FACTA UNIVERSITATIS
Series: **Mechanical Engineering**
https://doi.org/10.22190/FUME250425032M

Original scientific paper

AN OPTIMIZATION SCHEME FOR THE FUZZY COST-BASED ASSEMBLY LINE BALANCING PROBLEM FOR THE CASE OF NUZZLE PRODUCTION LINE IN PETROLEUM INDUSTRIES

Ali Mahmoodirad[1], **Dragan Pamucar**[2,3,4], **Sadegh Niroomand**[5]

[1]Department of Mathematics, Bab. C., Islamic Azad University, Babol, Iran
[2]Department of Operations Research and Statistics, Faculty of Organizational Sciences, University of Belgrade, Belgrade, Serbia
[3]Department of Industrial Engineering & Management, Yuan Ze University, Taoyuan City, Taiwan
[4]Department of Applied Mathematical Science, College of Science and Technology, Korea University, Sejong, Republic of Korea
[5]Department of Industrial Engineering, Firouzabad Higher Education Center, Shiraz University of Technology, Shiraz, Iran

Abstract. *In this study an assembly line configuration is obtained for the nuzzle production line in petroleum industries. This is an important product which is widely used petroleum industries. Therefore, applying an optimization scheme for obtaining an optimal configuration is necessary. For this aim, a cost-based mathematical formulation is proposed to obtain the optimal assembly line configuration. In this model, overall station establishment cost, fixed salary, and variable wages are optimized simultaneously. In order to be close to real-world situations, the problem is formulated in a triangular fuzzy environment, where the cost- and time-based parameters are represented by fuzzy values. The proposed fuzzy formulation is converted to a crisp form using a ME measure of fuzzy sets and numbers. Then, in order to evaluate the proposed crisp formulation, a case study from the petroleum industries of Iran is considered. Based on the performed experiments and obtained results, the best configuration of the assembly line is obtained, and a sensitivity analysis is performed as well.*

Key words: *Assembly line balancing, Nuzzle production, Petroleum industry, Fuzzy sets and numbers, Mathematical modeling*

Received: April 25, 2025 / Accepted October 07, 2025

Corresponding authors: Sadegh Niroomand, Dragan Pamucar
Shiraz University of Technology, Shiraz, Iran; Korea University, Sejong 30019, Republic of Korea
E-mail: niroomand@sutech.ac.ir, dpamucar@gmail.com

1. INTRODUCTION

Assembly line is a type of production systems that are widely used in rapid production of the products consisting of several sub-assemblies and raw parts. This type of production system is widely used in the discrete type of production systems in order to make a faster production with more qualitative products [1]. An assembly line consists of several consecutive workstations where in each workstation some operational tasks are to be performed by one or more operators using manual or semi-automated operational equipment. A main characteristic of an assembly line is its cycle time. This is the time that all tasks of each station should be performed as a cycle. For producing a product on an assembly line, when assigning the required operational tasks of the product to the stations of the line, the precedence relationships of the tasks should be respected in the consecutive stations too. Here, respecting the limitations of the cycle time and the precedence relationships of the tasks and optimizing an objective function like minimization of the number of stations forms a problem called assembly line balancing problem [2,3,4,5,6,7].

According to Boysen et al. [8] the assembly line balancing problem is divided into two types of (1) simple assembly line balancing problem (SALBP), and (2) general assembly line balancing problem (GALBP). The SALBPs are divided to some types such as SALBP-1, SALBP-2, SALBP-E, and SALBP-F, where, the GALBPs are divided to some types such as mixed model assembly line balancing problem (MALBP) and U-shaped assembly line balancing problem (UALBP). The SALBP-1 considers a constant cycle time and minimized the number of established stations. The SALBP-2 considers a constant number of stations and minimizes the cycle time. The SALBP-E simultaneously maximizes the line efficiency and minimizes the cycle time and number of the established stations. The SALBP-F generates a feasible assembly line for a combination of a given cycle time and a given number of stations. In the MALBP, more than one type of product is assembled on one assembly line, while in the UALBP, the physical configuration of assembly line is U-shaped.

