An innovative analytical approach for predicting the fundamental time period of moment-resisting frames

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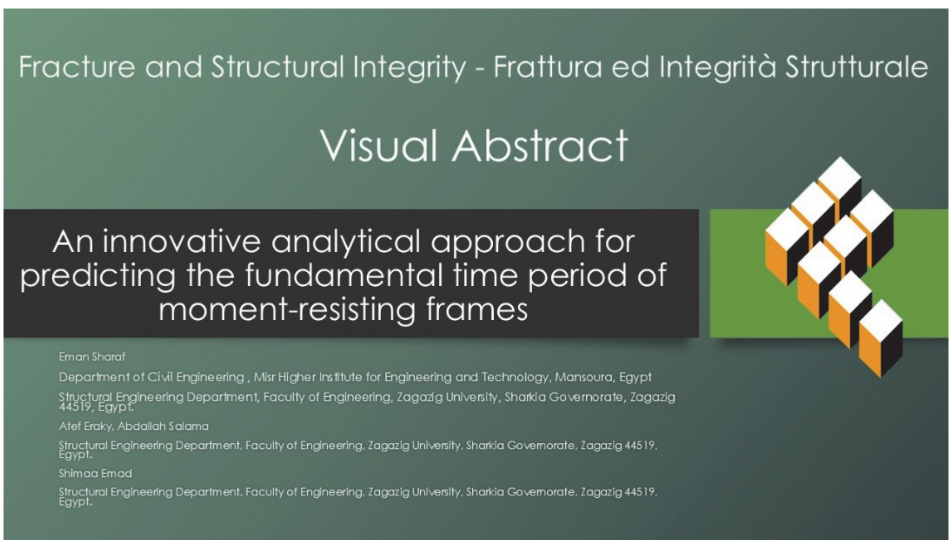

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INTRODUCTION

The fundamental period of vibration of a building is one of the important parameters appearing in the different building codes to compute the design base shear and lateral loads. The empirical formulae found in codes are mostly based on the building type (e.g., frame or shear wall), overall size, and construction material (e.g., steel or reinforced concrete). The determination of this period is critical since it has a direct impact on the calculation of seismic loads for improving both the accuracy and reliability of the building design process.

Almost all the formulas currently found in the "Uniform Building Code" (UBC) [1], the "Structural Engineers Association of California" (SEAOC) [2] recommendations, and the "Egyptian Code" (EGC) [3] are primarily derived from building vibration data recorded during the 1971 San Fernando earthquake. Later, in 1996 and 1997, Goel and Chopra [4, 5] evaluated existing empirical formulas and subsequently proposed new empirical formulas to estimate the fundamental vibration period of reinforced concrete moment-resisting frame (RC MRF) buildings. These methods were developed for use in equivalent lateral force analysis, as required by building codes, utilizing motion data from various structures recorded during multiple earthquakes. The data analyzed by [4] were obtained from earthquakes, including the 1971 San Fernando, 1984 Morgan Hill, 1986 Mt. Lewis and Palm Springs, 1987 Whittier, 1989 Loma Prieta, 1990 Upland, 1991 Sierra Madre, and 1994 Northridge events.

Salama [6] also updated formulas for the estimation of the fundamental vibration period by using regression analysis on data from eight recorded earthquakes in California. Their formulas account for both the building height and the number of stories in the moment-resisting reinforced concrete frames. Several other researchers have also proposed an adjustment to the Ct coefficient based on the number of stories, according to the formulas in American and Egyptian building codes. Similarly, Kalpan et al. [7] emphasized that the empirical formulas in building codes should be based on the particular region in which the typical design and architectural character of local construction are exhibited. In this regard, they derived a simplified formula to estimate the fundamental elastic period of mid-rise reinforced concrete buildings and compared it with the formula available in the "Turkish Earthquake Code" (TEC) [8]. Their study indicated that the empirical formula in the code yields non-conservative base shear values. They also investigated the contribution of infill walls to the lateral stiffness of buildings and offered preliminary recommendations based on their findings.

Several researchers, including [9, 10, 11, 12, 13, 14, 15, 16], have derived simplified equations based on building height by using ambient vibration measurements to estimate the fundamental period of buildings. Mohamed et al. [17] conducted nonlinear dynamic analyses to estimate the fundamental period of mid-rise moment-resisting RC frames. In their study, the results were compared with the period equations given by Salama and "American Society of Civil Engineers: Minimum Design Loads for Buildings and Other Structures" (ASCE 7-16) [6, 18]. They developed several formulas from the regression analysis of their test results to estimate the period of vibration. They also supported the application of rational approaches, like modal analysis, to calculate the fundamental period of RC frames and, in this way, remove some of the restrictive code-based limits.

Young and H. Adeli [19, 20] criticized the methodology for determining the estimated fundamental period of moment-resisting frames obtained from ASCE 7-16 [18] as being overly conservative. They emphasized that this approach doesn't fully account for geometric irregularity in structures, which may greatly affect the structural responses during seismic events. Due to this deficiency, an extensive study was carried out on the fundamental period of moment-resisting frames with various types of geometric irregularities. They developed several formulas that took into consideration both vertical and horizontal irregularities; hence, these formulas became more accurate and flexible methods of estimating the fundamental period.

The degree of irregularity in a stepped building frame is estimated in [21], using the Rayleigh approach , and suggested modifications to the empirical formula used in building codes for estimating the fundamental period of such stepped frames. However, the calculation of the regularity index, as proposed by them, is complex and time-consuming. Ricci et al. [22] studied the elastic vibration period of uncracked and infilled reinforced concrete moment-resisting frame buildings. They employed a modal analysis approach to assess the structural response and subsequently offered simplified formulas to estimate the fundamental period. These formulas were based on regression analysis of numerical data, providing a practical alternative for efficient computation.

An equation was proposed for estimating the fundamental vibration period of regular frame structures with a maximum of ten stories, aiming to improve the empirical expressions widely used in building codes [23]. They utilized structural features such as stiffness and mass distribution, which enable more accurate estimations to be obtained by getting a correction factor based on effective mass and height, and the effective lateral stiffness of the structure. They also saw its applicability to fairly irregular frame structures with minimal loss of accuracy.

Alrudaini [24] critically examined the estimation of the fundamental vibration period (T) of reinforced concrete moment-resisting frame (RC-MRF) buildings, noting the limitation in existing seismic design codes. Based on regression analysis of recorded motions from RC moment-resisting frame (RC-MRF) buildings during eight California earthquakes, an improved formula was proposed in [24] that implicitly accounts for the floor height (h) as a significant parameter by incorporating both the total building height (H) and the number of stories (N).The study suggests the modification of the coefficient Ct in the current code equations as a function of N to enhance the precision of T estimates.

Current studies have focused more on improving the precision of fundamental period estimation of steel and reinforced concrete structures, particularly in terms of complex structural properties such as height, irregularity, and bracing systems.

Kumar et al. [25] performed a comparative analysis of machine learning algorithms, including artificial neural networks (ANN), genetic programming (GP), and regression trees (RT), demonstrating that ANN models exhibited superior predictive accuracy, especially in the presence of geometric irregularities.

Likewise, Rahman et al. [26] employed interpretable machine learning methods with SHapley Additive exPlanations (SHAP) to determine the significant structural parameters influencing the fundamental period in steel-braced reinforced concrete buildings and the way its configurations have significant influences on its dynamic behavior. In order to accurately capture the uncertainties associated with aging and material degradation, Shan et al. [27] developed probabilistic machine learning techniques to forecast the natural period of existing high-rise reinforced concrete buildings. Moreover, Karampinis et al. [28] created hybrid analytical-machine learning models that used SHAP values to generate interpretable equations for framed structures. This allowed them to preserve engineering transparency while achieving prediction accuracy on par with purely data-driven methods. These developments highlight a growing movement to improve period estimation, especially for complex and irregular structures, by combining data-driven techniques with conventional analytical models.

Most seismic design codes recommend empirical equations for the estimation of the fundamental vibration period of buildings. However, it has been demonstrated through several studies that these code-based equations typically underestimate significantly the actual measured periods, as stipulated by Goel and Chopra [4]. They once again highlighted the inadequacy of the code provisions available by citing that the simple equations that are commonly used can vary as much as 100% from observed values. These broad variations can lead to excessively conservative estimates of base shear force, thus structurally over-strengthened and uneconomical designs. Although several equations have been presented both in design codes and in the literature, many are empirical and usually derived from regression analyses of small datasets. These formulations often neglect the physical parameters most critical to dynamic behavior. Although many recent studies have also looked into data-driven and machine learning strategies can provide high predictive accuracy, their applicability and reliability in engineering practice are limited by the need for large, high-quality datasets and the possibility of overfitting or a lack of physical interpretability. Furthermore, they can be computationally intensive and require specialized knowledge to implement, which makes them less useful for everyday design. Hence, there is an evident necessity for a more logical and physically based model, including the equivalent mass and stiffness, two of the most significant parameters that control the fundamental period, precisely to increase both accuracy and usability of period estimation techniques.

This paper proposes a novel analysis procedure for the accurate estimation of the fundamental period of vibration of reinforced concrete moment-resisting frames. The equivalent seismic mass and the building's lateral stiffness are both included in the proposed procedure for higher accuracy than the current procedure. This approach differs from the previous approaches because it provides an easier process to compute the equivalent stiffness and equivalent mass of a multi-story building and also computes its fundamental period with accuracy, which gives more accurate results, and the resulting equations are in the form of closed-form expressions. Sensitivity analysis was carried out to investigate the influence of significant design parameters such as building height, span length, beam-to-column stiffness, and material elasticity. This procedure not only serves to confirm the strength and validity of the proposed equation but also establishes the key parameters that need to be taken into consideration in the design. It confirms the proposed model as being stable and representative of a wide range of buildings, thus making it more generalizable and usable in practice.

FORMULAS FROM THE CODES AND LITERATURE

The empirical formulas for the fundamental vibration period of moment-resisting frames (MRFs), adopted by most design codes, including U.S. building codes, UBC [1], the "Applied Technology Council" (ATC,) [29], SEAOC, [2], the "National Earthquake Hazards Reduction Program" (NEHRP) [30], and the more recent EGC [3], are generally in the form:

$$T_a = C_t H^{0.9} \quad (1)$$

In this empirical formula, H represents the total height of the building in meters, which is measured as the vertical distance from the ground level or foundation. Ct represents a numerical coefficient related to the lateral force-resisting system. Values adopted for Ct by the above codes are 0.047 for RC MRF and 0.051 for Steel MRF buildings. For some design codes, such as the NEHRP [30] regulations and earlier versions of other seismic codes, including the EGC [3], an alternative formula is available for RC MRF buildings. This formula expresses the fundamental period as:

$$T_a = 0.1N \tag{2}$$

The simple formula is restricted to structures with a height limit of a story of 10 feet and a height of not more than 12 stories, where N denotes the number of stories. Salama [6] provides the estimation of the fundamental period of vibration (T) of reinforced concrete moment-resisting frame buildings, one of the more important parameters in seismic design. Conventional seismic codes, such as ASCE [18] and EGC [3], estimate T using empirical equations that account for either building height (H) or the number of stories (N) individually, not both together. An improved empirical formula that includes both H and N, based on regression analysis of the data for moment-resisting frame reinforced concrete buildings experiencing eight California earthquakes was suggested in [6]. The method is intended to give an improved estimate of T by considering variations in floor heights and the number of stories. The study recommends that the coefficient Ct of the current code formulas be modified for the effect of N in order to make the correlation between estimated and observed vibration periods even better.
Salama [6] adopted a theoretical form of the period equation that included both the number of stories (N) and the total height (H) of the building

$$T = a.N^b.H^c \tag{3}$$

in this approach, the constants a, b, and c as variables in an unconstrained regression analysis, allowing the optimal fit of statistical data to measured data. This method minimized the standard error of the estimate, resulting in:

$$T = 0.027.N^{0.17}.H^{0.74} \tag{3-a}$$

where T is the fundamental period of vibration (in seconds), N is the number of stories, and H is the height of the building (in meters). Aninthaneni et al.[23] derived an improved analytical method to forecast the fundamental period (T) of regular reinforced concrete (RC) moment-resisting frame buildings in the Bulletin of the New Zealand Society for Earthquake Engineering. The recommended equation estimates T more correctly than empirical equations by considering seismic weight, storey height, number of storeys, bays, and effective stiffness. The method is validated against eigenvalue analysis and experimental findings on low- to mid-rise RC frames, even with minor deviation.
The improved empirical formulas proposed are:

$$T = 0.47.\varphi_3.\sqrt{\frac{W_s.h}{3n_s[(1-\beta_d)^3 + \lambda(1-\beta_c)^3](n_b+1)E_c I_{cef}}} \tag{4}$$

In this equation, ϕ_3 is a correcting factor that takes into account the effective distribution of mass and height in the structure. The variable Ws is the total seismic weight of the frame, and h is the storey height, or the height between two consecutive floors. The storey number is denoted by n_s, meanwhile, n_b is the total number of bays in the frame. The parameters β_d and β_c are the depiction of the distribution of stiffness over the building height for the beam and column systems, respectively. The ratio of the beam-to-column stiffness is given by λ, and the material properties are described by E_c, the modulus of elasticity of the concrete, and I_{cef}, the effective moment of inertia of the columns. The model enables the prediction of the fundamental period of the building, considering both material and geometric properties, with better accuracy compared to standard empirical approaches for common moment-resisting frame-type structures.
Goel and Chopra [4] experimentally validated the equations provided in U.S. seismic codes for approximating the fundamental period of vibration (T) of buildings. Based on the assessment of observed periods of 106 buildings that experienced eight California earthquakes from 1971 through 1994, they discovered that the current code equations underestimate the fundamental time period. By regression analysis of this expanded database, [5] developed better empirical equations that more accurately estimate the natural periods of reinforced concrete (RC) and steel moment-resisting frame (MRF) buildings. These improved equations had a significant impact in revising seismic design provisions in, for example, the NEHRP [31] and ASCE 7-05 [32] standards, and are expressed as:

$$T = 0.0466H^{0.9} \tag{5}$$

Here, T is the fundamental period of vibration of the building in seconds, and H is the height of the building in meters.

Most current design codes specify that the design base shear should be calculated using the following equation:

$$V = CW \tag{6}$$

where V is the design base shear, W is the total seismic dead load, and C is the seismic coefficient. The seismic coefficient C depends on several parameters: the soil profile, the seismic zone factor, the importance factor, the building's fundamental period T, and a numerical coefficient representing overstrength and global ductility capacity inherent in the lateral load-resisting system. The fundamental period Ta calculated by the empirical formulas from equations (1) and (2) is typically designed to be shorter than the true period in order to provide a conservative estimate of the base shear, as structures are required to be designed for larger seismic forces. Thus, code formulas are intentionally calibrated to underestimate the fundamental period by about 10–20%. Furthermore, while building codes allow the determination of the fundamental period using rational methods, such as Rayleigh's method [33], they also specify that the period computed based on these methods shall not exceed the period obtained from the empirical formula by more than a certain factor. In this way, consistency in seismic design is achieved, while there is provision for making more accurate calculations when required.

