NON-LINEAR WAVE PROPAGATION OVER A SUBMERGED BAR

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Extensive laboratory experiments have been carried out to study the transformation of both monochromatic and random waves during the propagation over a smooth and impermeable bar, with the aim to study the mechanisms of interaction between waves and coastal structures, using spectral and bispectral analyses. It has been observed that the increase of the wave length leads to a greater energy transfer towards high frequencies and, in some cases, the secondary harmonic components become prevalent. The wave breaking acts by mainly reducing the energy of the primary components and it involves a redistribution of energy over a wider high-frequency range, with less pronounced peaks at the secondary harmonics. The observed spatial variations of the non-linearity parameters (bicoherence, skewness and asymmetry) indicate strong phase couplings between the primary component and its harmonics over the bar due to the non-linear triad interactions. Some of the main results of the laboratory experiments are presented.

Keywords: triad interactions, wave-structure interaction, submerged structures, spectral and bispectral analysis

INTRODUCTION AND THEORY

Submerged structures significantly modify the incident wave spectrum and the main wave parameters due to wave-structure interaction.

The growing demand to both utilize and preserve our coastal zones causes the need to better understand the coastal dynamics and to improve the capability to model such processes. According to the last report on climate change (IPCC, 2023), the increase of sea levels and the increase in frequency of extreme sea storms will have negative effects also on beaches protected by coastal structures, such as low-crested breakwaters (Marini et al., 2020, 2022). The efficiency of existing structures in dissipating incident wave energy will be reduced and it will be necessary to adapt structures to the new environmental conditions. This work aims to better understand the hydrodynamics induced by submerged structures, particularly the non-linear interaction processes which are responsible for the characteristics of the transmitted wave spectrum.

As surface gravity waves propagate from deep-water toward the shore, non-linear processes become relevant in the nearshore zone. The non-linear processes act by altering the wave profile shape so that symmetric wave profiles with oscillatory velocities in deep waters become asymmetric and skewed when they are close to wave-breaking. The transformation of the wave profile and of the wave spectrum is mainly due to the energy transfer between harmonic components of the wave field in a process called *non-linear triad interaction* (Phillips, 1960; Freilich and Guza, 1984; Elgar and Guza, 1985). Two harmonic components with frequencies f_1 e f_2 (for example close to the peak frequency $f_p \approx f_1 \approx f_2$) transfer energy both to the sum frequency ($f_1 + f_2 = f_3$) and to the lower frequency due to difference interaction ($f_1 - f_2 = f_3$).

As waves travel from deep to shallow water, the mechanism of the non-linear wave interactions is strongly influenced by the dispersion characteristics of the wave field. In deep water ($kh \gg O(1)$, where k is the wave number and h is the water depth), strong frequency dispersion allows resonant interactions among quartets of waves (Hasselmann, 1962), while triad interactions are negligible (Phillips, 1960). In intermediate depths ($kh = O(1)$), waves are weakly dispersive and they undergo substantial changes caused by non-resonant triad interactions. In shallow water depths ($kh \ll 1$), triad interactions are nearly resonant because waves are nondispersive and the mismatch between the bound wavenumber given by $|\boldsymbol{k_1} \pm \boldsymbol{k_2}|$ (where $\boldsymbol{k}$ are the vector wavenumbers) and the free wavenumber obtained from the linear dispersion relation $|\boldsymbol{k}(f_3)|$ is equal to zero. Zero wavenumber mismatch represents the limiting case in the interaction process, in which the interacting waves remain intact and in phase (resonant interaction) during evolution. Thus the magnitude of energy transfer is maximum and a continued one-way transfer takes place to the harmonics over relatively short evolution distance.

The phenomenon of non-linear wave interactions is enhanced by the presence of submerged obstacles, which strongly modify the hydrodynamics of waves propagating towards the shore. The generation of higher and lower frequency waves, due to the non-linearity in a wave field propagating over a shallow region, was first addressed from a theoretical point of view by Massel (1983) and Goda et al. (1999), who studied the propagation of Stokes waves over a submerged rectangular and impermeable step with second and third order theories, respectively. They addressed the topic as an extension of the propagation problem in which the wave field presents strong discontinuities, represented by a sudden variation in the bottom topography due to the presence of a bar. By following a perturbative

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approach to solve the system of equations of motion, they proposed solutions of fundamental importance to understand the complex mechanism of wave-structure interaction from a qualitative point of view. In the presence of the submerged obstacle, the wave motion is partly reflected and partly transmitted behind the obstacle. The presence of the discontinuity in the wave field due to the submerged barrier causes the decomposition of the incident primary wave (with frequency f) into different harmonics of multiple frequencies with respect to the main one. A part of these super-harmonics is linked to the fundamental (*bound waves*) while the other part travels as free oscillations with frequency $2f$ and celerity independent from the previous ones (*free waves*).