In recent years, the topic of assembly line balancing has gain many interests from in the areas of industrial engineering and mechanical engineering. Adeppa [9] studied the basic models of assembly line balancing problem. Sungur and Yavuz [10] studied the assembly line balancing problem with hierarchical worker assignment. Heydari et al. [11] proposed an entropy-based mathematical formulation for straight assembly line balancing problem. Abdullah Make et al. [12] performed a review of the two-sided assembly line balancing problems. Pereira and Álvarez-Miranda [13] proposed an exact approach for the robust assembly line balancing problem. A multi-objective assembly line balancing problem with worker's skill and qualification considerations in fuzzy environment was studied by Zamzam and Elakkad [14]. Fathi et al. [15] performed a study based on comparative evaluation of heuristics and computational assessment of objectives for the assembly line balancing problems. Abdous et al. [6] introduced an uncertain multi-objective assembly line balancing problem and solved it by a credibility-based fuzzy modeling approach. Şahin and Tural [16] Proposed an effective hybrid fuzzy programming approach for an entropy-based multi-objective assembly line balancing problem. Álvarez-Miranda and Pereira [13] provided study and focused on the complexity nature of the assembly line balancing problems. A systematic review of research themes and hot topics in assembly line balancing through the web of science within the years 1990–2017 was performed by Didden et al. [17]. Liu et al. [18] introduced some classical and hybrid meta-heuristic algorithms to solve a new cost-oriented assembly line balancing problem. El Abidine and Koltai [19] introduced a new and effective hybrid goal programming approach for multi-objective straight assembly line balancing problem with stochastic parameters. Boysen et al. [1] introduced a new hybrid artificial electric field algorithm for assembly line balancing problem with equipment model selection possibility.

Fink et al. [20] introduced several hybrid meta-heuristic algorithms for U-shaped assembly line balancing problem with equipment and worker allocations. Hou and Zhang [21] introduced some new criteria and mathematical formulations for workload smoothing in straight assembly line balancing problem. Li et al. [22] introduced a sustainable uncertain integrated supply chain network design and assembly line balancing problem with U-shaped assembly lines and multi-mode demand. Sheibani and Niroomand [23] developed an optimization model for sustainable multi-product multi-echelon supply chain networks with U-shaped assembly line balancing under uncertainty. For more about this topic, the study of Niroomand and Vizvari [24] for mathematical formulation, the study of Mahmoodirad and Niroomand [25] for uncertain modelling, the study of Sing et al. [26] for fuzzy modelling, the study of Imran et al. [27] for decision-making approaches, and Mishra and Rani [28] for fuzzy modeling and decision-making can be referred.

In this paper a novel study is performed on optimization of the assembly line of the nuzzle for petroleum industries. Generally, optimization of industry related problems is very important issue [29,30,31]. For this aim a cost-based mathematical formulation is proposed for such assembly line balancing problem. In this model, overall station establishment cost, fixed salary, and variable wages are optimized simultaneously. In order to be close to real-world situations, the problem is formulated in a triangular fuzzy environment, where the cost- and time-based parameters are represented by fuzzy values. The proposed fuzzy formulation is converted to a crisp form using a ME measure of fuzzy sets and numbers. This is for the first time in the literature that such measure is used in fuzzy assembly line balancing problems. Then, in order to evaluate the proposed crisp formulation, a case study from the petroleum industries of Iran is considered. Based on the performed experiments and obtained results, the best configuration of the assembly line is obtained, and a sensitivity analysis is performed as well.

The rest of this paper is organized in some sections. Section 2 represents the nuzzle production system and the case study. Section 3 presents the fuzzy and crisp mathematical formulations of the nuzzle production system. Section 4 includes the computational study and the obtained results. Section 5 represents some concluding remarks.

2. PROBLEM DESCRIPTION - NUZZLE PRODUCTION ASSEMBLY LINE

As mentioned earlier, a typical nuzzle production line is to be balanced in this study. The considered nuzzle is made of several parts and needs several assembly operations to be completed. As a case study this nuzzle is to be produced in petroleum industries of Iran and it has a wide range of applications in that environment. All information and data of this assembly line is obtained from the petroleum industries of Iran. This type of nuzzle requires the raw parts described by Table 1.

Table 1 The raw parts required for producing a unit of the nuzzle

No.	Raw part
1	Fuel connection
2	Mounting flange
3	Feed arm
4	Shroud
5	Deflector valve
6	Spring locking ring
7	Plastic part

On the other hand, some assembly operations (tasks) are required to complete one unit of the nuzzle on an assembly line. These operations and their operating times are described by Table 2. Furthermore, the precedence relationship graph of these assembly tasks is presented by Fig. 1.

Table 2 The assembly tasks for producing a unit of the nuzzle and their operating times

No.	Assembly task	Triangular fuzzy operating time (minutes)	Triangular fuzzy variable cost (wage) ($)
1	Fuel connection preparation	(4, 5, 6)	(0.1, 0.2, 0.3)
2	Mounting flange preparation	(5, 6, 7)	(0.1, 0.2, 0.3)
3	Feed arm preparation	(3, 4, 6)	(0.1, 0.2, 0.4)
4	Shroud preparation	(3, 4, 5)	(0.1, 0.3, 0.4)
5	Deflector valve preparation	(4, 6, 7)	(0.1, 0.2, 0.3)
6	Spring locking ring preparation	(5, 6, 8)	(0.2, 0.3, 0.4)
7	Plastic part preparation	(3, 4, 5)	(0.1, 0.2, 0.3)
8	Part 1 to part 2 assembly	(5, 7, 8)	(0.3, 0.4, 0.5)
9	Part 3 to part 2 assembly	(9, 10, 11)	(0.2, 0.3, 0.4)
10	Parts 4 and 5 to part 1 assembly	(4, 5, 7)	(0.2, 0.3, 0.4)
11	Part 6 to part 5 assembly	(2, 4, 5)	(0.3, 0.4, 0.5)
12	Part 7 to part 6 assembly	(4, 6, 8)	(0.3, 0.4, 0.5)