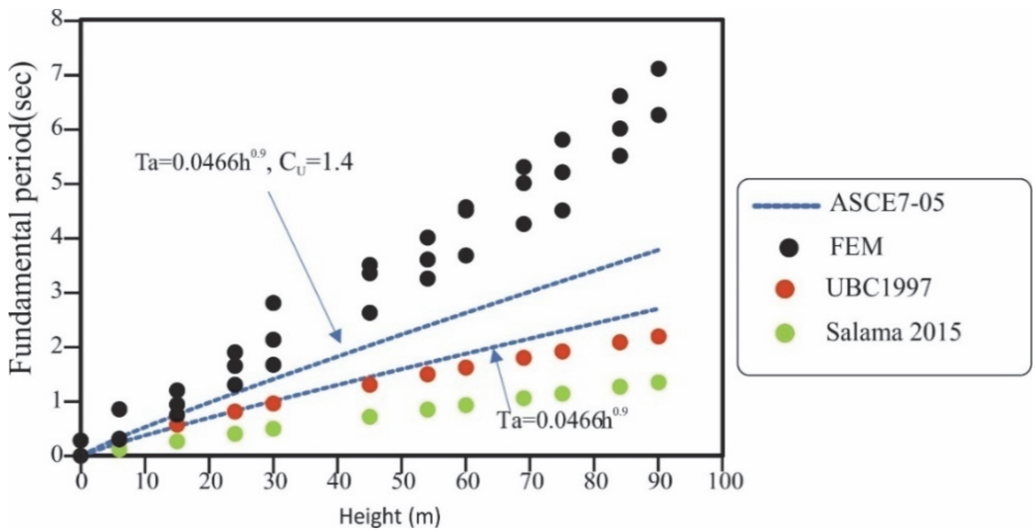

Figure 1: Comparaison of FEM and code periods for RC MRF buildings.

EFFECTIVENESS OF FORMULAS FROM CODES AND LITERATURE

To evaluate the validity of frequently used empirical formulas in identifying the fundamental period of structures, comparative study was performed between code expressions and FEM. The comparison given in Figure 1 shows a detailed assessment of accuracy of different empirical expressions to comprehensive finite element method (FEM) analysis for calculation of the fundamental period of structures for a broad range of heights. Specifically, the comparison incorporates empirical period formulae from UBC [1], ASCE 7-05 [32], and also Salama's equation [6] as written in Equations (1), (5) and (3) respectively. Figure 1 presents several important observations concerning the accuracy of these formulae relative to FEM results:

- FEM calculations consistently forecast longer fundamental periods than all empirical formulas, particularly for structures exceeding 30 meters in height. This reflects the enhanced flexibility represented in numerical models, which is typically underestimated in empirical formulations.
- ASCE 7-05 [32] calculates the period using the formula T_a=0.0466$h^{0.9}$, with an upper limit adjusted by C_u=1.4. While the upper bound curve somewhat aligns with the FEM results for mid-heights; nevertheless, it significantly underestimates periods for high-rise buildings, indicating inadequate applicability for tall structures.
- The ASCE 7-05 [32] code formula produces significantly shorter period estimates than those obtained from FEM analysis, particularly for buildings exceeding 45 m in height. At 90 m, the percentage error is approximately −45%, reflecting a rather large underestimation of the fundamental period and that the code may not capture accurately the dynamic behaviour of high-rise buildings.

- The UBC [1] predictions are relatively aligned with FEM results for low- to mid-rise structures (about 15–40 m), but diverge significantly as building height increases. It is obvious that the UBC formula is relatively adequate up to a height of 40 m, but is not reliable in the case of high-rise buildings.
- Salama [6] presents the most conservative of the three estimates, significantly underestimating the fundamental period for all heights of buildings. This shows that the model is probably missing the structural flexibility and cracking influences, and hence it overestimates the lateral stiffness.

The discrepancy between empirical predictions and FEM highlights the restrictions of generalized code equations, especially for irregular geometries or taller structures. This warrants calibration of empirical models with newer numerical data and consideration of FEM-based strategies in performance-based seismic design.

Estimation of the Fundamental Period of Vibration

Seismic design typically begins with estimating a trustworthy seismic force, usually based on the periods of the building's vibration. These periods are primarily influenced by the mass and stiffness of the building. In this case, seismic design often becomes a process of trial and error, since the relationship between seismic capacity, mass, and stiffness must be well recognized to ensure effective seismic performance. To avoid unnecessary trial and error, most standards include starting points for element size, construction orientation, number of levels, and material properties. These provide natural period calculations, and many formulas from codes and literature are used to calculate the building's seismic load. The fundamental period of the building, during which a significant portion of the mass participates in the associated mode of vibration, is by definition the natural period of vibration. While more complex formulas are available in building codes, simpler equations based on height or the number of stories are often recommended to streamline the calculation process.
Several studies have examined the effect of both structural and non-structural factors on the fundamental period of vibration in reinforced concrete buildings. Consequently, the empirical equations have revealed that the natural period is mostly determined by the building's height or number of stories. More specifically, the taller or higher a building is, the longer the period of vibration will be [34, 35]. These two properties, height and number of stories, are found to be the two most influential ones that determine a building's fundamental period. Consequently, most design codes rely on one of these two properties, preferably height or number of floors, in developing their empirical equations for the vibration period. Notable examples of such codes include the "building standard law of Japan" (BSLJ) [36], the National Building Code of Canada (NBCC, 1995) [37] the Uniform UBC, [1], and the ATC [29], all of which incorporate height or number of stories as key parameters in their period formulas.

A simplified technique for determining the fundamental period

The most practical approach to developing an analytical model of a real-frame structure is to model it as a skeleton with masses lumped at each floor level. In this section, a simplified technique is outlined for calculating the fundamental period of vibration of such structures. As previously mentioned, and as widely recognized, the period of vibration of a building depends on its mass and stiffness. For an idealized Single-Degree-of-Freedom (SDOF) system, the fundamental period is classically expressed by Equation (7), which defines the dynamic relationship between mass and stiffness.

$$T = 2\pi\sqrt{\frac{m}{K}} \tag{7}$$

Equation (7) forms the base of this study. An RC building can be modeled by a single-degree-of-freedom (SDOF) system using an equivalent lumped mass m_{eq} and equivalent stiffness k_{eq} as shown in Fig. 2. This idealization is concerned with the fundamental period of vibration and the interaction between the mass and stiffness in the system, making it much easier to analyze the dynamic behavior of the building.

Procedure for estimating the equivalent lateral stiffness of the frame (K_{eq})

The following explanation may help clarify the development of the current formula. Initially, the building is simplified to achieve a more accurate estimate of lateral stiffness. The proposed method begins by calculating the equivalent stiffness for a two-storey building, as shown in Fig. 3. Using the Rayleigh approach, the process for estimating the fundamental period of the building is formulated in this section.

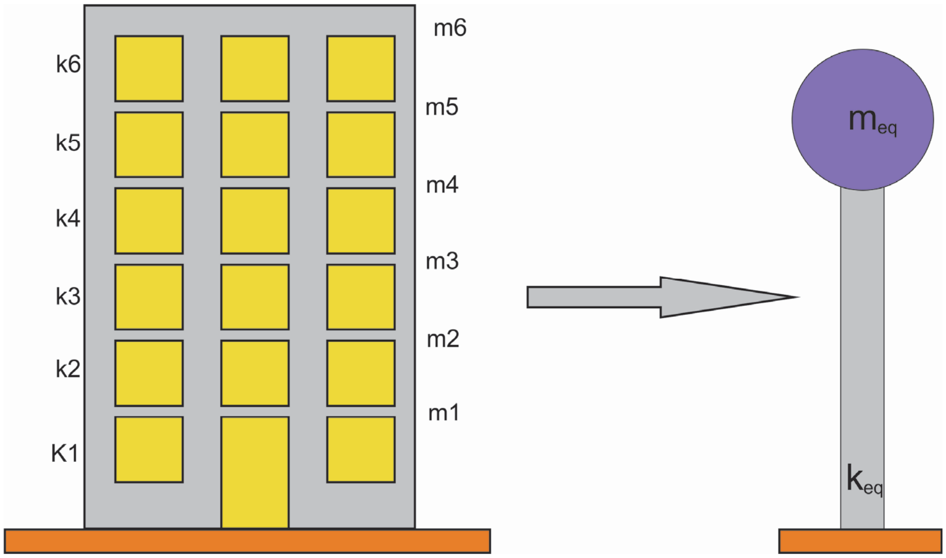

Figure 2: Equivalent cantilever column.

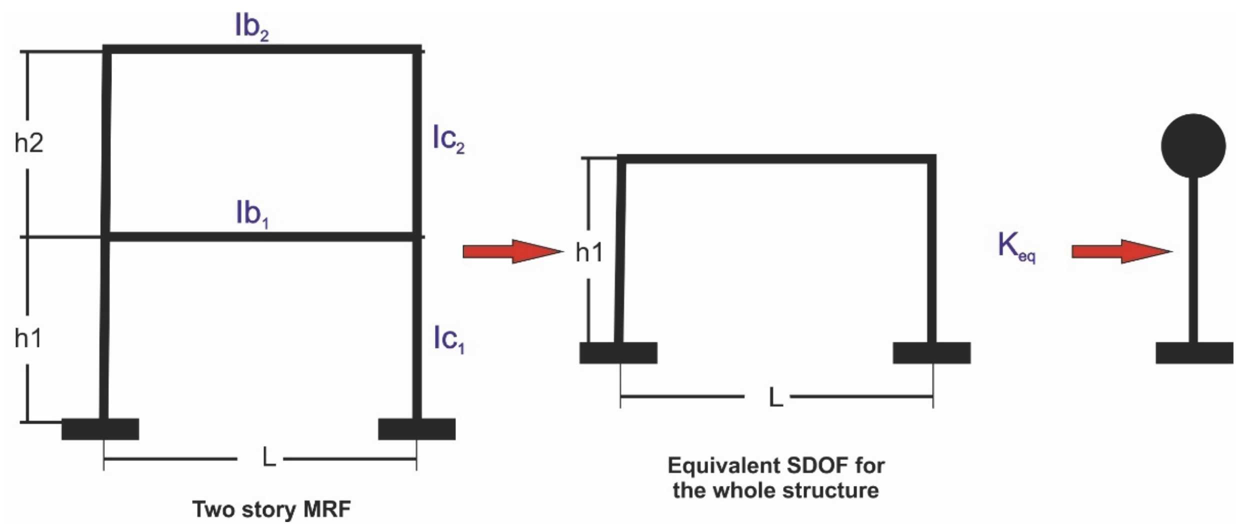

Figure 3: Configuration of MRF with simplification procedures.

To develop the shear stiffness matrix of the two-storey MRF, considering the stiffness of the beams and anti-symmetry, the flexibility matrix is developed first by applying two horizontal forces separately and calculating the horizontal deformations. The stiffness matrix of the frame due to translational and rotational degrees of freedom takes the form:

$$K=\begin{bmatrix} \frac{24EIc_1}{h_1^3}+\frac{24EIc_2}{h_2^3} & \frac{12EIc_1}{h_1^2}-\frac{12EIc_2}{h_2^2} & -\frac{24EIc_2}{h_2^3} & -\frac{12EIc_2}{h_2^3} \\ \frac{12EIc_1}{h_1^2}-\frac{12EIc_2}{h_2^2} & \frac{8EIb_1}{L}+\frac{8EIc_2}{h_2}+\frac{8EIc_1}{h_1} & \frac{12EIc_2}{h_2^2} & \frac{4EIc_2}{h_2} \\ -\frac{24EIc_2}{h_2^3} & \frac{12EIc_2}{h_2^2} & \frac{24EIc_2}{h_2^3} & \frac{12EIc_2}{h_2^2} \\ -\frac{12EIc_2}{h_2^3} & \frac{4EIc_2}{h_2} & \frac{12EIc_2}{h_2^2} & \frac{8EIb_2}{L}+\frac{8EIc_2}{h_2} \end{bmatrix} \quad (8)$$

To obtain the flexibility matrix, the horizontal deformations due to horizontal unit forces are obtained as follows:

$$f = T^T K T \tag{9}$$

where T is the transformation matrix that takes the form:

$$T_{trans} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \tag{10}$$

Then the shear stiffness matrix of the shown frame is obtained as follows:

$$K = f^{-1} \tag{11}$$

The structure mass is condensed to one mass at the first-floor level, then the structure mass matrix takes the form:

$$M = \begin{bmatrix} m_1 & 0 \\ 0 & 0 \end{bmatrix} \tag{12}$$

where m_1 is the equivalent mass of the structure.
Then the first natural frequency of this frame is obtained by:

$$\left| K - \omega_{eq1}^2 M \right| = 0 \tag{13}$$

Then the equivalent first-floor stiffness is obtained as follows:

$$K_{eq} = \omega_{eq1}^2 m_1 \tag{14}$$

which takes the following form:

$$K_{eq} = \frac{6EIc_1}{h_1^3}\left(\frac{Ic_1 Ic_2 L^2 + 4Ib_1 Ic_2 Lh_1 + 4Ib_2 Ic_1 Lh_2 + 4Ib_2 Ic_2 Lh_1 + 16Ib_1 Ib_2 h_1 h_2}{Ic_1 Ic_2 L^2 + Ib_1 Ic_2 Lh_1 + 4Ib_2 Ic_1 Lh_2 + Ib_2 Ic_2 Lh_1 + 4Ib_1 Ib_2 h_1 h_2}\right) \tag{15}$$

where K_{eq} is the lateral equivalent stiffness, E is the concrete modulus of elasticity, Ic_1 and Ic_2 are the moments of inertia of the first and second floors columns, respectively, L is the frame span, Ib_1 and Ib_2 are the moments of inertia of the first and second floors girders, respectively, and h_1 and h_2 are the heights of the first and second floors, respectively.
By substituting in equation (15) with $Ic_1=Ic_2$ and $Ib_1=Ib_2$ to obtain a more simplified form as follows:

$$K_{eq} = \frac{(6EIc_1(Ic_1^2 L^2 + 16Ib_1^2 h_1 h_2 + 8Ib_1 Ic_1 Lh_1 + 4Ib_1 Ic_1 Lh_2))}{(h_1^3(Ic_1^2 L^2 + 4Ib_1^2 h_1 h_2 + 2Ib_1 Ic_1 Lh_1 + 4Ib_1 Ic_1 Lh_2))} \tag{16}$$

By substituting into equation 16 with $I_{b1}=\alpha * Ic_1$.

$$K_{eq} = \frac{6EIb_1\alpha(16h_1 h_2 + L^2\alpha^2 + 8L\alpha h_1 + 4L\alpha h_2)}{(h_1^3(4h_1 h_2 + L^2\alpha^2 + 2L\alpha h_1 + 4L\alpha h_2))} \tag{17}$$

where α is defined as the beam-to-column moment of inertia ratio.
This method is applied on a broader scale to include multi-story frames, as shown in Fig. 4. The equivalent lateral stiffness is derived using the same equation, based on the sections and heights of the first and second floors. This indicates that the

upper floors have a negligible impact on determining the lateral stiffness, allowing for a simplified analysis focused primarily on the lower floors. This assumption helps streamline calculations while maintaining reasonable accuracy for structural behavior under lateral loads. It becomes clear that the previous equation for K_{eq}, expressed in terms of the properties and behaviors of the first two stories, can be generalized for multi-story structures with high accuracy.

