MODELING INFRA-GRAVITY WAVES USING SCHISM-WWMIII BASED ON IMPROVED FORMULAS AND COUPLING APPROACH

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INTRODUCTION

The two-way coupled phase-resolved long wave (LW) model and phase-averaged spectrum wave (SW) model is often employed to simulate the coastal flooding due to superimposed astronomical high tides and storm surges. The LW model solves the coupled momentum equation

$$\frac{\partial U_j}{\partial t} + U_i \frac{\partial U_j}{\partial x_i} = \frac{\bar{p}}{\rho(\bar{\zeta}+h)} \frac{\partial h}{\partial x_j} - g \frac{\partial \bar{\zeta}}{\partial x_j} - \frac{1}{\rho(\bar{\zeta}+h)} \frac{\partial S_{ij}}{\partial x_i} + F \quad (1)$$

and continuity equation

$$\frac{\partial \bar{\zeta}}{\partial t} + \frac{\partial}{\partial x_i}[U_i(\bar{\zeta}+h)] = 0 \quad (2)$$

where U_j is the depth and time-averaged horizontal velocity with subscript denoting x or y direction, $\bar{\zeta}$ is the time-averaged surface elevation, $\bar{p}$ is the dynamic pressure at seabed, S_{ij} is the stress tensor representing excess momentum flux due to short waves, F represents effects due to surface and bottom shear stresses, g is the gravitational acceleration and h is the local water depth. It simulates relatively long waves including tides and surges in phase-resolved approach. Meanwhile, the SW model solves the wave action balance equation with a range of source terms to describe the evolution of the spectrum of wind waves and swells in phase-averaged approach. The local wave spectrum obtained in the SW model is used to calculate S_{ij} based on the formula of radiation stresses (RS). However, the RS only considers the self-interactions of short waves producing a water level set up/down that corresponds to the zero-frequency wave component. In operational practice, the SW model adopts a cut-off frequency of, e.g., f_c = 0.03 Hz, hence, the waves of frequency $0 < f < f_c$ corresponding to the infra-gravity (IG) waves are ignored during the simulations. Recently, some studies improved the SW model to account for the free IG waves by adjusting the shoreline boundary conditions (Ardhuin, et al., 2014; Rijnsdorp, et al., 2021; Reniers & Zijlema, 2022). This study suggests an alternative approach by proposing a new formula for the RS term to consider the IG waves. The new formula becomes space-time dependent, and it can reduce to the original form of RS and that corresponding to the 'surf-beats' (Longuet-Higgins & Stewart, 1964). The new formula is programmed and implemented in the model suite SCHISM-WWMIII (Zhang, et al., 2016), which is used to simulate short wave groups induced bound and free long waves on variable water depth. The good agreement between numerical results and theoretical predictions (Liu, 1989) indicates that the new formula and coupling approach can be accurately used to describe the scattering of the IG waves in coastal waters.

FORMULATIONS

The new formula for the improved RS term is given as (Wang & Liu, 2022):

$$S_{ij} = \frac{\rho g}{4} \sum_{m,l} A_{m,l}^{i,j} a_m a_l \cos\left[\int (\boldsymbol{k_m} - \boldsymbol{k_l}) \cdot \mathrm{d}\boldsymbol{x} - (\omega_m - \omega_l)t + \varphi_m - \varphi_l\right] \quad (3)$$

where a, $\boldsymbol{k}$, ω and φ are the amplitude, wavenumber, circular frequency, and phase of the short wave component, respectively, with the indices m and l to account for self-interactions $(m = l)$ and difference-interactions $(m \neq l)$. The interaction coefficient $A_{m,l}^{i,j}$ follows as:

$$A_{m,l}^{i,j} = \frac{2k_m^i k_l^j(\omega_m^2-\omega_l^2)}{\omega_m\omega_l(k_m^2-k_l^2)} + \delta_{ij}\left\{\frac{2k_m^i(k_m^i-k_l^i)k_l^2}{\omega_m\omega_l(k_m^2-k_l^2)}\left[\left(\frac{\omega_m^2\omega_l^2}{g^2k_l^2} - 1\right)gh - \frac{\omega_l^2}{k_l^2} + \frac{2(\omega_m^2-\omega_l^2)}{(k_m^2-k_l^2)}\right] + 1 - \frac{2(k_m^2\omega_l^2-k_l^2\omega_m^2)}{\omega_m\omega_l(k_m^2-k_l^2)}\right\} \quad (4)$$

where δ_{ij} is the Kronecker delta function. For $m = l$, Eq.(3) reduces to the RS of Longuet-Higgins & Stewart (1964).