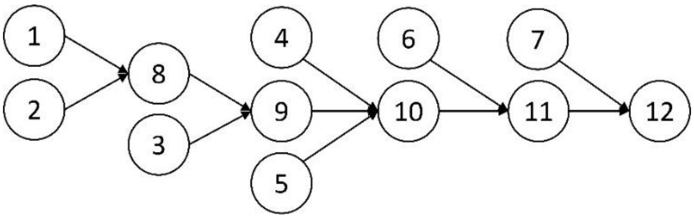

Fig. 1 The precedence diagram of the nuzzle

The triangular fuzzy cycle time of (20, 23, 27) minutes is considered for this assembly line where in each station of the line one operator with approximate triangular fuzzy salary of $(1000, 1100, 1300) per month will work. On the other hand, each station can be established by triangular fuzzy average cost of $(10000, 11000, 12000). The managers aim to balance and establish this line by considering establishment cost, operators salaries, and workload smoothness.

This is notable to mention that the fuzzy data of this case study are estimated from the historical data of the existing production systems of the considered product.

In the next section, a mathematical formulation is proposed for balancing this assembly line according to the preferences of the managers.

3. PROPOSED MATHEMATICAL FORMULATIONS

In this section, the problem of Section 2, first, is formulated as a fuzzy model. Then its equivalent crisp formulation is derived.

3.1. Fuzzy Formulation

In order to formulate the assembly line of the nuzzle production line described in Section 2, the below assumptions and the notations of Table 3 are considered in advance.

- The overall cost such as station establishment cost, fixed salary, and variable wage of all stations is to be minimized.
- Each task is assigned to only one station.
- The variable wage of the worker of a station is rated based on the task with highest rate in that station.
- All cost and time based parameters are represented by triangular fuzzy values.

Table 3 The notations used in the formulations of the paper

Notation	Nature	Description
$i(I)$	Index	Index used for task (number of tasks)
$k(K)$	Index	Index used for station (maximum number of stations)
$\tilde{t}_i = \left(t_i^{(1)}, t_i^{(2)}, t_i^{(3)}\right)$	Parameter	Triangular fuzzy processing time of task *i*
$\widetilde{ct} = \left(ct^{(1)}, ct^{(2)}, ct^{(3)}\right)$	Parameter	Triangular fuzzy cycle time
$\widetilde{stb} = \left(stb^{(1)}, stb^{(2)}, stb^{(3)}\right)$	Parameter	Triangular fuzzy establishment cost of each station
$\widetilde{sal} = \left(sal^{(1)}, sal^{(2)}, sal^{(3)}\right)$	Parameter	Triangular fuzzy salary paid to the worker of each station
$\tilde{c}_i = \left(c_i^{(1)}, c_i^{(2)}, c_i^{(3)}\right)$	Parameter	Triangular fuzzy variable processing cost of task *i*
PR_i	Parameter	Predecessor set of task *i*
X_{ik}	Binary variable	1, if task *i* is assigned to station *k* 0, otherwise
W_k	Binary variable	1, if station *k* is opened 0, otherwise
VC_k	Variable	Variable cost (wage) of station *k*

Therefore, based on the above-mentioned assumptions and the notations of Table 3, the below fuzzy formulation is presented assembly line balancing of the nuzzle production line of Section 2.

$$OF = \min \sum_{k=1}^{K} \left(\left(stb + sal \right) W_k + VC_k \right) \tag{1}$$

subject to

$$X_{ik} \le \frac{\sum_{j \in PR_i} \sum_{r \le k} X_{jr}}{\left| PR_i \right|}, \forall i,k \tag{2}$$

$$\sum_{k=1}^{K} X_{ik} = 1, \forall i,k \tag{3}$$

$$W_k \ge X_{ik}, \forall i,k \tag{4}$$

$$\sum_{i=1}^{I} \tilde{t}_i X_{ik} \le ct, \forall k \tag{5}$$

$$\tilde{c}_i X_{ik} \le VC_k, \forall i,k \tag{6}$$

$$X_{ik}, W_k \in \left\{ 0,1 \right\}, \forall i,k \tag{7}$$

$$VC_k \ge 0, \forall k \tag{8}$$

In the above formulation, the objective function of Eq. (1) minimizes overall cost of the line including establishment cost of each station, salary paid to the worker of each station, and variable wage paid to the worker of each station. The constraint presented by Eq. (2) guarantees that a task can be assigned to a station if its predecessors are assigned to either that station or previous stations. The constraint presented by Eq. (3) respects the assumption that a task can be assigned to only one station. The constraint of Eq. (4) ensures that a station is established if it contains at least one task. The constraint given by Eq. (5) guarantees that the workload of each station cannot exceed the cycle time of the line. The constraint presented by Eq. (6) calculates the variable wage rate of each station according to the above-mentioned assumptions. The constraints provided by Eqs. (7-8) are the sign constraints of the model.