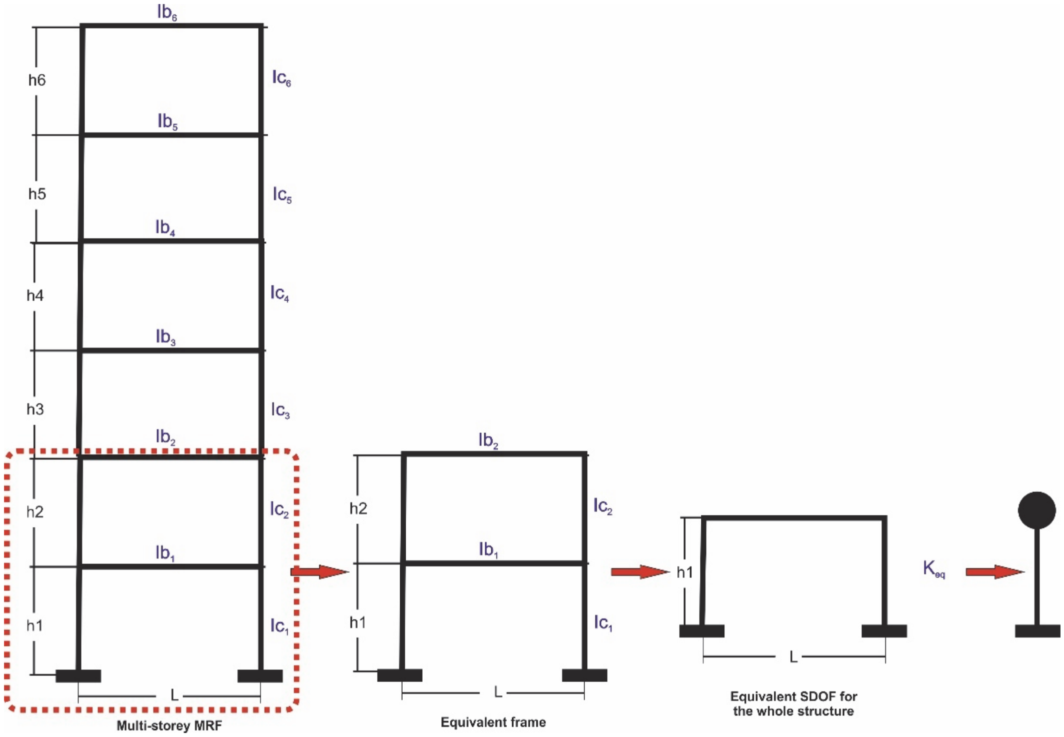

Figure 4: The simplification of a multi-storey frame into an SDOF.

Equivalent seismic mass of the multi-storey buildings

Equivalent seismic mass is a simplified representation of how each floor's weight contributes to the building's overall seismic response. It considers the distribution of mass and how it affects the structure's dynamic behavior during an earthquake. The sequence of the method is as illustrated in Fig. 5, where the following building consists of three floors. Two iterations are performed to obtain an equivalent mass concentrated at the level of the first floor. The first iteration involves shifting the mass of the third floor to the level of the second floor to obtain m_{eq1}. Then, the mass obtained from the previous iteration, m_{eq1}, is shifted to the level of the first floor to calculate m_{eq2}, resulting in a single equivalent mass at the first-floor level.

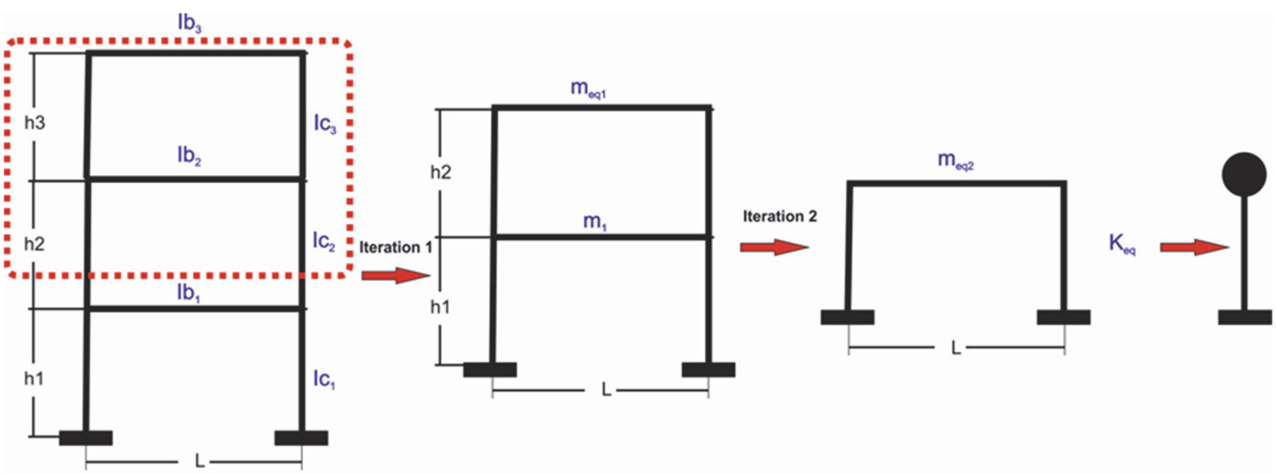

Figure 5: Procedures for obtaining equivalent seismic mass.

To perform this task, return again to the two-storey frame shown in Fig. 3, but the mass of the second floor is set by m_2, and the mass of the first floor is omitted; then the structure mass matrix takes the form:

$$M = \begin{bmatrix} 0 & 0 \\ 0 & m_2 \end{bmatrix} \tag{18}$$

Again, by substituting into the eigen equation:

$$\left| K - \omega_{eq2}^2 M \right| = 0 \tag{19}$$

Then the equivalent mass is expressed as:

$$m_{eq} = \frac{K_{eq}}{\omega_{eq2}^2} \tag{20}$$

To get the mass contribution factor, apply the following equation:

$$f_0 = \frac{m_{eq}}{m_2} = \frac{K_{eq}}{m_2 \omega_{eq2}^2} \tag{21}$$

By substituting into Eqn. (21) for $\alpha = I_b/I_c$ and $\beta_2 = L/h_2$, obtaining:

$$f_0 = \frac{(4\alpha^2 h_1^4 + 4\alpha^2 h_1 h_2^3 + 3\alpha\beta_2 h_1^4 + 4\alpha\beta_2 h_1^3 h_2 + 6\alpha\beta_2 h_1^2 h_2^2 + 8\alpha\beta_2 h_1 h_2^3 + \beta_2^2 h_1^3 h_2 + 3\beta_2^2 h_1^2 h_2^2 + 3\beta_2^2 h_1 h_2^2 + \beta_2^2 h_2^4)}{(h_1^3(4\alpha^2 h_1 + \beta_2^2 h_2 + 2\alpha\beta_2 h_1 + 4\alpha\beta_2 h_2))} \tag{22}$$

To simplify calculations, the formula can be modified using $h_1 = n \cdot h_2$ in Eqn. (22), where h_2 is the floor height, .h_1 is the height at which the mass has accumulated from the previous floor, and n is the floor number to which the mass was moved to simplify the calculation.

$$f_0 = \frac{(4\alpha^2 n^4 + 4\alpha^2 n + 3\alpha\beta_2 n^4 + 4\alpha\beta_2 n^3 + 6\alpha\beta_2 n^2 + 8\alpha\beta_2 n + \beta_2^2 n^3 + 3\beta_2^2 n^2 + 3\beta_2^2 n + \beta_2^2)}{(\mathrm{n}^3(4\alpha\beta_2 + 4\alpha^2 n + \beta^2 + 2\alpha\beta_2\,\mathrm{n}))} \tag{23}$$

The following equation is obtained by replacing $\gamma = \dfrac{\beta_2}{\alpha}$, in Eqn. (23):

$$f_0 = \frac{(4n^4 + 4n + 3 * \gamma n^4 + 4\gamma n^3 + 6\gamma n^2 + 8\gamma n + \gamma^2 n^3 + 3\gamma^2 n^2 + 3\gamma^2 n + \gamma^2)}{(n^3(4\gamma + 4n + \gamma^2 + 2\gamma n)} \tag{24}$$

A simplified form of Equation (22) can be obtained by setting the stiffness ratio α=1, which corresponds to the case where the beam and column sections are identical. This assumption leads to the following simplified expression:

$$f_0 = \frac{(\beta_2^2 h_1^3 h_2 + 3\beta_2^2 h_1^2 h_2^2 + 3\beta_2^2 h_1 h_2^3 + \beta_2^2 h_2^4 + 3\beta_2 h_1^4 + 4\beta_2 h_1^3 h_2 + 6\beta_2 h_1^2 h_2^2 + 8\beta_2 h_1 h_2^3 + 4h_1^4 + 4h_1 h_2^3)}{(h_1^3(4h_1 + 2\beta_2 h_1 + 4\beta_2 h_2 + \beta_2^2 h_2))} \tag{25}$$

By substituting in Eq. (24) with $h_1 = n * h_2$

$$f_0 = \frac{(\beta_2^2 n^3 + 3\beta_2^2 n^2 + 3\beta_2^2 n + \beta_2^2 + 3\beta_2 n^4 + 4\beta_2 n^3 + 6\beta_2 n^2 + 8\beta_2 n + 4n^4 + 4n)}{(n^3(4\beta_2 + 4n + 2\beta_2 n + \beta_2^2))} \tag{26}$$

The equivalent mass in an SDOF system can be calculated using the following formula:

$$m_{eq[\mathrm{n,(n-1)}]} = m_{(m-1)} + f_{0[\mathrm{n,(n-1)}]} \tag{27}$$

where n represents the number of the floor whose mass is to be shifted, and f_0 is the mass-shifting coefficient to the lower floor, which was derived in its original form from Eqn. (22). Meanwhile, m_{eq} is defined as the total equivalent mass of the frame.

A multi-degree-of-freedom (MDOF) frame structure may have many modes of vibration. Usually, we use the first mode of vibration in orthogonal directions to estimate seismic demand. The fundamental period of the equivalent SDOF system is given by:

$$T = 2\pi\sqrt{\frac{m_{eq}}{K_{eq}}} \tag{28}$$

A step-by-step flowchart of the proposed method for the computation of the fundamental time period of the RC MRFs is provided in Fig. 6.

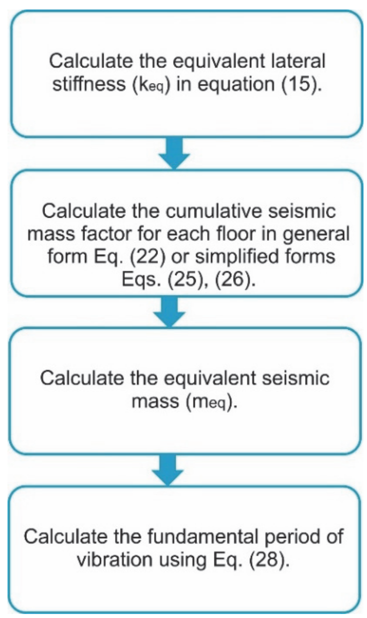

Figure 6: Proposed procedure flowchart.

NUMERICAL MODELING FRAMEWORK

Assumptions and scope

The assumption of this study mainly examines bare frame buildings, excluding the stiffness contribution of infill walls. The building models examined range from 3 to 30 stories in height. For each number of stories studied, different storey heights were chosen, which vary from 2.5m to 4m. The suggested formula has mainly been derived through estimating the fundamental time period of RC moment-resisting frames under elastic conditions.

Modeling approach

The frame structure shown was modeled using frame elements in the ETABS environment. The beams and columns were idealized as linear elastic elements. Each member (beams and columns) was discretized into a single frame element between joints, given the simple geometry, and to maintain computational efficiency. The finite element model (FEM) was used to validate the proposed analytical framework for seismic response prediction.

Element type and meshing strategy
Beam-column members were modeled by [Timoshenko beam elements] to account for both bending and shear deformations. A uniform structured mesh was employed for the model, with local refining in significant regions, particularly in beam–column joints and positions of supports, for better local stress accuracy and realistic distribution of stiffness. Mesh density was selected following initial convergence tests to ensure negligible variation in global response parameters such as the fundamental period and the maximum story drift with additional refinement.

Mesh sensitivity and convergence
A detailed mesh sensitivity analysis was not performed. The initial mesh density was selected by trial, starting with element lengths of approximately 1.0 m, and subsequently refined to about 0.6 m, at which point the variations in fundamental frequency and maximum storey drift became negligible. Local refinement was applied at beam–column connections and support areas, where element lengths were further reduced to the range of 0.20 m to 0.35 m to improve accuracy in regions with high stress concentrations. The final mesh meets the minimum convergence criteria, with global response parameters remaining stable under further refinement.

Mass and boundary condition modeling with diaphragm assumptions
In order to achieve a simulated fixed base condition, boundary conditions were provided in the form of fixed supports on column foundations that restrained all the translational and rotational degrees of freedom. By removing the influence of soil-structure interaction, this assumption makes dynamic analysis easier and reflects idealized conditions typical in initial design and verification studies. To provide equal lateral displacement in each story, rigid diaphragms were also provided at every level to impose in-plane stiffness. For dynamic analysis, a 5% damping ratio was applied overall to account for inherent structural and material damping, but no other damping elements were specifically simulated.

Visualization of FEM setup
For enhanced transparency and reproducibility, schematic illustration of the finite element model has been provided (Fig.7), showing mesh distribution, boundary conditions, and areas of mesh refinement. Boundary conditions are indicated to mark fixed and restrained DOF at support nodes. All joints were assigned two degrees of freedom (DOF) per node, namely horizontal translation (*u*) and rotation (*θ*). The figure also indicates local mesh refinement where regions of interest such as beam–column joints and in the regions near the zones of support are higher in stress gradient. Such graphical inspection ensures that the model's constraints, degrees of freedom, and discretization scheme accurately portray the intended real-life phenomenon.

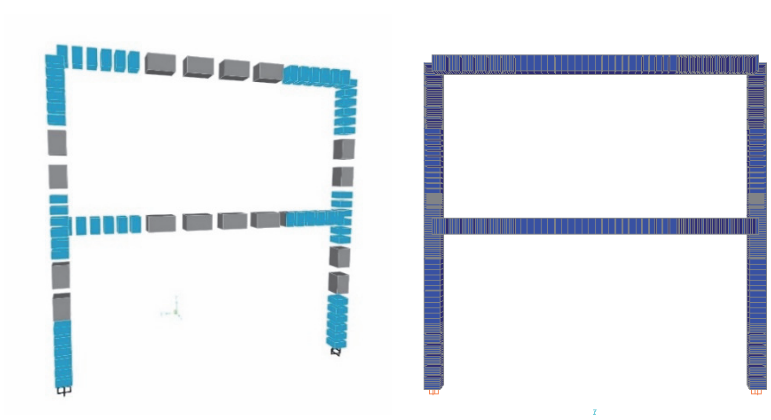

Figure 7: Finite element meshing strategy

Loading conditions
Dead load is the weight of the structural and non-structural elements themselves, i.e., beams, columns, and finishes. Dead load was automatically computed from cross-section dimensions and assigned material densities (25 kN/m³ for reinforced concrete). Live loads are temporary occupancy-related forces. A uniform live load was placed on every beam in accordance

with code requirements, using a value of 2.0 kN/m². For seismic combination, a smaller portion of the live load (typically 25% or according to code) was included with the mass for dynamic analysis.
The seismic lateral forces were applied in the present study through the response spectrum analysis (RSA) method. A code-based design response spectrum was applied, taking into account parameters that include site class, seismic zone, importance factor, and structural damping ratio (assumed to be 5% for reinforced concrete frames).