For unidirectional short incident waves on constant water depth, the resulted surface elevation of the forced IG waves that are bounded to the short waves can be expressed as:

$$\bar{\zeta} = -\frac{g}{4} \sum_{m,l} A_{m,l}^{xx} \frac{a_m a_l}{gh - c_g^2} \cos[(k_m - k_l)x - (\omega_m - \omega_l)t + \varphi_m - \varphi_l] \quad (5)$$

which is equivalent to the expression of 'surf beat' if the short waves are composed of only two components (Longuet-Higgins & Stewart, 1964).

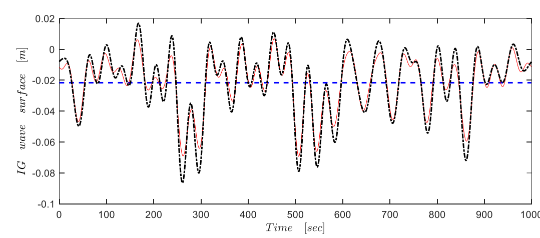

Figure 1 - Comparisons of IG wave surface. Red: Present theory; Black: Second-order potential flow theory; Blue dash line: Radiation stresses induced mean set-down.

Let's now consider constant water depth of 50m and short waves based on a JONSWAP spectrum with significant height 4.8m, peak frequency 0.1Hz and peak enhancement factor 3. The resulted long wave surface is compared with that derived based on the second-order potential flow theory (Okihiro, Guza, & Seymour, 1992), and they are displayed in Figure 1. It shows that the wave surfaces are comparable, though the present theory yields slightly smaller wave amplitudes. The difference may be due to the shallow water assumption being adopted to derive Eq. (5).

Regarding different short wave spectra, the spectra of the bound IG waves can be estimated, and they are presented in Figure 2(a). It shows that the shapes of the IG wave spectra largely depend on the energy density distribution of the short waves. In addition, Figure 2(b) displays the IG waves spectra subject to different water depth given the same input short wave spectrum. It is found that the energy of the IG waves is enhanced when water depth reduces. Therefore, Eqs. (3) - (5) can be used to estimate the forced IG waves that are bounded to the short waves. Whereas on a variable water depth, free IG waves can be excited when short wave groups propagate over a rapidly changing bathymetry. This will be studied in the following section by using numerical simulations.

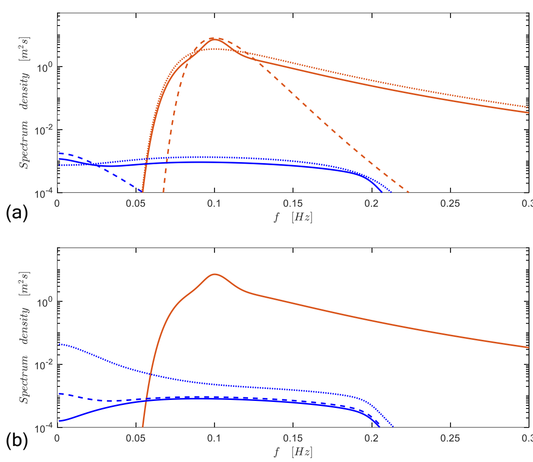

Figure 2 - Spectra of free/input waves (red line) and the bound IG waves (blue lines). In (a): Dot - PM spectrum; Solid - JONSWAP with γ = 3; Dash - Wallops with m = 20. In (b): Solid - h=100m; Dash - h=50m; Dot - h=20m.