As formulation presented by Eqs. (1-8), is a fuzzy formulation, we cannot directly solve it. Therefore, first we obtain its crisp form, and then the crisp form is solved by any optimization solver. The equivalent crisp formulation of fuzzy formulation presented by Eqs. (1-8) is introduced in the next section.

3.2. Equivalent Crisp Formulation

The possibility theory is an effective approach to deal with optimization problems with fuzzy objective function and constraints [32]. In this theory, there are three classical measures of possibility (POS), necessity (NEC), and credibility (CR) measures for converting a fuzzy constraint to its equivalent crisp form. These measures are of pessimistic, optimistic, and average points of view, where decision makers cannot be flexible when using these measures (see [1]). Instead, a more flexible measure of this theory called ME measure [33] is defined as below, where the weighted average of the possibility and necessity measures of the fuzzy constraint A are considered there.

$$ME\{A\} = \gamma POS\{A\} + (1-\gamma) NEC\{A\} \tag{9}$$

In the above formula, the value of $\gamma = 0$, results in $ME\{A\}=NEC\{A\}$, the value of $\gamma = 1$ results in $ME\{A\}=POS\{A\}$, and $\gamma = 0.5$ results in $ME\{A\}=CR\{A\}$. The below theorem can clearly explain the use of ME measure for a fuzzy constraint with triangular fuzzy parameters. As an advantage, this measure can crisp a fuzzy event from any possibility degree in addition to the necessity, possibility, and credibility measure degrees.

Theorem 1. For triangular fuzzy variable $\xi = (a, b, c)$ and real number r, the following inequalities are defined for any confidence levels β (from the range of $0 < \beta \le 1$) and γ (from the range of $0 < \gamma \le 1$) (see [32]).

$$\begin{gathered}
ME\{\xi \le r\} \ge \beta \Leftrightarrow \begin{cases} \left(\dfrac{\gamma-\beta}{\gamma}\right)a + \left(\dfrac{\beta}{\gamma}\right)b \le r, & \beta \le \gamma \\ \left(\dfrac{1-\beta}{1-\gamma}\right)b + \left(\dfrac{\beta-\gamma}{1-\gamma}\right)c \le r, & \beta > \gamma \end{cases} \\
ME\{\xi \ge r\} \ge \beta \Leftrightarrow \begin{cases} \left(\dfrac{\beta}{\gamma}\right)b + \left(\dfrac{\gamma-\beta}{\gamma}\right)c \ge r, & \beta \le \gamma \\ \left(\dfrac{\beta-\gamma}{1-\gamma}\right)a + \left(\dfrac{1-\beta}{1-\gamma}\right)b \ge r, & \beta > \gamma \end{cases}
\end{gathered} \tag{10}$$

Therefore, the fuzzy model of Eqs. (1-8) is converted to a crisp form using the above-mentioned ME measure. For this aim, first the below model is obtained. It is notable to mention that objective function given by Eq. (1) is considered as a constraint, and the ME measure of its constraint form is considered then.

$$OF = \min f \tag{11}$$

subject to

$$\sum_{k=1}^{K}\Big(\big(stb + sal\big)W_k + VC_k\Big) \le f \tag{12}$$

$$X_{ik} \le \frac{\sum_{j\in PR_i}\sum_{r\le k} X_{jr}}{|PR_i|}, \forall i,k \tag{13}$$

$$\sum_{k=1}^{K} X_{ik} = 1, \forall i,k \tag{14}$$

$$W_k \ge X_{ik}, \forall i,k \tag{15}$$

$$\sum_{i=1}^{I}\tilde{t}_i X_{ik} \le ct, \forall k \tag{16}$$

$$\tilde{c}_i X_{ik} \le VC_k, \forall i,k \tag{17}$$

$$X_{ik}, W_k \in \{0,1\}, \forall i,k \tag{18}$$

$$VC_k \ge 0, \forall k \tag{19}$$

Therefore, the below model is obtained by considering the ME measure of the fuzzy constraints of formulation presented by Eqs. (11-19).