VALIDATION OF THE PROPOSED EQUATION

Overview of validation methodology

In this study, a number of examples are presented to assess the validity of the suggested equation to estimate buildings' fundamental period. Initially, two cases of moment-resisting frame (MRF) structures of reinforced concrete with uniform cross sections are investigated. Then, applying more models with varying section properties along their height. The fundamental periods derived from the suggested equation are compared with numerical model eigenvalue analysis solutions, which represent the benchmark solution. Comparisons are made using ETABS 2018 and MATLAB 2023 [38, 39] for reliability and accuracy.

Estimation of the fundamental period for a six-story reinforced concrete frame with uniform member sections

The following frame consists of six levels, each with a height of 3 meters and a span of 6 meters. The sections of both beams and columns are uniform throughout the building's entire height, as shown in Fig. 8 and Tab.1. This example was solved using the proposed method by first determining the equivalent lateral stiffness and then computing the mass transfer coefficient from one floor to the level of the next. This method is carried out in five iterations. The result of applying the proposed method is compared with the values from FEM as shown in Tab. 2.

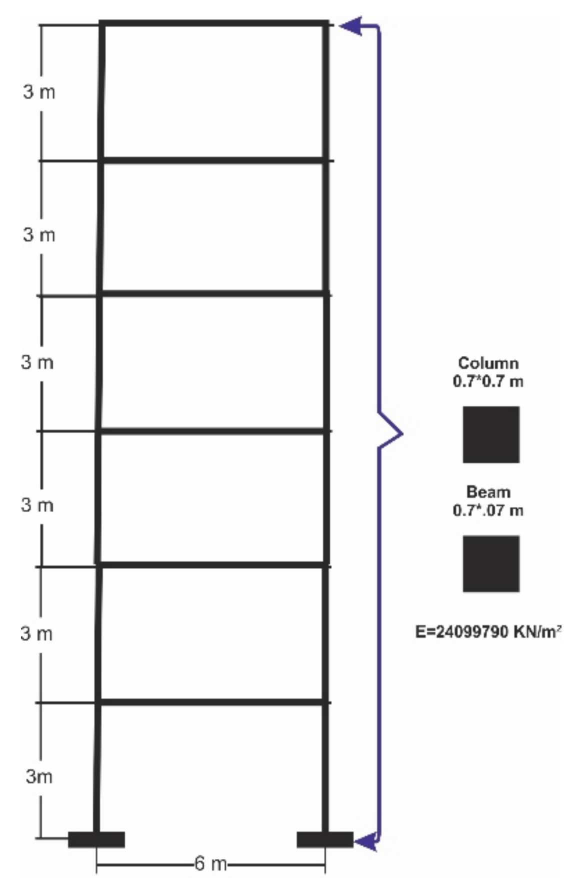

Figure 8: Configuration of reinforced concrete MRF.

Floor	Column			Beam			$(\alpha)=I_b/I_c$	$(\beta_2)=L/h_2$
	Section (m)	I_C (m^4)	h_2 (m)	Section (m)	I_b (m^4)	L (m)		
1^{st} to 6^{th}	0.7× 0.7	0.02	3	0.7× 0.7	0.02	6	1.0	2

Table 1. Geometric and mechanical properties of frame elements adopted for analytical validation.

m_{eq} (KN.s²/m)	k_{eq} (KN/m)	ω (rad/sec)	T_{FEM} (sec)	$T_{calculated}$ (sec) (Eq.28)	Error%
709.8	243810	18.7	0.32	0.33	4%

Table 2: The comparison between the fundamental period by the proposed equation with that calculated by FEM.

By comparing the results, it was found that the value of the fundamental period obtained from the suggested equation has an error margin of 4% compared to the results from the FEM analysis. This indicates the validity of the derived conclusions and the equations used.

Estimation of the fundamental period for a ten-story reinforced concrete frame with uniform member sections

In this example, the comparison is conducted on a 10-storey frame, where the floor height for the first five stories is 3.5 meters, and for the remaining five stories, it is 3 meters as shown in Fig.9. The geometric dimensions of the element of the structure are given in Tab. 3. The simplifying process involved nine iterations to convert the building into an equivalent single-story structure. As shown in Tab. 4, the error margin using the equation is approximately 3%, confirming the accuracy of the proposed equation. It can be stated that the proposed formula produces results with high accuracy.

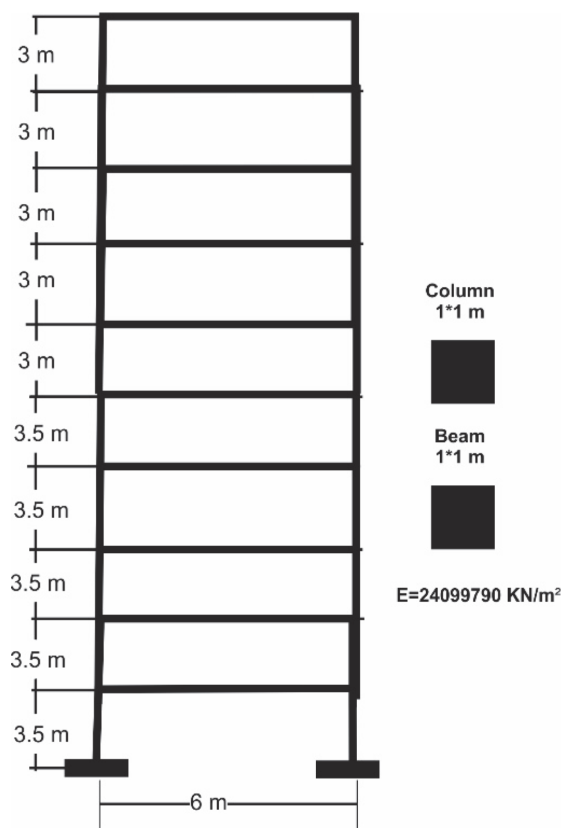

Figure 9: Configuration of RC MRF building.

Floor	Column			Beam			$(\alpha)=I_b/I_c$	$(\beta_2)=L/h_2$
	Section (m)	I_C (m^4)	H (m)	Section (m)	I_b (m^4)	L (m)		
1st to 5th	1×1	0.0833	3.5	1×1	0.0833	6	1.0	1.71
6th to 10th	1×1	0.0833	3	1×1	0.0833		1.0	2

Table 3: Geometric and Section Properties of Columns and Beams for frame.

m_{eq} (KN.s^2/m)	k_{eq} (KN/m)	ω (rad/sec)	T_{FEM} (sec)	$T_{calculated}$ (sec) (Eq.28)	Error%
4019.4	596090	12.178	0.52	0.54	3%

Table 4: Comparison between the fundamental period by the proposed equation with FEM.

SIMPLIFIED EQUIVALENT MASS CALCULATIONS WITH VALIDATION EXAMPLES

The equivalent mass calculation involves performing several iterations to reach the final equivalent mass for the SDOF (Single Degree of Freedom) system. This process represents a burden for the user and a waste of time. The proposed method has been simplified by avoiding multiple iterations for determining the equivalent mass. Instead, an equivalent curve has been derived that allows for a single-step calculation of the equivalent mass in terms of the storey numbers. This ultimately led to the curve shown in Fig. 10. This curve is used to determine the equivalent mass coefficient of the frame (f_0) and convert the system into an SDOF (Single Degree of Freedom) based on the number of floors of the frame, as demonstrated in the following examples.

Tab. 5 presents the results of five models of frames with different story numbers, heights, and sections. The natural period of all the models was calculated one by one from the proposed equation and compared to the value taken from the ETABS software. Upon comparing the difference between the two values, we observe that the proposed equation has an error margin of no more than 6%, indicating its accuracy.

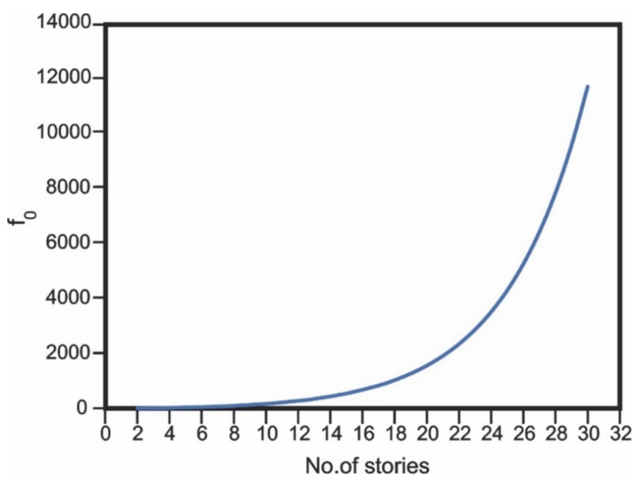

Figure 10: The cumulative seismic mass factor of the RC MRF system.

NUMERICAL VALIDATION OF THE PROPOSED PERIOD FORMULA FOR FRAMES WITH NON-UNIFORM CROSS-SECTIONAL PROPERTIES

To validate the proposed Equation (28) for estimating the fundamental period of frame buildings with non-uniform cross-sectional properties along the height, a seven-storey frame was analyzed. Each storey had a height of 3 meters, and the span length was 6 meters, as shown in Figure 11.

A comparison of the equation's results is made using five different configurations in terms of beam-to-column cross-sections. Each model has a different beam cross-section compared to the column cross-section; however, this difference remains consistent throughout the building's height, as shown in Tab. 6.
The analysis of models in Tab.7 demonstrates that a reduction in the alpha coefficient value, which represents the beam-to-column inertia ratio, results in a greater difference between the value derived from the proposed equation and the value calculated by FEM. When the alpha value was 0.14, the difference between the two values reached 10%. Therefore, a correction was made to the seismic mass coefficient to obtain the corrected ratio (f/f_0) as a function of alpha to achieve more accuracy. Fig. 12 shows the corrected seismic mass coefficient as a function of the alpha ratio used. To simplify the steps and obtain results quickly and with high accuracy, an equation was derived as a function of alpha to facilitate access to results faster and with higher accuracy, as stated in Eqn. (29).

Model number	1	2	3	4	5
Number of storeys	10	15	20	25	30
I_b (m^4)	0.0341	0.0833	0.02	1.33	1.33
I_c (m^4)	0.0341	0.0833	0.02	1.33	1.33
L (m)	8.0	7.0	6.0	8.0	8.0
m_{eq} (KN.s²/m)	3420	16140	32701.5	526309	1309360
E (KN/m²)	24099790	24099790	24099790	24099790	24099790
K_{eq} (KN/m) (Eq.16)	371262	937408	684556	12107600	14946270
ω(rad/s)	10.417	7.62	4.57	5.24	3.37
T(sec) (Eqn.28)	0.6	0.82	1.373	1.2	1.85
T_{FEM} (sec)	0.57	0.78	1.39	1.3	1.95
Error (%)	5%	5%	2%	6%	5%

Table 5: Accuracy assessment of the proposed period estimation formula for frames with varying numbers of storeys.

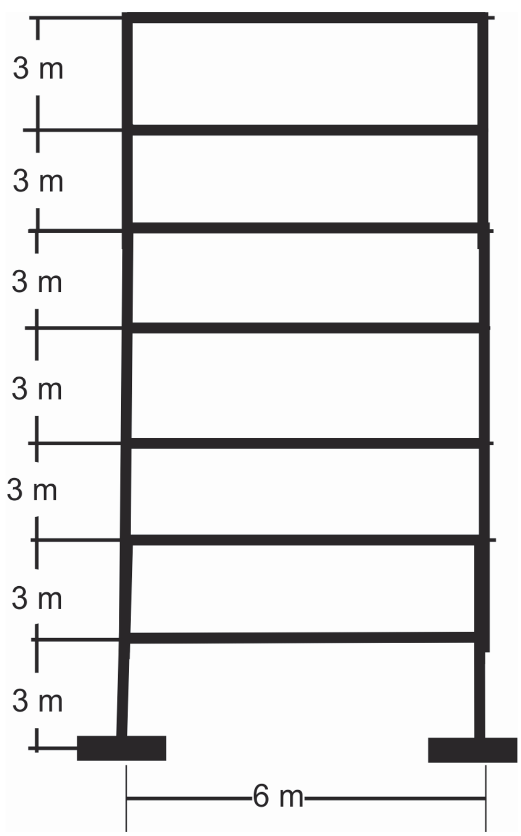

Figure 11: Configuration of MRF concrete structures.

Model number	Column cross section(m)	Beam cross section(m)	Alpha (I_b/I_c)	E(KN/m)
Model 1		0.8× 0.8	0.4096	
Model 2		0.7× 0.7	0.24	
Model 3	1× 1	0.68× 0.68	0.178	24099790
Model 4		0.65× 0.65	0.21	
Model 5		0.62× 0.62	0.14	

Table 6: Geometric properties and flexural stiffness ratio (I_b/I_c) for five analytical models with varying beam cross sections and constant column dimensions.

	Column cross-section(m)	Beam cross-section(m)	α (I_b/I_c)	m_{eq} (KN.sec²/m)	k_{eq} (KN/m) (Eq.15)	ω (rad/sec)	T proposed equation (sec)	T_{FEM} (sec)	Error%
Model 1	1× 1	0.8× 0.8	0.4096	35604	778590.5	4.67	1.34	1.282	5%
Model 2	1× 1	0.7× 0.7	0.24	32701.5	684556.8	4.57	1.37	1.399	2%
Model 3	1× 1	0.65× 0.65	0.178	31393.44	640356.4	4.51	1.39	1.492	7%
Model 4	1× 1	0.68× 0.68	0.21	32167.44	666569.1	4.55	1.38	1.433	4%
Model 5	1× 1	0.62× 0.62	0.14	30650.4	615260.5	4.48	1.40	1.56	10%

Table 7: Comparative evaluation of fundamental time periods derived from the proposed analytical equation and FEM results for five structural models with varying beam to column flexural stiffness.

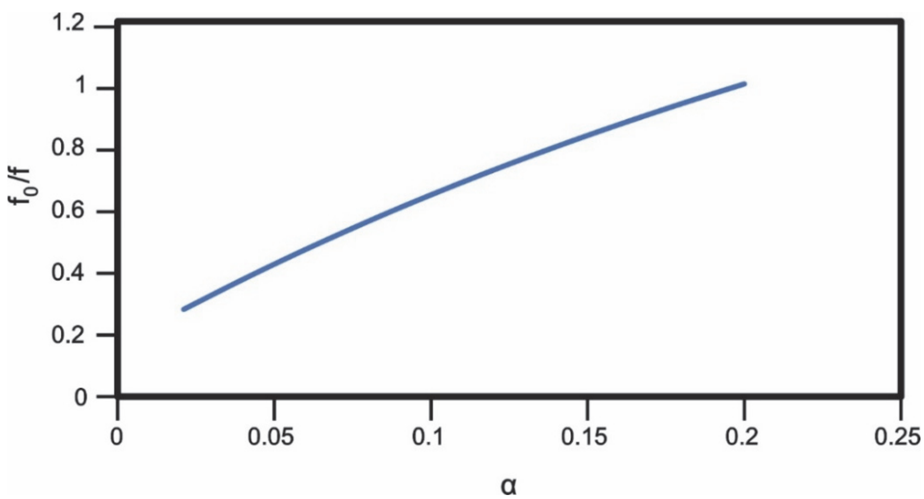

Figure 12: Correction factor of a cumulative seismic mass factor.

$$\frac{f_0}{f} = 2.3679\alpha^{0.549} \tag{29}$$

where α is the beam-to-column inertia ratio (I_b/I_c), f_0 is the cumulative factor for shifting the story masses when α equals 1, and f is the cumulative mass factor adjusted after correction for differences between the cross-sections of beams and columns. The comparison was conducted following the correction of the seismic mass coefficient since the alpha value was less than 1.0 for five different models, as indicated in Tab. 8. These comparisons were executed as shown in Tab. 9 using the revised f value. Upon doing the evaluation, favorable results were achieved in relation to the outputs of the FEM.