The problem of frequency generation over a submerged bar was also studied experimentally by many researchers. Liberatore and Petti (1992) performed random wave tests in a flume to investigate wave transformation above a submerged bar built on a horizontal bottom. Experimental results were interpreted using a second-order analytical model, able to separate first- and second-order components from measured variance spectra. Beji and Battjes (1993, 1994) studied the wave propagation process over a submerged bar with laboratory experiments and numerical simulations. They observed the amplification of the bound harmonics during the shoaling process and their release in the deeper region after the bar crest. For Beji and Battjes (1993) wave breaking plays a secondary role in the process of high frequency energy generation, contributing by dissipating the overall wave energy without changing the relative spectral distribution significantly. Masselink (1998) investigated the generation of secondary waves on a barred beach in field measurements and observed a decomposition of breaking incident swell into several smaller and shorter waves in the deeper region after the bar. Sénéchal et al. (2002) concluded that although wave breaking appears to weaken the non-linear wave-wave couplings, the generation of high frequency energy is hardly affected by wave breaking on a sandy, barred beach. The effect due to the permeability of a submerged structure on the process of generation of super-harmonics was investigated in laboratory by Losada et al. (1997). They found that, under non-breaking wave conditions, the permeability of the structure reduces the amplitude of wave harmonics modes considerably through the dissipation within porous media and by increasing the effective relative water depth above the structure. In agreement with the results of Losada et al. (1997), Corvaro et al. (2014) found that a lower transfer of energy to higher harmonics occurs when waves propagate over a permeable bed and, hence, a porous medium reduces the capacity to generate harmonics. Garcia et al. (2004) and Lara et al. (2006) performed laboratory experiments and numerical simulations in order to study the propagation of regular and irregular waves over a wide crested structure. They observed that the submerged structure induces spectral energy decay due to wave breaking over the crest and a broadening of the incident spectrum related to energy transfer to high frequencies. The transformation of the energy spectra behind low-crested structures has been studied by several authors (van der Meer et al., 2000, 2005; Lamberti et al., 2007; Calabrese and Buccino, 2009; Carevic et al., 2013), as well as the 105 piling-up and the filtration (Cappietti et al., 2007; Zanuttigh et al., 2008). In van der Meer et al. (2000) and Lamberti et al. (2007) a simplified spectral shape behind low-crested structures is proposed, with a percentage of the transmitted energy shifted towards higher frequencies.

The non-linear interactions, which cause energy transfers among triads with frequencies f_1, f_2 and f_3 (where $f_1 \pm f_2 = f_3$), can be evaluated with higher order statistical tools, such as the bispectral analysis (Elgar and Guza, 1985; Eldeberky, 1996). Elgar and Guza (1985) used the bispectral analysis to examine the skewness and the asymmetry of shoaling non-breaking surface gravity waves. They observed that shoaling gradually leads to more intense triad interactions involving higher harmonics and an evolution of the phase relations consistent with the wave profile that is pitched shoreward relative to the vertical axis. They also observed that shoreward propagating low-frequency energy has a significant coupling to higher-frequency modes within the spectral peak, which suggests a difference interaction between primary frequencies. Elgar and Guza (1986) used the one-dimensional non-linear Boussinesq model of Freilich and Guza (1984) to model the bispectral evolution in shoaling non-breaking waves. They found that the bispectral evolution is insensitive to mild bottom slopes. They also concluded that the trends in the non-linear evolution of the bispectrum, skewness and asymmetry do not depend critically on the initial phase coupling. Elgar et al. (1997) performed field measurements to study the evolution of a gravity wave spectrum that propagates over a natural beach profile with sandy bars. They obtained that the skewness and the asymmetry increase from the offshore value of 0 (symmetric waves) to values of about 0.5 over the bar. In the horizontal part the skewness keeps constant while asymmetry goes to 0. Zou and Peng (2011) found that wave skewness is a primary wave non-linearity indicator and it varies across a low-crested structure. Their results show that wave skewness decreases slightly above the seaward slope, increases rapidly up to a maximum value above the structure crest, and then decreases above the leeward slope to a value close to the incident one. Moreover, they studied the influence of different controlling factors on the evolution of the wave shape. They found that the extent of the effects of structures on the evolution of wave skewness and asymmetry increases with structure crest width, while the maximum absolute values of skewness and asymmetry are hardly affected by the crest width. The effect of large porosity is also equivalent to the effect of small relative structure crest width.