$$OF = \min f \tag{20}$$

subject to

$$ME\left\{\sum_{k=1}^{K}\left(\left(stb + sal\right)W_k + VC_k\right) \le f\right\} \ge \alpha \tag{21}$$

$$X_{ik} \le \frac{\sum_{j\in PR_i}\sum_{r\le k} X_{jr}}{\left|PR_i\right|}, \forall i,k \tag{22}$$

$$\sum_{k=1}^{K} X_{ik} = 1, \forall i,k \tag{23}$$

$$W_k \ge X_{ik}, \forall i,k \tag{24}$$

$$ME\left\{\sum_{i=1}^{I}\tilde{t}_i X_{ik} \le ct\right\} \ge \lambda_k, \forall k \tag{25}$$

$$ME\left\{\tilde{c}_i X_{ik} - VC_k \le 0\right\} \ge \theta_k, \forall i,k \tag{26}$$

$$X_{ik}, W_k \in \{0,1\}, \forall i,k \tag{27}$$

$$VC_k \ge 0, \forall k \tag{28}$$

Now, applying the relationships given by Theorem 1, the below crisp formulation is obtained, which is an equivalent crisp formulation of the fuzzy formulation presented by Eqs. (1-8).

$$OF = \min f \tag{29}$$

subject to

$$\frac{\gamma-\alpha}{\gamma}\left(\sum_{k=1}^{K}\left(\left(stb^{(1)} + sal^{(1)}\right)W_k + VC_k^{(1)}\right)\right) + \frac{\alpha}{\gamma}\left(\sum_{k=1}^{K}\left(\left(stb^{(2)} + sal^{(2)}\right)W_k + VC_k^{(2)}\right)\right) \le f, \alpha \le \gamma \tag{30}$$

$$\frac{1-\alpha}{1-\gamma}\left(\sum_{k=1}^{K}\left(\left(stb^{(2)}+sal^{(2)}\right)W_k+VC_k^{(2)}\right)\right)+\left(\frac{\alpha-\gamma}{1-\gamma}\right)\left(\sum_{k=1}^{K}\left(\left(stb^{(3)}+sal^{(3)}\right)W_k+VC_k^{(3)}\right)\right)\leq f,\alpha>\gamma \tag{31}$$

$$X_{ik}\leq\frac{\sum_{j\in PR_i}\sum_{r\leq k}X_{jr}}{\left|PR_i\right|},\forall i,k \tag{32}$$

$$\sum_{k=1}^{K}X_{ik}=1,\forall i,k \tag{33}$$

$$W_k\geq X_{ik},\forall i,k \tag{34}$$

$$\frac{\gamma-\lambda_k}{\gamma}\left(\sum_{i=1}^{I}t_i^{(1)}X_{ik}-ct^{(3)}\right)+\frac{\lambda_k}{\gamma}\left(\sum_{i=1}^{I}t_i^{(2)}X_{ik}-ct^{(2)}\right)\leq 0,\forall k,\lambda_k\leq\gamma \tag{35}$$

$$\frac{1-\lambda_k}{1-\gamma}\left(\sum_{i=1}^{I}t_i^{(2)}X_{ik}-ct^{(2)}\right)+\frac{\lambda_k-\gamma}{1-\gamma}\left(\sum_{i=1}^{I}t_i^{(3)}X_{ik}-ct^{(1)}\right)\leq 0,\forall k,\lambda_k>\gamma \tag{36}$$

$$\frac{\gamma-\theta_k}{\gamma}\left(c_i^{(1)}X_{ik}\right)+\frac{\theta_k}{\gamma}\left(c_i^{(2)}X_{ik}\right)\leq VC_k,\forall i,k,\theta_k\leq\gamma \tag{37}$$

$$\frac{1-\theta_k}{1-\gamma}\left(c_i^{(2)}X_{ik}\right)+\frac{\theta_k-\gamma}{1-\gamma}\left(c_i^{(3)}X_{ik}\right)\leq VC_k,\forall i,k,\theta_k>\gamma \tag{38}$$

$$X_{ik},W_k\in\{0,1\},\forall i,k \tag{39}$$

$$VC_k\geq 0,\forall k \tag{40}$$

The crisp formulation presented by Eqs. (29-40) is considered as the crisp form of the fuzzy formulation presented by Eqs. (1-8). In order to implement this formulation for the case study of Section 2, the flowchart of Fig. 2 is presented. This notable to mention that, according to the relationship between the confidence levels α, λ_k, Θ_k, and γ, only one constraint from each pair of the constraints presented by Eqs. (30-31), (35-36), and (37-38) is selected in the final model.

4. RESULTS AND SENSITIVITY ANALYSIS

As mentioned earlier, the crisp formulation presented by Eqs. (29-40) is solved for the nuzzle production line data of Section 2 in order to obtain the results and configure the related assembly line. For this aim, the formulation presented by Eqs. (29-40) is coded in GAMS and solved by its CPLEX solver. All required experiments are run on a PC with Core i7-1165G7@ 2.80GHz processor and 16 GB RAM.