Model number	Column cross-section (cm)	Beam cross-section (cm)	α (I_b/I_c)	M_n (KN.sec²/m)
Model 1	100× 100	80× 80	0.4096	23
Model 2	100× 100	70× 70	0.24	21.125
Model 3	100× 100	65× 65	0.178	20.28
Model 4	100× 100	68× 68	0.21	20.78
Model 5	100× 100	62× 62	0.14	19.8

Table 8: Geometric properties and mass values (Mn) for structural models with varying beam-to-column flexural stiffness ratios.

Model number	f_0 (Fig.9)	f (Fig.11)	m_{eq} (KN.sec²/m)	k_{eq} (KN/m) (Eq.15)	ω (rad/sec)	$T_{proposed\ equation}$(sec) (Eq.28)	T_{FEM} (sec)	Error (%)
Model 1	1548	1583.69	35604	778590.5	4.67	1.34	1.282	5%
Model 2	1548	1880.789	32701.5	684556.8	4.52	1.389	1.399	1%
Model 3	1548	1693.86	31393.44	640356.4	4.09	1.53	1.492	3%
Model 4	1548	2098.70	32167.44	666569.1	4.35	1.44	1.433	1%
Model 5	1548	2098.70	30650.4	615260.5	3.84	1.63	1.56	5%

Table 9: Comparative analysis of fundamental frequencies and time periods computed using the proposed analytical model and FEM for five structural models with corresponding mass and stiffness parameters.

In order to check the validity of the suggested formula for estimation of the fundamental time period of buildings, comparisons were also made with FEM results for different frame configurations, as presented in Figs. 13a and 13b. Fig. 13a shows the comparison for a height of 10 with different spans of 6 m, 7 m, 8 m, 9 m, 10 m, 12 m, and 15 m. Here, the maximum error that was found was 5%. The proposed equation shows good concordance with FEM predictions, as most data points are in proximity to the 45° line. Statistical analysis also confirms this observation, as a high coefficient of determination (R^2) of 0.86 and a low root mean square error (RMSE) of 0.0165 seconds indicate strong predictive accuracy. Fig. 13b shows the evaluation of a 20-story frame of constant 6-meter span, in which the ratio of alpha changes continuously from 0.24 to 0.21, 0.17, and 0.14. As in the former case, data points are very close to the 45° line, indicating a high degree of agreement between FEM results and the provided equation. This case yields even more impressive statistical results, with R^2 = 0.962 and RMSE = 0.0308 seconds, demonstrating the powerful ability of the equation to represent the dynamic response of taller buildings. These results overall validate that the equation provided herein provides a very accurate estimate of the fundamental time period for a range of building configurations.

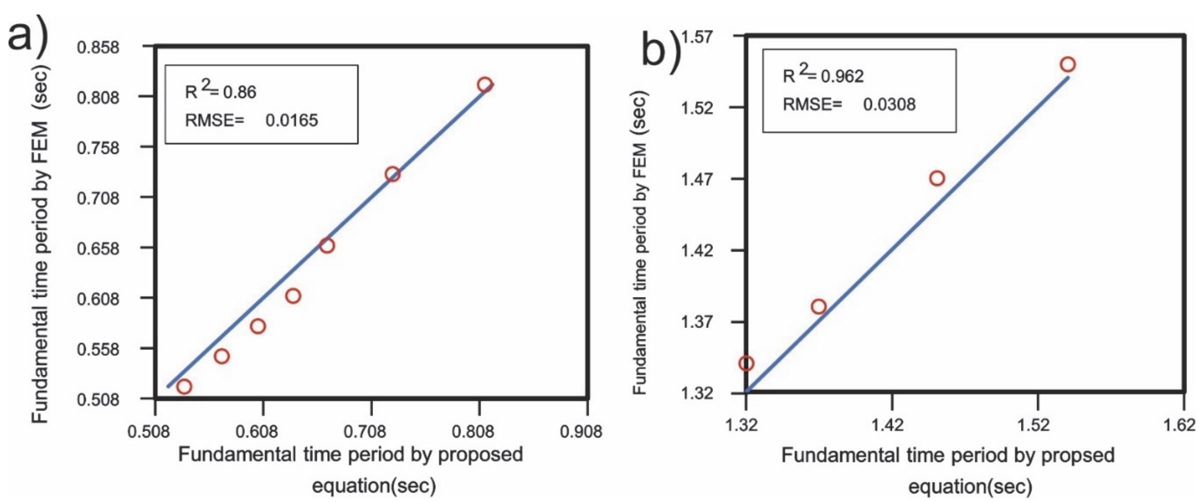

Figure 13: Scatter plots of the proposed analytical and estimated fundamental periods, a) 10-storey building, and b) 20-storey building.

Comprehensive validation of the proposed equation against FEM results and established formulas from literature

The accuracy of the suggested formula for estimating the fundamental vibration period of reinforced concrete moment-resisting frame (RC MRF) structures is confirmed in Fig. 14 by comparing it with existing formulations from Goel and Chopra [5], Salama [6], Aninthaneni and Dhakal [23], as well as seismic design codes such as ASCE 7-05 [32] and UBC [1], using Finite Element Method (FEM) results as reference. Two structural layouts are considered: frames with a 5 m span (Fig. 14a) and frames with a 7 m span (Fig. 14b). The comparison is based on two important statistical indicators: the coefficient of determination (R^2), which measures the degree of correlation between predicted and reference values, and the root mean square error (RMSE), which quantifies the average prediction error.

For the 5 m span illustration case (Fig. 14a), the new equation is well correlated with the FEM solution with $R^2 = 0.998$ and RMSE = 0.039s. Both indicators confirm the high accuracy and reliability of the new model in estimating RC MRF frames' dynamic behavior. For comparison, Aninthaneni and Dhakal's equation [23] gives fair agreement ($R^2 = 0.81$, RMSE = 0.50 s), whereas Goel and Chopra's equation [5] is poor ($R^2 = 0.14$, RMSE = 1.09 s) with considerable departure from FEM-based periods. Likewise, Salama's equation [6] exhibits poor predictive strength ($R^2 = 0.62$, RMSE = 0.71 s) due to overestimation of stiffness. Of the seismic code equations, ASCE 7-05 [32] is moderately consistent with FEM solutions ($R^2 = 0.80$) but tends to underestimate the fundamental period for higher structures, which may result in overly conservative seismic force estimates. The most conservative estimates overall are given by UBC [1], with lower predicted periods at every height and relatively high $R^2 = 0.86$, indicating trend conformity more than actual accuracy.

In the case of 7 m span (Fig. 14b), the proposed equation maintains the predicting proficiency with $R^2 = 0.999$ and RMSE = 0.027 s, and makes it robust and span-insensitive. Aninthaneni and Dhakal's model [23] in the present case shows enhanced performance ($R^2 = 0.93$, RMSE = 0.29 s), depicting higher applicability for wide spans. Goel and Chopra's equation [5] still performs poorly ($R^2 = 0.016$, RMSE = 1.12 s), whereas Salama's equation [6] still has moderate accuracy ($R^2 = 0.56$, RMSE = 0.75 s). The ASCE 7-05 [32] model once more has moderate correlation ($R^2 = 0.80$) but still underestimates periods for high-rise structures. The UBC [1] formula has $R^2 = 0.84$ but is still too conservative, severely underestimating the fundamental period for all heights.

Overall, for both span cases, the new equation outperforms any other empirical and code-based formula in terms of prediction accuracy (lowest RMSE) and magnitude of correlation (highest R^2). Its excellent agreement with FEM solutions, regardless of span length, demonstrates its utility in the seismic analysis and design of RC MRF buildings in practice. These outcomes validate its usage as a rational alternative to conventional empirical period estimation formulas.

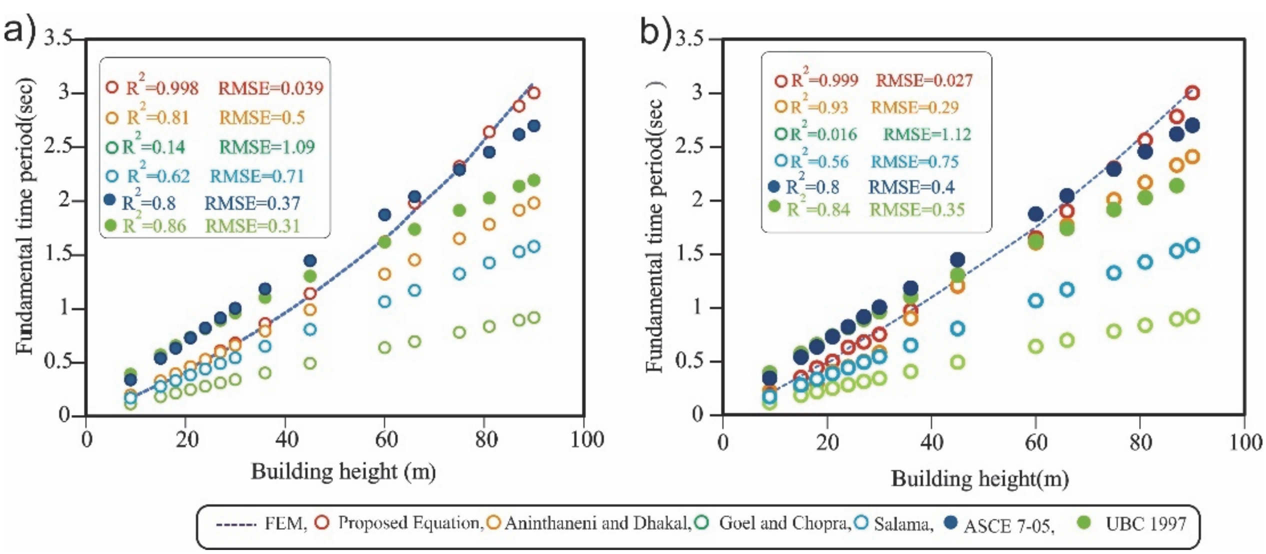

Figure 14 : Comparison of fundamental vibration periods estimated using the proposed equation with those predicted by FEM analysis and existing formulas for reinforced concrete moment-resisting frame (RC MRF) buildings : (a) frames with a 5 m span ; (b) frames with a 7 m span.

Summary of recent studies on fundamental period estimation and their methodological characteristics

In addition to the comparison with empirical code-based formulas and FEM results, it is important to situate the proposed analytical model within the context of recent research. Tab.10 is an overview of recent advances in fundamental period prediction methods, comparing structural scope, methodologies, parameters treated, and main findings across selected studies. The development of research provides a clear trend toward the integration of data-driven approaches, namely machine learning (ML), with traditional analytical models both to enhance accuracy and enable interpretability. Kumar et al. [25] explored RC moment-resisting frames via comparative analysis of ML algorithms (ANN, GP, RT), where ANN was most accurate. Their results emphasize the requirement for geometric parameters such as number of bays and bay length to influence period predictions. Rahman et al. [26] extended their scope to steel-braced, RC structures and employed ML models that were enhanced with SHAP-based interpretability. Shan et al. [27] addressed the issue of prediction of existing high-rise RC buildings by integrating probabilistic ML with explicit accounting for material deterioration and structural aging. Karampinis et al. [28] bridged analytical and ML approaches, explaining the influence of plan irregularities and stiffness ratios through SHAP values, while also producing analytical equations with comparable performance to ML models.

Relative to recent advances in fundamental period predictive techniques (Tab.10), the deterministic analytical model developed herein offers a novel compromise between predictability, computational efficiency, and transparency. While several recent studies have applied machine learning (ML) techniques either alone or in combinations to enhance predictability, these approaches are often require large amount of data (with high-quality data sets), model-intensive (with sophisticated training), and may be infested with interpretability challenges in the lack of specific explainability tools (e.g., SHAP) [25, 26, 27, 28]. Compared to the previous, the new model is an analytical model based on merely the basic structural parameters of mass, height, and stiffness distribution and therefore can be applied without the computational overhead or data dependence of ML-based methods. Moreover, unlike probabilistic or data-driven methods, the present approach gives closed-form solutions that are inherently interpretable and therefore can be applied directly in early design and code development.

Study & year	Structural Scope	Methodology	Key Parameters Considered	Main Findings
Kumar et al. [25]	RC moment-resisting frames	Algorithms for comparative machine learning (ANN, GP, RT)	Height, length of the bays, number of bays, and material	The influence of geometric irregularities was highlighted by ANN, which achieved the highest accuracy
Rahman et al. [26]	RC buildings with steel bracing	Interpretable machine learning (SHAP values)	Height, bracing arrangement, and stiffness distribution	Period was greatly shortened by bracing; important stiffness variables were identified by ML interpretability
Shan et al. [27]	RC high-rise buildings	Probabilistic machine learning	building height, and material deterioration	For older structures, probabilistic machine learning captured uncertainty and enhanced prediction.
Karampinis et al. [28]	Structures with frames	ML + Analytical hybrid with SHAP	Height, stiffness ratios, and plan irregularities	Created comprehensible analytical formulas with precision on par with the ML
Proposed Model	RC moment-resisting frames	Deterministic Analytical Model	Height, mass and stiffness distribution,	Easy to understand, transparent, and computationally efficient; similar accuracy for a standard configuration

Table 10. Summary of recent studies on fundamental period prediction, highlighting structural scope, methodology, key parameters, and main findings, with emphasis on the positioning of the proposed deterministic analytical model relative to contemporary machine learning and hybrid approaches.

SENSITIVITY INVESTIGATIONS

Sensitivity analysis was conducted to evaluate the influence of the main design parameters on the fundamental vibration period of RC moment-resisting frame (MRF) buildings. This investigation is based on numerical results of 125 building models, where the primary parameters are shown in Tab.11. They include the number of stories, total building height, typical storey height, span length, cross-sectional beam and column sizes, model mass, and concrete modulus of elasticity. The results of the sensitivity analysis are outlined in the following subsections.

Geometric parameters

The building's geometrical configuration is essential in determining its dynamic features, specifically its fundamental time period. The building's height and span length are among the most significant characteristics, as they directly affect the structure's overall stiffness and mass distribution. This subsection provides a comprehensive examination of how various elements affect the fundamental time period.