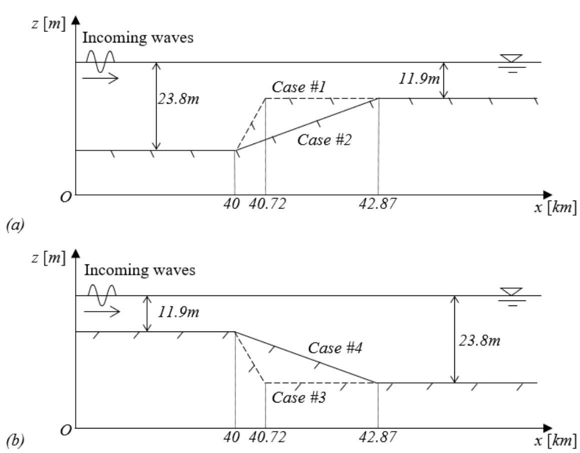

Figure 3 - Bathymetry of the plane slope beaches.

NUMERICAL SIMULATIONS AND VERIFICATIONS

The two-way coupled SCHISM-WWMIII model (Zhang, et al., 2016) is employed here. A computer program based on the new formulas for the improved RS term is introduced to the model. The model is then used to simulate the scattering of long waves when short wave groups propagate over a plane beach. Four scenarios are considered, and the slope profiles are depicted in Figure 3. The wave group consists of two wave components with the same amplitude of 2.42 m and frequencies of 0.1 Hz and 0.105 Hz, respectively. The propagation of wave groups is simulated in the WWMIII, and the resulted bound and free long waves are modelled by SCHISM.

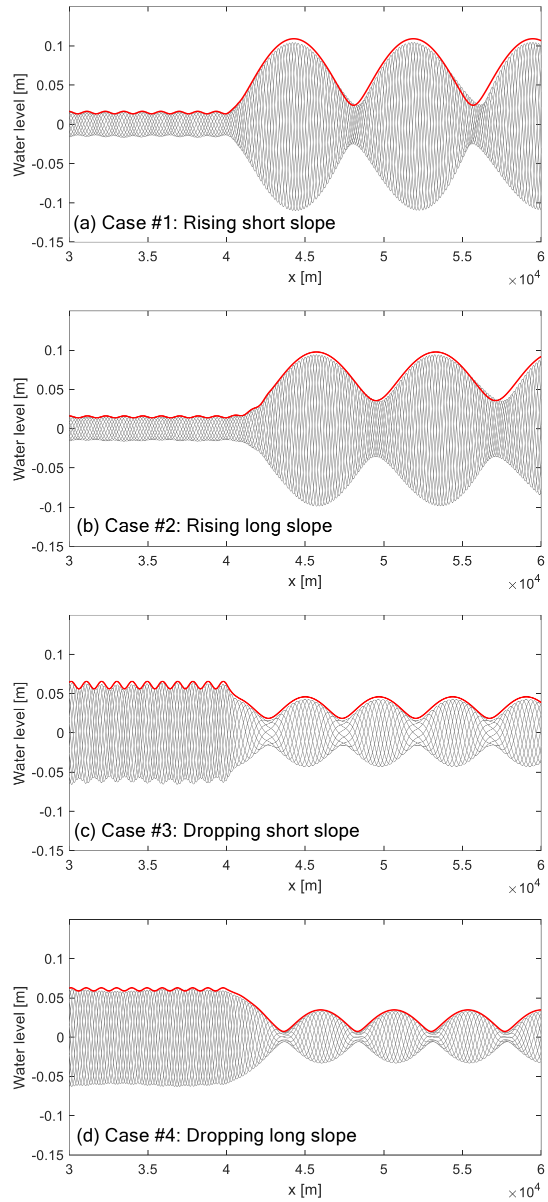

Figure 4 - Comparisons between the simulated long waves and the theory for plane slopes. Red: Envelope height of the wave surface based on the theory of Liu (1989). Black: Simulated wave surface elevations.

After the simulations, the snapshots of the long wave surfaces are extracted and displayed in Figure 4. They are compared with the wave surface envelope as predicted based on the theory of Liu (1989). Over the plane beach, free long waves will be generated and superimpose with the bound long waves, creating long wave groups of different lengths before and after the plane beach. The agreement between the numerical results and theory indicates that the scattering of free long waves is successfully captured in the present numerical simulations.