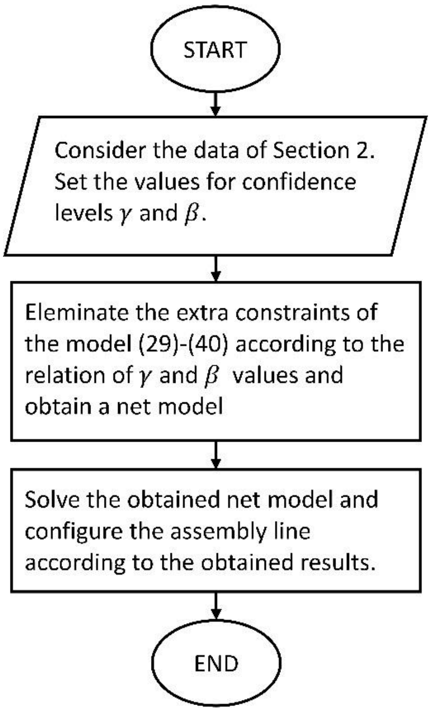

Fig. 2 The flowchart of implementing the proposed formulations for balancing the nuzzle production line

In order to obtain the assembly line configuration, according to the preferences of the managers, the values of $\alpha,\lambda_k,\Theta_k = 0.8$ and $\gamma = 0.6$ are considered. Based on these values, the final crisp formulation consisting of Eqs. (29), (31-34), (36), and (38-40) is obtained and solved for the data of Section 2 and the results of Table 4 are obtained. According to the obtained results, the schematic representation of Fig. 3 is considered to show the related assembly line configuration. In Table 4, the tasks assigned to each established station is marked. The variable wage of each station is reported. The value of f is the optimal value obtained for objective function (29) and the value of $\tilde{f}$ is obtained by the objective function presented by Eq. (1) and the values of the variables obtained in this solution.

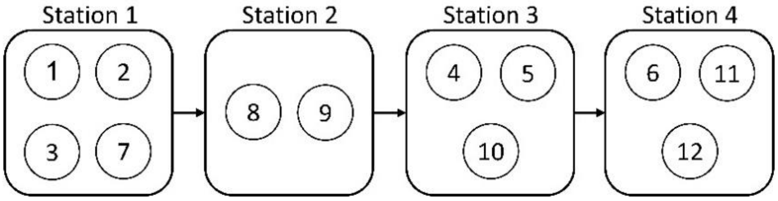

Fig. 3 Schematic optimal configuration of the assembly line for the nuzzle production line of Section 2

Table 4 The optimal results obtained for the nuzzle production line for $\alpha,\lambda_k,\Theta_k = 0.8$ and $\gamma = 0.6$

Task	Established stations				Optimal	Equivalent $\tilde{f}$
	1	2	3	4	f	
1	✓				70485	(61050, 67155, 73815)
2	✓					
3	✓					
4			✓			
5			✓			
6				✓		
7	✓					
8		✓				
9		✓				
10			✓			
11				✓		
12				✓		
Variable wage	0.30	0.45	0.35	0.45		

Furthermore, the sensitivity of the crisp formulation of Eqs. (29-40) to the confidence level values is studied here. For this aim, the data of Section 2 is considered, and the confidence level values are changed (totally 16 experiments are considered). The results for these experiments are obtained by solving the crisp formulation of Eqs. (29-40) are represented by Table 5.

According to the results presented by Table 5, the sensitivity of formulation of Eqs. (29-40) to the confidence level values are studied. The following remarks can be made for this aim.

- In the experiments with the same value of γ, by increasing the value of α, λ_k, and Θ_k, the value of f is increased. This issue happens for each of the elements of the fuzzy objective function value $\tilde{f}$ separately.
- Considering the results of Table 5, by increasing the value of γ, the value of f is increased. This issue happens for each of the elements of the fuzzy objective function value $\tilde{f}$ separately.
- The results obtained by the experiments 1-4, is related to the necessity measure of formulation presented by Eqs. (29-40).
- The results obtained by the experiments 9-12, is related to the credibility measure of formulation presented by Eqs. (29-40).
- The results obtained by the experiments 21-24, is related to the possibility measure of formulation presented by Eqs. (29-40).

From the managerial point of view, the proposed problem of this study is developed and solved to help the managers of the industry to design an optimal assembly line for the nuzzle considering the important costs. On the other hand, the fuzziness of the proposed model can help the managers for more robust and near real-world situation decisions.