Effect of building height

Relative comparison provided in Fig. 15 (a-c) contrasts accuracy of different formulations in estimating the fundamental time period of moment-resisting frames having different spans (5 m, 6 m, and 7 m), with Finite Element Method (FEM) results. The equations considered are those given by [5, 6, 23] and a new expression developed in the present work.
In Fig. 15a, for 5 m span frames, the proposed equation is in very good agreement with FEM-based calculations with R^2 = 0.999 and RMSE = 0.02 s. Aninthaneni and Dhakal's equation [23], however, has R^2 = 0.60 and RMSE = 0.67 s, indicating a moderate relationship with some scatter. Goel and Chopra's formula [5] exhibits suboptimal performance, yielding an R^2 of 0.36 and a root mean square error (RMSE) of 0.84 s, whereas Salama's formula [6] is more accurate (R^2 = 0.78, RMSE = 0.49 s) but less than the proposed model. For Fig. 15b, for a frame with a span of 6 m, the proposed model maintains high predictive fidelity (R^2 = 0.999, RMSE = 0.013 s), underscoring its robustness across structural configurations. Aninthaneni and Dhakal's equation [23] shows some improvement (R^2 = 0.66, RMSE = 0.55 s), but yet maintains a very high margin of error. Goel and Chopra [5] approach is still imprecise (R^2 = 0.38, RMSE = 0.76 s), while Salama's formula shows quite good agreement (R^2 = 0.83, RMSE = 0.40 s), though not as good as the suggested expression.

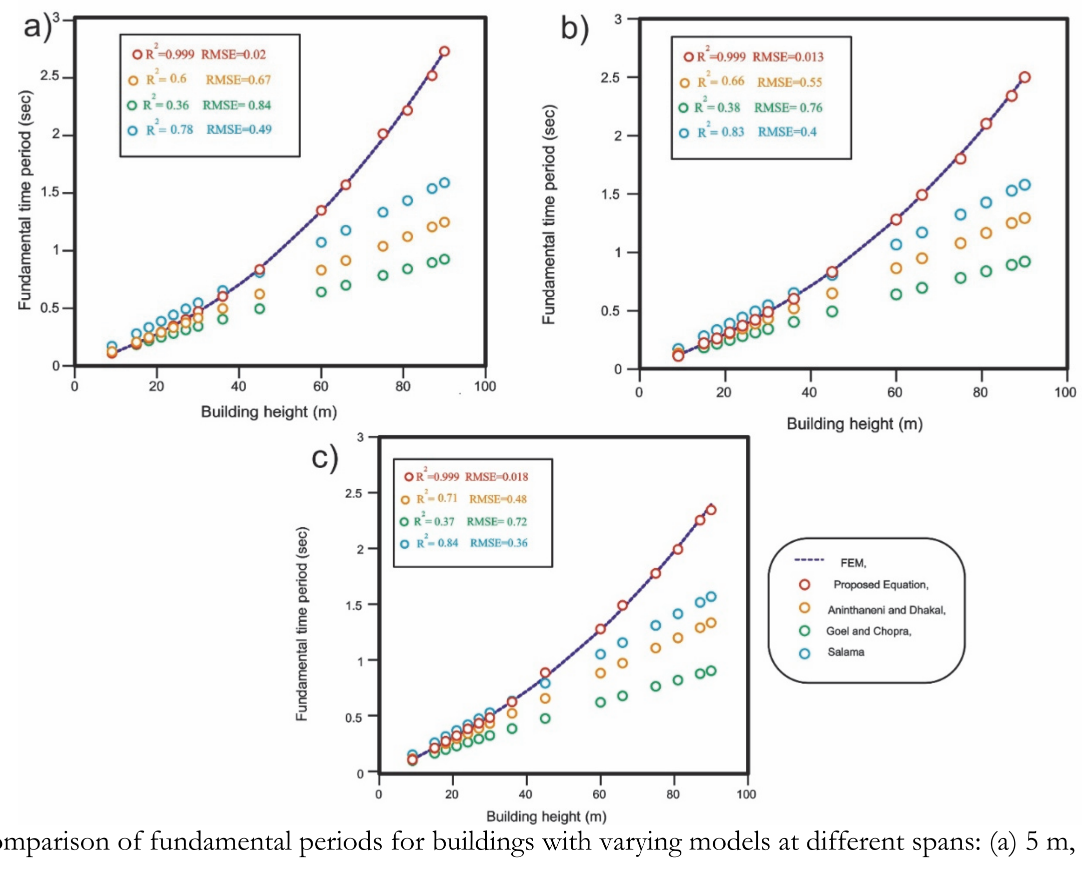

Figure 15: Comparison of fundamental periods for buildings with varying models at different spans: (a) 5 m, (b) 6 m, and (c) 7 m.

Model number	Number of stories	Building height (m)	Storey height (m)	Frame span (m)	Column Dimension (m)	Beam dimension (m)	T equation (sec)	T FEM (sec)	Error %
1	3	9	3	5	0.5*0.5	0.5*0.5	0.169	0.17	1%
2	5	15	3	5	0.5*0.5	0.5*0.5	0.31	0.304	2%
3	6	18	3	5	0.5*0.5	0.5*0.5	0.36	0.373	3%
4	7	21	3	5	0.5*0.5	0.5*0.5	0.45	0.445	1%
5	8	24	3	5	0.5*0.5	0.5*0.5	0.52	0.519	0%
6	9	27	3	5	0.5*0.5	0.5*0.5	0.61	0.596	2%
7	10	30	3	5	0.5*0.5	0.5*0.5	0.68	0.675	1%
8	12	36	3	5	0.5*0.5	0.5*0.5	0.84	0.843	0%
9	15	45	3	5	0.5*0.5	0.5*0.5	1.1	1.121	2%
10	20	60	3	5	0.5*0.5	0.5*0.5	1.67	1.666	0%
11	22	66	3	5	0.5*0.5	0.5*0.5	1.98	1.915	3%
12	25	75	3	5	0.5*0.5	0.5*0.5	2.3	2.325	1%
13	27	81	3	5	0.5*0.5	0.5*0.5	2.62	2.623	0%
14	29	87	3	5	0.5*0.5	0.5*0.5	2.95	2.942	0%
15	30	90	3	5	0.5*0.5	0.5*0.5	3.05	3.109	2%
16	3	9	3	6	0.5*0.5	0.5*0.5	0.18	0.188	4%
17	5	15	3	6	0.5*0.5	0.5*0.5	0.32	0.335	4%
18	6	18	3	6	0.5*0.5	0.5*0.5	0.4	0.411	3%
19	7	21	3	6	0.5*0.5	0.5*0.5	0.48	0.489	2%
20	8	24	3	6	0.5*0.5	0.5*0.5	0.58	0.568	2%
21	9	27	3	6	0.5*0.5	0.5*0.5	0.66	0.649	2%
22	10	30	3	6	0.5*0.5	0.5*0.5	0.73	0.731	0%
23	12	36	3	6	0.5*0.5	0.5*0.5	0.9	0.903	0%
24	15	45	3	6	0.5*0.5	0.5*0.5	1.2	1.18	2%
25	20	60	3	6	0.5*0.5	0.5*0.5	1.75	1.702	3%
26	22	66	3	6	0.5*0.5	0.5*0.5	1.98	1.934	2%
27	25	75	3	6	0.5*0.5	0.5*0.5	2.3	2.311	0%
28	27	81	3	6	0.5*0.5	0.5*0.5	2.6	2.582	1%
29	29	87	3	6	0.5*0.5	0.5*0.5	2.98	2.87	4%
30	30	90	3	6	0.5*0.5	0.5*0.5	3	3.02	1%
31	3	9	3	7	0.5*0.5	0.5*0.5	0.2	0.207	3%
32	5	15	3	7	0.5*0.5	0.5*0.5	0.35	0.368	5%
33	6	18	3	7	0.5*0.5	0.5*0.5	0.46	0.45	2%
34	7	21	3	7	0.5*0.5	0.5*0.5	0.53	0.534	1%
35	8	24	3	7	0.5*0.5	0.5*0.5	0.61	0.62	2%
36	9	27	3	7	0.5*0.5	0.5*0.5	0.7	0.706	1%
37	10	30	3	7	0.5*0.5	0.5*0.5	0.78	0.794	2%
38	12	36	3	7	0.5*0.5	0.5*0.5	0.98	0.974	1%
39	15	45	3	7	0.5*0.5	0.5*0.5	1.3	1.259	3%
40	20	60	3	7	0.5*0.5	0.5*0.5	1.7	1.78	4%
41	22	66	3	7	0.5*0.5	0.5*0.5	2	2.006	0%

Table 11: Comprehensive Sensitivity Analysis of Fundamental Time Periods Considering Variations in Building Height, Span Length, Beam-to-Column Flexural Stiffness Ratio (I_b/I_c), and Concrete Modulus of Elasticity (E)identify the most sensitive parameters affecting the fundamental period.

Model number	Number of stories	Building height (m)	Storey height (m)	Frame span (m)	Column Dimension (m)	Beam dimension (m)	T equation (sec)	T FEM (sec)	Error %
42	25	75	3	7	0.5*0.5	0.5*0.5	2.35	2.368	1%
43	27	81	3	7	0.5*0.5	0.5*0.5	2.6	2.626	1%
44	29	87	3	7	0.5*0.5	0.5*0.5	2.89	2.896	0%
45	30	90	3	7	0.5*0.5	0.5*0.5	3	3.036	1%
46	20	60	3	5	0.6*0.6	0.5*0.5	1.63	1.639	1%
47	3	9	3	7	0.7*0.7	0.5*0.5	1.67	1.666	0%
48	5	15	3	7	0.8*0.8	0.5*0.5	1.7	1.719	1%
49	6	18	3	7	0.9*0.9	0.5*0.5	1.75	1.782	2%
50	7	21	3	7	1*1	0.5*0.5	1.83	1.848	1%
51	20	60	3	7	0.6*0.6	0.5*0.5	1.78	1.78	4%
52	20	60	3	7	0.7*0.7	0.5*0.5	1.85	1.782	0%
53	20	60	3	7	0.8*0.8	0.5*0.5	1.92	1.833	1%
54	20	60	3	7	0.9*0.9	0.5*0.5	1.94	1.905	1%
55	20	60	3	7	1*1	0.5*0.5	2	1.983	2%
56	20	60	3	4	1*1	1*1	1.35	1.382	2%
57	20	60	3	4.5	1*1	1*1	1.3	1.299	0%
58	20	60	3	5	1*1	1*1	1.25	1.239	1%
59	20	60	3	5.5	1*1	1*1	1.23	1.196	3%
60	20	60	3	6	1*1	1*1	1.2	1.167	3%
61	20	60	3	6.5	1*1	1*1	1.1	1.148	4%
62	20	60	3	7	1*1	1*1	1.13	1.138	1%
63	20	60	3	7.5	1*1	1*1	1.12	1.135	1%
64	20	60	3	8	1*1	1*1	1.13	1.137	1%
65	18	54	3	4	1*1	1*1	1.1	1.139	3%
66	18	54	3	4.5	1*1	1*1	1.09	1.076	1%
67	18	54	3	5	1*1	1*1	1.05	1.032	2%
68	18	54	3	5.5	0.5*0.5	0.5*0.5	1.01	1.001	1%
69	18	54	3	6	1*1	1*1	0.99	0.982	1%
70	18	54	3	6.5	1*1	1*1	0.97	0.972	0%
71	18	54	3	7	1*1	1*1	0.965	0.968	0%
72	18	54	3	7.5	1*1	1*1	0.96	0.97	1%
73	20	60	3	6	1*1	1*1	1.3	1.268	3%
74*	20	60	3	6	1*1	1*1	1.13	1.134	0%
75**	20	60	3	6	1*1	1*1	1.06	1.035	2%
76***	20	60	3	6	1*1	1*1	0.95	0.959	1%
77	10	30	3	6	1*1	1*1	0.421	0.448	6%
78*	10	30	3	6	1*1	1*1	0.4	0.401	0%
79**	10	30	3	6	1*1	1*1	0.36	0.366	2%
80***	10	30	3	6			0.34	0.342	1%
81	3	9	3	5	0.8*0.8	0.8*0.8	0.11	0.106	4%
82	5	15	3	5	0.8*0.8	0.8*0.8	0.19	0.195	3%
83	6	18	3	5	0.8*0.8	0.8*0.8	0.24	0.242	1%

Table 11:. continued …..

Model number	Number of stories	Building height (m)	Storey height (m)	Frame span (m)	Column Dimension (m)	Beam dimension (m)	T equation (sec)	T FEM (sec)	Error %
84	7	21	3	5	0.8*0.8	0.8*0.8	0.292	0.293	0%
85	8	24	3	5	0.8*0.8	0.8*0.8	0.345	0.348	1%
86	9	27	3	5	0.8*0.8	0.8*0.8	0.4	0.405	1%
87	10	30	3	5	0.8*0.8	0.8*0.8	0.47	0.467	1%
88	12	36	3	5	0.8*0.8	0.8*0.8	0.6	0.603	0%
89	15	45	3	5	0.8*0.8	0.8*0.8	0.83	0.84	1%
90	20	60	3	5	0.8*0.8	0.8*0.8	1.34	1.333	1%
91	22	66	3	5	0.8*0.8	0.8*0.8	1.56	1.566	0%
92	25	75	3	5	0.8*0.8	0.8*0.8	2	1.955	2%
93	27	81	3	5	0.8*0.8	0.8*0.8	2.2	2.24	2%
94	29	87	3	5	0.8*0.8	0.8*0.8	2.5	2.547	2%
95	30	90	3	5	0.8*0.8	0.8*0.8	2.71	2.708	0%
96	3	9	3	6	0.8*0.8	0.8*0.8	0.11	0.117	6%
97	5	15	3	6	0.8*0.8	0.8*0.8	0.22	0.212	4%
98	6	18	3	6	0.8*0.8	0.8*0.8	0.26	0.262	1%
99	7	21	3	6	0.8*0.8	0.8*0.8	0.31	0.315	2%
100	8	24	3	6	0.8*0.8	0.8*0.8	0.37	0.37	0%
101	9	27	3	6	0.8*0.8	0.8*0.8	0.42	0.427	2%
102	10	30	3	6	0.8*0.8	0.8*0.8	0.488	0.488	0%
103	12	36	3	6	0.8*0.8	0.8*0.8	0.6	0.618	3%
104	15	45	3	6	0.8*0.8	0.8*0.8	0.83	0.839	1%
105	20	60	3	6	0.8*0.8	0.8*0.8	1.28	1.285	0%
106	22	66	3	6	0.8*0.8	0.8*0.8	1.49	1.493	0%
107	25	75	3	6	0.8*0.8	0.8*0.8	1.8	1.838	2%
108	27	81	3	6	0.8*0.8	0.8*0.8	2.1	2.09	0%
109	29	87	3	6	0.8*0.8	0.8*0.8	2.34	2.361	1%
110	30	90	3	6	0.8*0.8	0.8*0.8	2.5	2.503	0%
111	3	9	3	7	0.8*0.8	0.8*0.8	0.126	0.128	2%
112	5	15	3	7	0.8*0.8	0.8*0.8	0.23	0.231	0%
113	6	18	3	7	0.8*0.8	0.8*0.8	0.29	0.284	2%
114	7	21	3	7	0.8*0.8	0.8*0.8	0.34	0.34	0%
115	8	24	3	7	0.8*0.8	0.8*0.8	0.4	0.397	1%
116	9	27	3	7	0.8*0.8	0.8*0.8	0.45	0.456	1%
117	10	30	3	7	0.8*0.8	0.8*0.8	0.5	0.517	3%
118	12	36	3	7	0.8*0.8	0.8*0.8	0.64	0.647	1%
119	15	45	3	7	0.8*0.8	0.8*0.8	0.9	0.862	4%
120	20	60	3	7	0.8*0.8	0.8*0.8	1.29	1.283	1%
121	22	66	3	7	0.8*0.8	0.8*0.8	1.5	1.475	2%
122	25	75	3	7	0.8*0.8	0.8*0.8	1.785	1.792	0%
123	27	81	3	7	0.8*0.8	0.8*0.8	2	2.022	1%
124	29	87	3	7	0.8*0.8	0.8*0.8	2.26	2.268	0%
125	30	90	3	7	0.8*0.8	0.8*0.8	2.35	2.397	2%

Table 11: continued …..