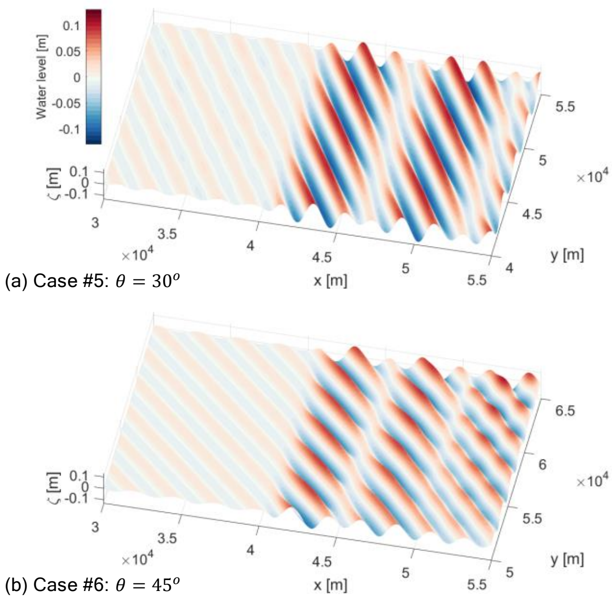

Figure 5 – Simulated long wave surfaces for obliquely incident short wave groups.

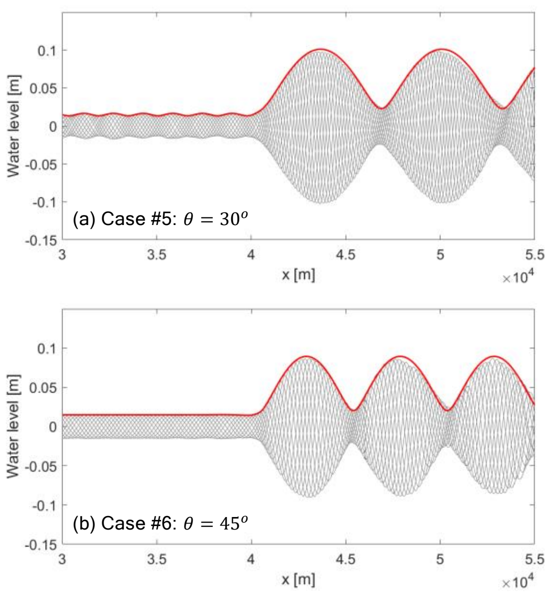

Figure 6 – Comparisons between the simulated water levels and the theory for obliquely incident wave groups over a plane beach. Red: Envelope height of the wave surface based on the theory of Liu (1989). Black: Simulated wave surface elevations.

In addition, long wave scattering due to obliquely incident short wave groups are simulated with incident angles of 30º and 45º, respectively. The beach profile is the same with Case #1. The simulated long wave surfaces after the scattering being created are plotted in Figure 5. It shows that the patterns of scattered waves are successfully captured in the simulations, though the results in Figure 5(b) is slightly polluted by reflected waves, which can be improved by adopting a larger simulation domain. The sectional profiles along x-axis are extracted and displayed in Figure 6. Good agreement between the numerical results and theory is observed, implying that the new formulas and the coupling approach work well for modelling the scattering of IG waves in variable depth.

CONCLUSIONS

This study proposed a new formula and coupling approach to consider the infra-gravity waves in coastal flooding models. The analysis shows that the infra-gravity waves and their spectra can be well described by using the new formulas, and the characteristics of the infra-gravity waves depend on the incident short wave conditions and the water depth. Moreover, the scattering of free long waves over plane beaches are also simulated by using the SCHISM-WWMIII model with modifications based on the new formulas for the improved radiation stresses term. The agreement between the numerical results and theoretical predictions indicates that the newly suggested formulas and coupling approach can be used to describe the scattering of free long waves in variable water depth. Future investigations will be focused on the infra-gravity waves generations and scattering in realistic coastal waters.

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