Table 5 The results obtained for the sensitivity analysis

Experiment	$\alpha,\lambda_k,\theta_k$	γ	Optimal f	Equivalent $\tilde{f}$
1	0.1	0	64155	(57750, 63525, 69825)
2	0.4	0	68686	(60060, 66066, 72618)
3	0.7	0	73499	(62480, 68728, 75544)
4	1	0	93100	(77000, 84700, 93100)
5	0.1	0.3	43193	(41800, 45980, 50540)
6	0.4	0.3	65739	(58928, 64821, 71250)
7	0.7	0.3	71965	(61914, 68105, 74860)
8	1	0.3	93100	(77000, 84700, 93100)
9	0.1	0.5	42187	(41360, 45496, 50008)
10	0.4	0.5	46807	(43340, 47674, 52402)
11	0.7	0.5	68686	(60060, 66066, 72618)
12	1	0.5	93100	(77000, 84700, 93100)
13	0.1	0.7	41759	(41171, 45288, 49780)
14	0.4	0.7	45019	(42585, 46844, 51490)
15	0.7	0.7	48400	(44000, 48400, 53200)
16	1	0.7	93100	(77000, 84700, 93100)
17	0.1	0.9	41522	(41066, 45173, 49653)
18	0.4	0.9	44551	(42655, 46921, 51574)
19	0.7	0.9	46631	(43266, 47593, 52313)
20	1	0.9	93100	(77000, 84700, 93100)
21	0.1	1	29108	(28820, 31702, 34846)
22	0.4	1	43700	(42020, 46222, 50806)
23	0.7	1	46020	(43010, 47311, 52003)
24	1	1	48400	(44000, 48400, 53200)

5. CONCLUSIONS

In this study a fuzzy mathematical model was introduced and solved for balancing and configuring the nuzzle production line in petroleum industries. This is an important product which is widely used petroleum industries. For this aim, first a cost-based mathematical formulation considering overall station establishment cost, fixed salary, and variable wages was proposed to obtain the optimal assembly line configuration. In order to be close to real-world situations, the problem was formulated in a triangular fuzzy environment with triangular fuzzy cost- and time-based parameters. The proposed fuzzy formulation was converted to a crisp form using a ME measure of fuzzy sets and numbers. Then, in order to evaluate the proposed crisp formulation, a case study from the petroleum industries of Iran was considered. Based on the performed experiments and obtained results, the best configuration of the assembly line was obtained, and a sensitivity analysis was performed as well.

The most important limitation of this study was the high uncertainty nature of the problem that applying fuzzy theory and its extensions, a framework was developed for this difficulty.