Note for Table 11: (*), (**), (***) denote models with different modulus of elasticity values, specifically 25 GPa, 30 GPa, and 35 GPa, respectively.

Fig. 15c, relative to a 7 m span, still confirms the goodness of the new model, which provides R^2 = 0.999 and RMSE = 0.018 s. Aninthaneni and Dhakal's formulation [23] has some improvement (R^2 = 0.71, RMSE = 0.48 s) but is still less accurate than that of the new model. Goel and Chopra's method [5] remains the least accurate (R^2 = 0.37, RMSE = 0.72 s), whereas Salama's equation [6] provides the most accurate estimate of the available models (R^2 = 0.84, RMSE = 0.36 s).
In general, the statistical analysis for all three cases exhibits the continual outperformance of the new equation in predicting the fundamental time period of moment-resisting frames. Its R^2 values close to unity and substantially lower RMSE values than those of the existing models show both high precision and low error of prediction. This implies that the new model provides a more precise and generalized tool for the dynamic analysis of frame structures for a broad spectrum of changing span lengths.

Effect of span variation

The sensitivity study in Fig. 16a and Fig. 16b presents a comprehensive evaluation of the effect of building span length on the fundamental time period of reinforced concrete (RC) moment-resisting frames of two various building heights, 60 m and 54 m, respectively. The performance of the suggested analytical formula is compared to the established formulations of [5, 6, 23] with the benchmark being the finite element method (FEM).
As can be seen from the two figures, the FEM results show a reducing trend for the period over the increasing span length. This is due to the fact that there is more lateral stiffness in buildings with larger widths, and therefore, the natural period decreases. The equation we have developed predicts this trend accurately and agrees very well with the FEM results for the range of spans considered whereas the other models have large differences. To determine the predictive capability of both models, statistical measures in the form of the coefficient of determination (R^2) and the Root Mean Square Error (RMSE) are provided. These measures criticize the correlation as well as predictive precision of both methods when compared to FEM results. Statistical comparison firmly confirms the higher performance of the new formula with the highest R^2 values (0.91 and 0.94) and the lowest RMSE values (0.025 and 0.016) for both building heights. These are indicators of high correlation with FEM results and low error in prediction and suggest the ability of the formula to capture the influence of variable span length on dynamic response.
In stark contrast, the other existing formulas all have negative R^2 values, meaning that their predictions deviate so significantly from the FEM results. This is most evident for [5, 23], who have RMSE values above 0.4 and R^2 values below -40. Salama's formula [6] has a slightly better fit, but once more, it performs badly in comparison to the suggested method, having low correlation and moderate error. In conclusion, the comparison indicates the inability of the equations from the literature to effectively account for the influence of span length, especially in mid- to high-rise buildings. The suggested method is, however, invariably correct irrespective of height and span.

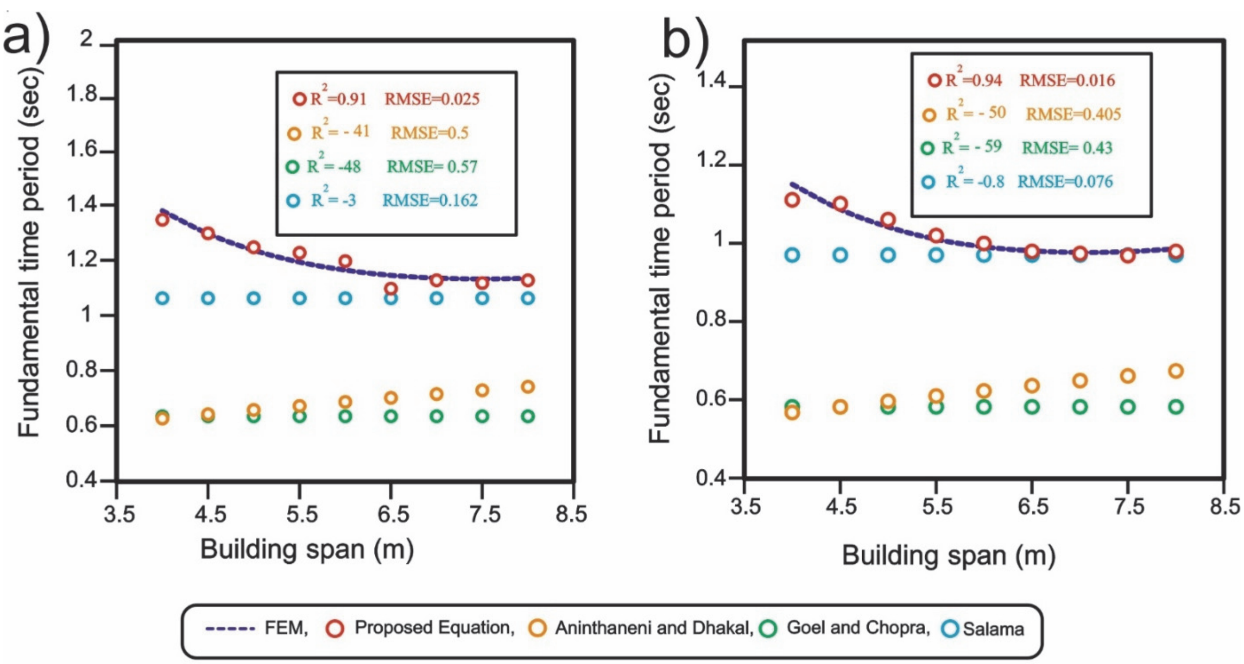

Figure 16: Variation of fundamental time period with span length for different building heights: (a) 60 m height; (b) 54 m height.

Stiffness-related parameters

The member structural properties play an important role in the global lateral resistance and dynamic response of moment-resisting frames. The beam to column inertia ratio and elastic modulus of members are parameters that have a direct influence on the global stiffness and thus the fundamental time period. Each of these parameters is treated separately in the following subsections to assess its own influence.

Effect of the beam to column inertia ratio

The sensitivity analysis shown in Fig. 17a and Fig. 17b evaluates the impact of the beam-to-column inertial ratio (α) on the fundamental time period of reinforced concrete (RC) moment-resisting frames with spans of 5 m and 7 m, respectively. The work compares the performance of a new proposed equation with three others [5, 6, 23] based on results from finite element method (FEM) simulations as a benchmark reference.

In order to determine the validity of the proposed equation, two statistical measures are employed: the coefficient of determination (R^2) and the Root Mean Square Error (RMSE). These statistical findings ratify the excellence of the new equation that in both ranges consistently provides the best approximation of FEM data. In Fig. 17a, the quasi-perfect R^2=0.95 and very low RMSE of 0.017 s reveal excellent agreement with numerical simulations. In Fig. 17b, although the correlation is slightly poorer (R^2=0.85), the RMSE is still small at 0.045 s, which is still much better than all other models. Conversely, the existing equations, especially for the 7 m span, show extreme statistical deterioration, as reflected by large negative R^2 values and large RMSEs. Negative R^2 values of [5, 6, 23] in Fig. 17b show that these models cannot capture the trend in the FEM results. Their RMSE values (0.83 to 1.25 s) also reflect their unsuitability for frames with larger spans and other stiffness properties. In conclusion, statistical analysis offers very strong evidence in favor of the adoption of the given method that possesses very high predictive power and good performance regarding the frame geometries. It takes into account the beam-to-column stiffness ratio, making it a better predictor of real structural behavior, a good and reliable tool for estimation of time periods in seismic frame design and analysis of RC frames.

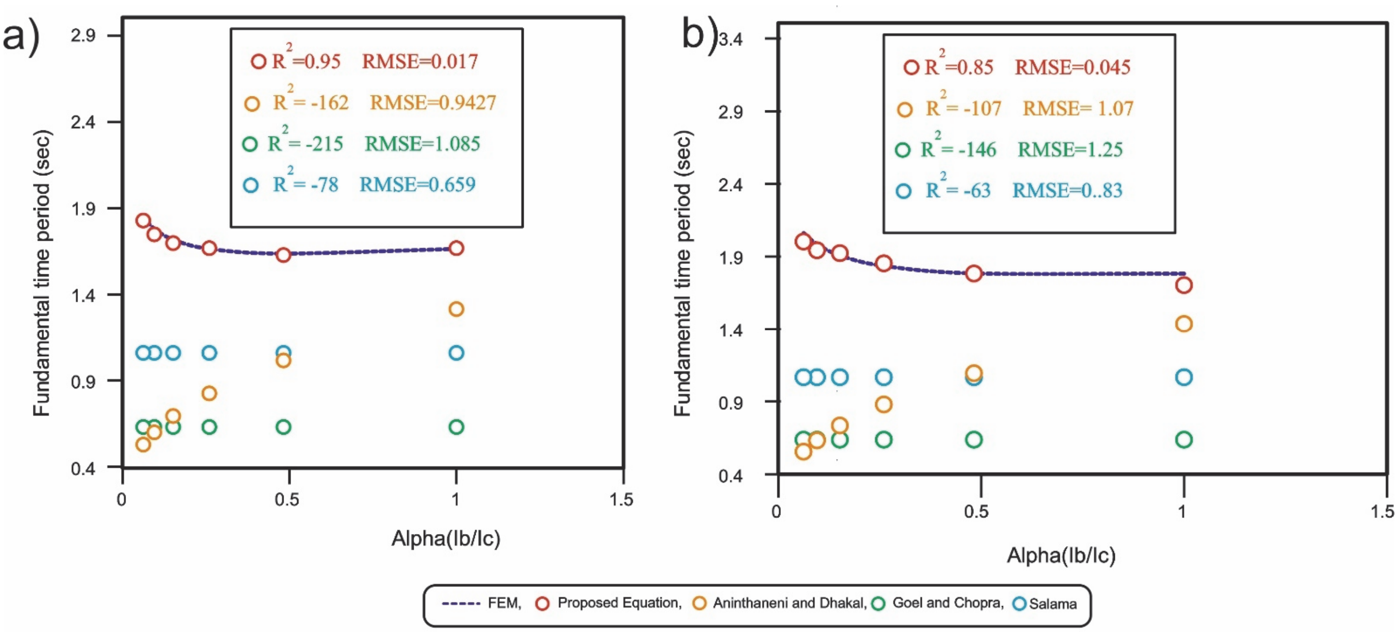

Figure 17: Variation of the fundamental time period with respect to beam-to-column inertia ratio ($\alpha = I_b/I_c$) for frames with spans of: (a) 5 m and (b) 7 m.

Effect of member elasticity

To investigate the influence of material stiffness, a sensitivity analysis is conducted by changing the value of Young's modulus over a reasonable range. For instance, the analysis of the frame model can be conducted using moduli of 20 GPa, 25 GPa, 30 GPa, and 35 GPa to model various material stiffnesses. A comparison between fundamental time periods from FEM analyses and the suggested equation in addition to the equations from literature [5, 6, 23] assists in determining the sensitivity of the equation to the variation of material properties.

Fig. 18 shows the variation in the fundamental time period of moment-resisting frames as a function of the modulus of elasticity (E), for 10-story and 20-story frames with a span of 6 m. The performance of the proposed equation is evaluated against FEM results and benchmarked against existing equations by [5, 6, 23]. The statistical parameters coefficient of determination (R^2) and root mean square error (RMSE) are used to assess the predictive accuracy of each equation.

In Fig. 18a, corresponding to the 10-storey frame, the new formula demonstrates good consistency with FEM results with $R^2 = 0.91$ and low RMSE = 0.0146 s, which indicates high reliability and accuracy in capturing the role of material stiffness in dynamic response. The traditional formulas are of poor performance, like Aninthaneni and Dhakal's equation[23] provides a negative R^2 value of 1.71 and RMSE of 0.064 s, reflecting poor fit and inability to track the observed trend. Goel and Chopra's equation [5] gives $R^2 = -1.3$ and RMSE = 0.062 s, whereas Salama's equation [6] is the poorest with $R^2 = -14.4$ and RMSE = 0.16 s. The negative R^2 for both of these models indicates that they are poorer in prediction, which further establishes their weakness.

In Fig. 18b, for the 20-storey frame, the superiority of the selected equation is clearer. It achieves an R^2 of 0.97 and an RMSE of 0.021 s, confirming its strength and versatility for taller structural configurations. Conversely, the remaining models show considerable disagreement with FEM results, with [5, 23] recording R^2 of –14.8 and –15.9, and RMSE of 0.45 and 0.47 s, respectively. Although Salama's equation [6] shows comparatively better performance ($R^2 = -0.08$, RMSE = 0.12 s), it still fails to provide satisfactory accuracy. Generally, statistical comparison of 10 and 20-story frames confirms that the new formula derived herein correctly outmatches existing empirical formulas in modeling the effect of modulus of elasticity on the fundamental time period. Its considerably higher R^2 values and significantly lower RMSEs confirm its adequacy for practical use in seismic analysis and design, especially where the change of material stiffness is of paramount concern to structural dynamics.

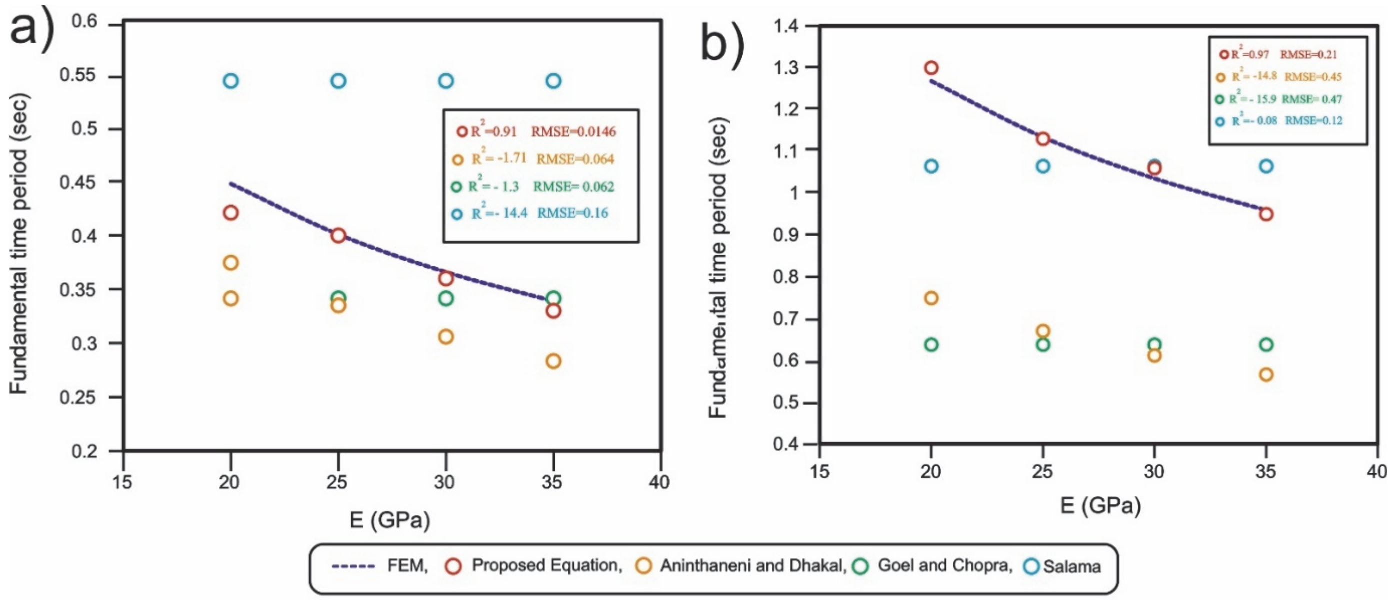

Figure 18: Variation of fundamental time period with respect to modulus of elasticity (E) for frames with 6 m span: (a) 10-story frame, and (b) 20-story frame, comparing FEM results with the proposed equation.

