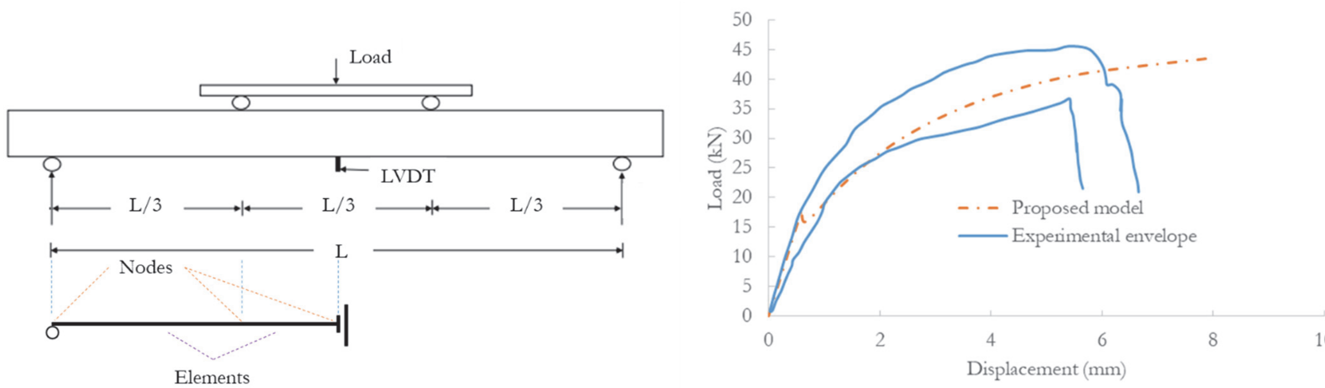

Figure 6: Test set-up [32], mathematical model and numerical results compared to the experimental ones.

Conclusions

This paper proposes a novel lumped damage model for bamboo-reinforced concrete (BRC) beams. Its accuracy was verified by comparing it with experimental results of BRC beams submitted to four-point bending tests available in the technical literature. The experimental load vs. displacement curves were compared with the results obtained with the proposed model, where the good agreement of the curves demonstrates the model's accuracy. The numerical analyses ended when the damage variable reached its ultimate value, characterising the bamboo slippage. The ultimate damage value of each beam was calculated, and it was quite close to the experiments regarding load vs. displacement curves. Note that only two finite elements were necessary to achieve a good agreement with the experiments, showing the model's feasibility for practical applications. Such characteristics are advantageous when considering structural reliability analysis, such as Monte Carlo.

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APPENDIX A

Lumped damage models for conventional (steel) reinforced concrete elements usually uses the following relation for the cracking resistance function:

$$R(d) = R_0 + q\frac{\ln(1-d)}{(1-d)} \tag{A.1}$$

where R_0 and q are model parameters (see e.g. [20] for a brief review). On the other hand, the novel lumped damage model for bamboo-reinforced concrete beams presented in this paper uses (10) as the cracking resistance function, written again as follows just for convenience:

$$R(d) = R_0 + k\frac{\ln(1-d)}{(1-d)}\left[\exp(d)\right]^5 \tag{A.2}$$

where R_0 and k are model parameters.
Firstly, note that R_0 is the same term for both equations, which can be defined as the initial cracking resistance of the gross cross-section, i.e. R_0 is related to the first cracking moment (M_r). The terms q and k from the previous equations are related to the additional cracking resistance introduced by both types of reinforcement. Note that both terms, q and k, are related to reinforcement ratio and resistance. Now, focusing on (A.2), it is well-known that the bamboo reinforcement does not yield and does not reach its ultimate strength since slippage and debonding from concrete usually occur first. Therefore, the parameter k is also related to the ultimate bending moment (M_u) of the bamboo-reinforced cross-section, which accounts for slippage and debonding. Finally, yet importantly, since bamboo-reinforced concrete beams usually present more ductility than equivalent beams with conventional (steel) reinforcement [8], an exponential term based on the damage variable is added to penalise the reinforcement parameter k to reproduce the ductility behaviour of bamboo-reinforced concrete beams [8]. Therefore, for a simple comparison, assume that a bamboo-reinforced concrete beam presents exactly the first cracking moment (M_r) and ultimate bending moment (M_u) of another beam with conventional reinforcement. Once R_0, q, and k are properly obtained, the ultimate damage for the conventional reinforced concrete beam is around 0.6-0.65, while the bamboo-reinforced concrete beam presents the ultimate damage at around 0.8-0.9. The initial damage propagation for the bamboo-reinforced concrete beam explains this difference (Fig. A.1).

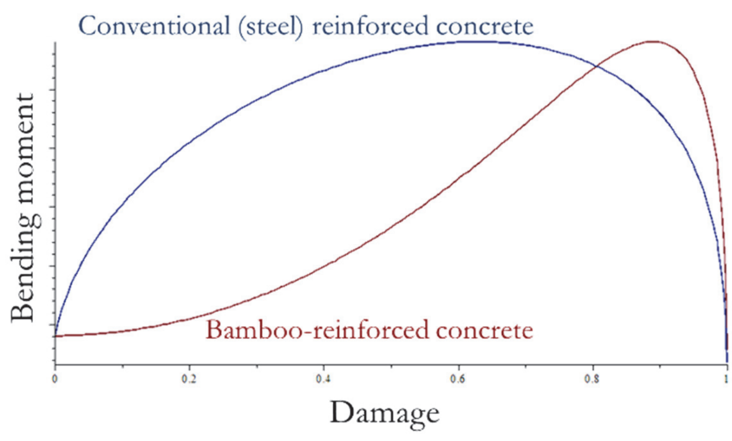

Figure A.1: Bending moment vs. damage for steel- and bamboo-reinforced concrete beams.

Now, in order to present a better interpretation of the parameter k, the following relation is obtained by rearranging (13):

$$k = \frac{\frac{L}{6EI}\left[\frac{M_u^2}{(1-d_u)^2} - M_r^2\right]}{\frac{\ln(1-d_u)}{(1-d_u)}\left[\exp(d_u)\right]^5} \tag{A.3}$$

Note that the parameter k is related to M_u, M_r and d_u. The numerator presents a difference between the square of the effective ultimate bending moment and the square of the first cracking moment. Since d_u is related to the neutral axis position, both concrete compressive resistance and reinforcement ratio also play a role in parameter k.

Finally, differently from the lumped damage models for conventional reinforced concrete elements, the proposed model for bamboo-reinforced concrete beams does not consider plastic deformations. This assumption was made to avoid unnecessary uncertainties since mono-sign cyclic tests were not found, however, it introduces limitations to the proposed model. Nevertheless, the proposed methodology may be useful for practical applications, where the actual behaviour of bamboo-reinforced concrete beams can be estimated with satisfactory accuracy.