REFERENCES

1. Boysen, N., Schulze, P., Scholl, A., 2022, *Assembly line balancing: What happened in the last fifteen years?*, European Journal of Operational Research, 301(3), pp. 797-814.
2. Chutima, P., 2023, *Assembly line balancing with cobots: An extensive review and critiques*, International Journal of Industrial Engineering Computations, 14(4), pp. 785-804.
3. Chutima, P., 2022, *A comprehensive review of robotic assembly line balancing problem*, Journal of Intelligent Manufacturing, 33(1), pp. 1-34.
4. Saracoglu, I., Ozen, F. 2025, *Simulation optimisation of ultrasonography resource scheduling with machine learning*, International Journal of Simulation Modelling, 24(3), pp. 437-448.
5. Keshvarparast, A., Katiraee, N., Finco, S., Calzavara, M. 2025, *Integrating collaboration scenarios and workforce individualization in collaborative assembly line balancing*, International Journal of Production Economics, 279, 109450.
6. Bu, L., Sugirbay, A., Li, T., 2025, *Optimal Combination of Picking Motions for Robotic Apple Harvesting Based on Experimental and Simulation Analysis*, Tehnički Vjesnik, 32(5), pp. 1607-1613.
7. Katiraee, N., Calzavara, M., Finco, S., Battaïa, O., Battini, D., 2023, *Assembly line balancing and worker assignment considering workers' expertise and perceived physical effort*, International Journal of Production Research, 61(20), pp. 6939-6959.
8. Boysen, N., Fliedner, M., Scholl, A., 2007, *A classification of assembly line balancing problems*, European Journal of Operational Research, 183(2), pp. 674-693.
9. Adeppa, A., 2015, *A study on basics of assembly line balancing*. International Journal on Emerging Technologies, 6(2), 294.
10. Sungur, B., Yavuz, Y., 2015, *Assembly line balancing with hierarchical worker assignment*. Journal of Manufacturing Systems, 37, pp. 290-298.
11. Heydari, A., Mahmoodirad, A., Niroomand, S., 2016, *An entropy-based mathematical formulation for straight assembly line balancing problem*, International Journal of Strategic Decision Sciences, 7(2), pp. 57-68.
12. Abdullah Make, M. R., Ab. Rashid, M. F. F., Razali, M. M., 2017, *A review of two-sided assembly line balancing problem*, The International Journal of Advanced Manufacturing Technology, 89, pp. 1743-1763.
13. Álvarez-Miranda, E., Pereira, J., 2019, *On the complexity of assembly line balancing problems*, Computers & Operations Research, 108, pp. 182-186.
14. Zamzam, N., Elakkad, A., 2021, *Time and space multi-manned assembly line balancing problem using genetic algorithm*, Journal of Industrial Engineering and Management, 14(4), pp. 733-749.
15. Fathi, M., Fontes, D.B.M.M., Urenda Moris, M., Ghobakhloo, M., 2018, *Assembly line balancing problem: A comparative evaluation of heuristics and a computational assessment of objectives*, Journal of Modelling in Management, 13(2), pp. 455-474.
16. Şahin, M.C., Tural, M.K., 2023, *Robotic stochastic assembly line balancing*, Flexible Services and Manufacturing Journal, 35(4), pp. 1076-1115.
17. Didden, J.B., Lefeber, E., Adan, I.J., Panhuijzen, I.W., 2023, *Genetic algorithm and decision support for assembly line balancing in the automotive industry*, International Journal of Production Research, 61(10), pp. 3377-3395.
18. Liu, X., Yang, X., Lei, M., 2021, *Optimisation of mixed-model assembly line balancing problem under uncertain demand*, Journal of Manufacturing Systems, 59, pp. 214-227.
19. El Abidine, Z.Z., Koltai, T., 2024, *The effect of learning on assembly line balancing: a review*, Periodica Polytechnica Social and Management Sciences, 32(1), pp. 90-102.
20. Fink, C., Bodin, U. Schelen, O., 2025, *Why decision support systems are needed for addressing the theory-practice gap in assembly line balancing*, Journal of Manufacturing Systems, 79, pp. 515-527.
21. Hou, W., Zhang, S., 2024, *Assembly line balancing and optimal scheduling for flexible manufacturing workshop*, Journal of Mechanical Science and Technology, 38(6), pp. 2757-2772.
22. Li, Z., Janardhanan, M., Tang, Q., Zhang, Z., 2023, *Models and algorithms for U-shaped assembly line balancing problem with collaborative robots*, Soft Computing, 27, pp. 9639-9659.
23. Sheibani, M., Niroomand, S., 2024, *An optimization model for sustainable multi-product multi-echelon supply chain networks with U-shaped assembly line balancing under uncertainty*, Supply Chain Analytics, 5, 100057.
24. Niroomand, S., Vizvári, B., 2013, *A mixed integer linear programming formulation of closed loop layout with exact distances*, Journal of Industrial and Production Engineering, 30(3), pp. 190-201.
25. Mahmoodirad, A., Niroomand, S., 2020, *A belief degree-based uncertain scheme for a bi-objective two-stage green supply chain network design problem with direct shipment*, Soft Computing, 24, pp. 18499-18519.

26. Sing, P., Rahaman, M., Sankar, S.P.M., 2024, *Solution of fuzzy system of linear equation under different fuzzy difference ideology*, Spectrum of Operational Research, 1(1), pp. 64-74.
27. Imran, R., Ullah, K., Ali, Z., Akram, M., 2024, *A multi-criteria group decision-making approach for robot selection using interval-valued intuitionistic fuzzy information and aczel-alsina bonferroni means*, Spectrum of Decision Making and Applications, 1(1), pp. 1-32.
28. Mishra, A.R., Rani, P., 2025, *Evaluating and Prioritizing Blockchain Networks using Intuitionistic Fuzzy Multi-Criteria Decision-Making Method*, Spectrum of Mechanical Engineering and Operational Research, 2(1), pp. 78-92.
29. Zhang, L., Wang, Q., Xia, Y., Xia, Y., Zhong, L., 2025, *Design and performance analysis of dual straight beam MEMS micro mirrors for enhanced scanning capabilities*, Tehnički Vjesnik, 32(5), pp. 1631-1638.
30. Du, J.J., Liu, B.R., Yang, X.D., Chen, Y.G., Zhang, W., 2025, *Dynamic analysis of rotor and machine structure in ultra-high-speed PMSM*, International Journal of Simulation Modelling, 24(3), pp. 497-508.
31. Jin, X., Zheng, J., Hua, S., Tong, X., 2025, *Influencing factors of deformation and failure of porous coal under conventional loading*, Facta Universitatis-Series Mechanical Engineering, 23(1), pp. 161-170.
32. Dehghan, E., Shafiei Nikabadi, M., Amiri, M., Jabbarzadeh, A., 2018, *Hybrid robust, stochastic and possibilistic programming for closed-loop supply chain network design*, Computers & Industrial Engineering, 123, pp. 220-231.
33. Xu, J., Zhou, X., 2013, *Approximation based fuzzy multi-objective models with expected objectives and chance constraints: Application to earth-rock work allocation*, Information Sciences, 238, pp. 75-95.