Mass-related effects

The total seismic mass of a building, comprising the weight of the structure and non-structural materials in addition to superimposed live loads, directly influences its fundamental period. An increase in mass is expected to result in a longer period, as it lowers the natural frequency of vibration for a given stiffness. To quantify this influence, a sensitivity analysis was performed for varying seismic mass values. Fig. 19 illustrates the chart of the fundamental period as a function of seismic mass comparing predictions from the suggested equation, the Aninthaneni & Dhakal model [23], Goel & Chopra model [5], and Salama model [6] with FEM results. As would be expected from fundamental vibration theory, the FEM computation does exhibit a clear-cut increasing trend for fundamental period with added seismic mass. The proposed equation accurately follows this trend and shows very good correlation with FEM computations of (R^2=0.998, RMSE = 0.039), implying that it well-represents the dynamic mass-dependent behavior.

On the other hand, the Aninthaneni & Dhakal model [23], which is positively correlated (R^2=0.81), substantially underestimates the period for heavy masses. This shows that the model is not entirely considering the influence of mass on the period for heavy structures. The Goel & Chopra [5] (R^2=0.14) and Salama (R^2=0.62) [6] models are considerably less responsive to mass change. Their predictions are relatively flat across the tested mass range, indicating that their models implicitly perhaps are postulating a more rigid stiffness-dominated or geometry-governed period control with minimal explicit reference to seismic mass. As such, these models widely deviate from FEM projections for larger structures, leading to larger RMSE values (1.09 and 0.71 respectively). Overall, the discussion highlights the need for explicit and clear mass-dependence in empirical period equations, particularly in buildings whose seismic mass can change significantly by occupancy loads, non-structure elements, or large machinery. The proposed equation has this capability best, with performance prediction being accurate over a broad range of mass

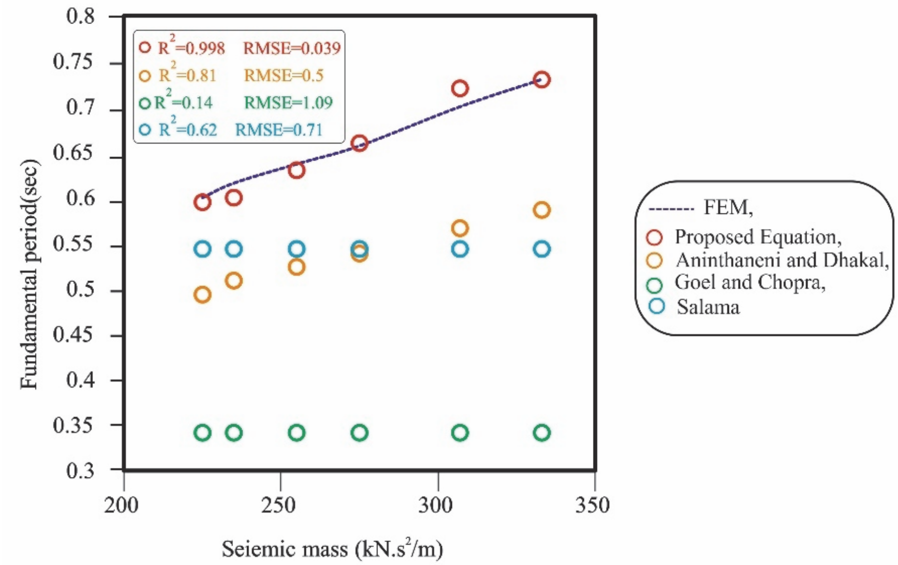

Figure 19: Comparison of fundamental period variation with seismic mass as predicted by the proposed equation, Aninthaneni & Dhakal model, Goel & Chopra model, and Salama model against FEM results.

Key influencing parameters on the fundamental period: insights from sensitivity analysis

Sensitivity analysis highlighted that the fundamental period of moment-resisting frames is mostly governed by geometric parameters, in which height of the building is the most prominent parameter. Increase in height leads to a considerable increase in the period due to the significant reduction in the lateral stiffness and the associated increase in the lateral displacements. Seismic mass was the second most important parameter, since greater mass would elevate the system inertia and consequently the fundamental period. This consequence is always more definite in heavy structures, where code-based empirical relations simplifying or disregarding mass dependence do significantly deviate from finite element computations. Span length also had a measurable, though lesser, effect; longer spans reduce global stiffness, hence prolonging the period. Modulus of elasticity and beam-to-column inertia ratio, which are stiffness parameters, had fairly small effects in the typical ranges for reinforced concrete frames. The results generally indicate that seismic mass and height must be considered explicitly and accurately in predictive models and other parameters are second-order effective on the fundamental period.

Visual enhancement of sensitivity analysis results

To facilitate interpretation of the sensitivity analysis outcomes, the results are presented in the form of heat maps that allow for a direct visual comparison across different parameter combinations. Fig. 20 presents two heat maps that visually summarize the ranking of many parameter combinations according to their agreement with FEM results. To analyze the influence of different structural parameters on the accuracy of fundamental period prediction, heat maps were generated to separately evaluate the effect of the beam-to-column moment of inertia ratio (α) and the span length, with variations examined across different numbers of stories. Fig. 20a illustrates the sensitivity of the fundamental period prediction error, classified into five levels (1 = minimum error, 5 = maximum error), as a function of the beam-to-column moment of inertia ratio (α) and the number of stories. The results indicate that high values of α (stiffer beams) give low errors for low- and mid-rise MRFs, but the error increases moderately with height. Very small values of α (flexible beams) are always associated with higher errors, especially in tall MRFs, indicating that lower beam stiffness exaggerates discrepancies with FEM solutions. Intermediate α values exhibit mixed behavior, performing well for mid- to high-rise cases but producing large errors in low-rise buildings. Overall, the trend reflects a non-linear, height-dependent relationship of α with prediction accuracy, highlighting the requirement for proposed equations to explicitly account for stiffness ratios, particularly especially for low-rise models.

The heatmap shows in Fig. 20b how the level of prediction error for the fundamental period varies with frame span and building height (number of stories). Shorter spans (6–7 m) at lower error levels (1–2) are clustered, as well as at moderate heights (≤10 stories), whereas taller buildings at moderate spans (9–12 m) perform erratically. The most significant error levels (4–5) occur mainly at long spans (≥12 m) with minimal stories or very tall structures, implying that both the extreme span–height ratios lead to greater deviation in period estimation.

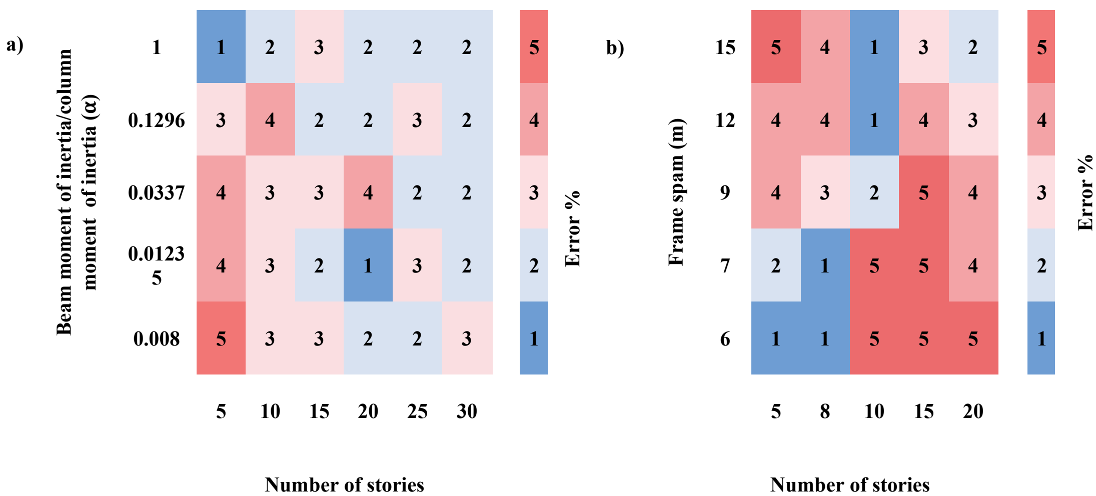

Figure 20: Heat maps showing the sensitivity of fundamental period prediction error to variations in (a) α versus number of stories and (b) span length versus number of stories.

Statistical indicators of agreement with FEM

To verify the efficacy of the provided analytical formula in estimating the fundamental time period, a comparison was also made using FEM results. Tab. 11 illustrates the comparison for a number of structural configurations that involve various span lengths, column and beam cross-sections, and building heights. The result shows a very good agreement between the values computed by the new approach and those computed using numerical analysis. FEM/proposed ranges from 0.96 to 1.04, with an average value of approximately 1.00. This reveals that the proposed model is not biased toward overestimation or underestimation of the fundamental period.

Besides, the coefficient of variation (COV) is just 2%, and the standard deviation of ratios is 0.02, reflecting the uniformity and consistency of the proposed formulation. For the different structural configurations, the close agreement between numerical and analytical solutions verifies the capability of the proposed equation in simulating the dynamic properties of the framed buildings. This accuracy renders the proposed method an effective tool in early design and seismic analysis

Span (m)	Col cross section (m)	Beam cross section (m)	Building height (m)	Fundamental time period		
				FEM (sec)	Proposed Equation (sec)	FEM/proposed
4	0.4*0.4	0.3*0.3	30	0.99	0.95	1.04
4	0.4*0.4	0.4*0.4	30	0.775	0.77	1.006
4	0.5*0.5	0.4*0.4	30	0.756	0.78	0.969
5	0.4*0.4	0.4*0.4	30	0.822	0.82	1.002
5	0.5*0.5	0.5*0.5	45	1.12	1.1	1.018
6	0.6*0.6	0.5*0.5	45	1.181	1.22	0.96
6	0.6*0.6	0.6*0.6	45	1.023	0.99	1.03
7	0.7*0.7	0.5*0.5	60	1.833	1.82	1.007
7	0.7*0.7	0.6*0.6	60	1.536	1.55	0.99
		Mean			1	
		St.deviation			0.02	
		COV%			2	

Table 12: Comparison of the predicted fundamental periods using the proposed model and numerical modal analysis.

LIMITATIONS AND SCOPE OF APPLICABILITY

Although the proposed analytical model is of very high accuracy when used to estimate the fundamental time period of common RC moment-resisting frames, certain limitations of methodology should be taken into account in order to conclude its applicability range. First, the model has been calibrated and validated for regular configurations with evenly distributed stiffness and mass distribution. Its applicability to irregular buildings like those with setbacks, reentrant corners, torsional eccentricities, or mixed lateral systems (shear walls, braced frames) has not been proven and may vary from the results presented unless re-calibrated. This aligns with findings of Suthar and Purohit [40], who reported erratic deviations in the plan-irregular RC MRF durations from conventional height-based expected formulas, and proposed customized formulations with irregularity cases.

The FEM validation presumed fixed-base conditions, neglecting soil-structure interaction (SSI). Flexible foundation conditions, particularly in soft soils, can increase the period, alter lateral displacements, and modify damping characteristics. Camayang et al. [41] emphasized that SSI effects are typically underestimated in analytical equations, while they have a great influence on seismic response, and recommended site-specific assessments where foundation flexibility is expected.

Finally, the calibration process employed a linear-elastic FEM approach, which, although suitable for first-order approximations, cannot capture material nonlinearity, cracking, stiffness reduction, or energy dissipation processes that occur under strong seismic shaking. Mohamed et al. [17] demonstrated that cracking and nonlinear effects can significantly alter the fundamental period of RC moment-resisting frames compared to predictions in linear terms, which poses an imperative to consider these effects in model application. These limitations need to be overcome when the proposed model is applied in practice, and further study is encouraged to extend its application to irregular configurations, SSI conditions, and nonlinear structural responses.

CONCLUSIONS

It has been observed that many forms of fundamental vibration period equations with varying period predictions have been found in the literature, ranging from complex to simple. Because of the significant scatter and the structural differences among the available equations, most of which are based on regression analysis, this paper tries to develop a more reliable analytical formula incorporating both mass and stiffness parameters. The results show that the proposed equation provides good estimates of the fundamental period of MRFs. In the case of models analyzed herein, the maximum and average errors in the fundamental period obtained by using the proposed formula compared to those derived from eigenvalue analysis of the numerical model are no more than 5% and 6%, respectively. Thus, this equation gives an effective closed-form solution for estimating the natural vibration period. The proposed equation demonstrates very good agreement with the FEM results, as evidenced by high R^2 values and root mean square error (RMSE). These statistical metrics are important in assessing the predictive accuracy and general performance of the proposed model since they measure the extent of deviation and strength of correlation between predicted results and reference values.

The following significant conclusions are reached from the present study:

- The current study proposes a simplified analytical approach to estimate the fundamental vibration period of reinforced concrete moment-resisting frame (RC-MRF) buildings. By idealizing the frame as an equivalent single-degree-of-freedom (SDOF) system, dynamic analysis is considerably simpler.
- It was found that the first two storeys contribute predominantly to the overall lateral stiffness, which enabled the derivation of a simplified expression for equivalent frame rigidity. In addition, the study proposed a practical formula to compute the building's equivalent seismic mass, supported by a correction factor to account for differences in cross-sectional properties between beams and columns.
- To enhance the usability of the mass estimation process, a curve was formulated to enable the usage of the proposed equation without sacrificing any accuracy. The final proposed model was validated through a comprehensive sensitivity study that demonstrated near correspondence with finite element method (FEM) outcomes. The model also demonstrated versatility across a range of structural configurations, including building height, span length, member stiffness ratio, and material elasticity.
- Overall, the proposed formula is a robust and efficient method for seismic preliminary assessment and could be employed as a foundation for further detailed analytical studies or code applications. The proposed equation predicts the fundamental time period with good accuracy a mean FEM/proposed value of 1.0 and with small scatter (COV 2%).

The small variations due to span, section dimensions, and height have negligible influences, demonstrating the robustness of the equation. The proposed equation is of higher predictive accuracy compared to existing equations, and therefore, it is a reliable technique for the estimation of natural vibration periods. The technique has also been tested to be applicable in high-rise buildings up to 30 stories. The formula takes into account seismic mass and the strength of concrete in the considered direction. Vertical frame irregularities are also taken into account. In future work, refining the suggested formulas will be developed to incorporate the effect of the number of bays in high-rise buildings and soil-structure interaction parameter variations.

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