This work investigates the wave evolution over a submerged bar by means of experimental tests performed in the wave-flume of the Laboratorio di Idraulica e Costruzioni Marittime of the Università Politecnica delle Marche (Ancona, Italy). Regular and random waves in both non-breaking and breaking conditions and two different bar submergences are studied. The submerged bar has a larger berm width and gentler slopes with respect to the typical geometries of submerged breakwaters in order to better analyse the energy exchange between wave harmonics over a longer distance. The aim is to increase the knowledge and the physical insight on the mechanism of non-linear triad interactions and to better understand the influence of different parameters on this phenomenon also through the bispectral analysis tool. A better understanding of the main parameters involved in this phenomenon could be useful to improve the method for the reconstruction of the transmitted energy spectra, taking into account the incident wave characteristics and the structure geometry. The knowledge of the transmitted spectrum characteristics is of great interest for design purposes, in order to estimate for instance the run-up and the morphological response of the beach protected by the considered structure.

EXPERIMENTAL MODEL

Laboratory setup

The experiments were carried out in the wave-flume of the Laboratorio di Idraulica e Costruzioni Marittime of the Università Politecnica delle Marche (Ancona, Italy). The flume (Fig. 1) has a length of 50m, width 1.0m and height 1.3m. The wave motion was forced by a pyston-type wave-maker that operates up to a maximum run of 0.5m (semi-stroke) and a maximum velocity of 0.8m/s. The bottom profile used in the experiments is shown in Fig. 2. A smooth submerged trapezoidal bar made of rigid aluminium sheets was constructed, consisting of upslope and downslope ramps of 1:10 and a 3m long horizontal crest. The height of the bar was 0.41m above the bottom of the flume. At the end of the flume a beach with a 1:20 slope mainly made of coarse gravel was used as a wave absorber. The wave reflection of the beach was estimated from previous experiments and it was lower than about 4-5%.

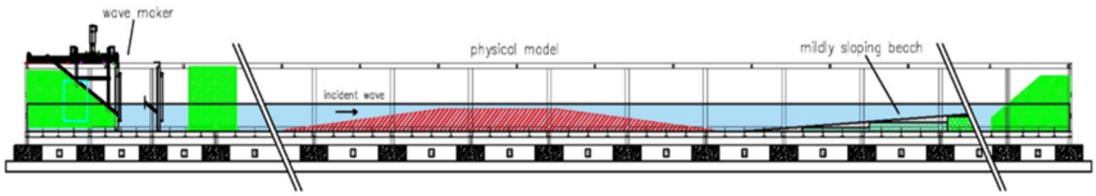

Figure 1. Global sketch of the wave-flume with the physical model realized in the Laboratorio di Idraulica e Costruzioni Marittime of Università Politecnica delle Marche (Ancona, Italy).

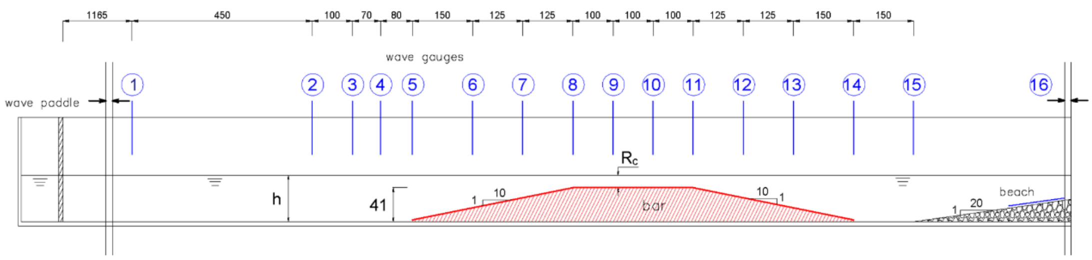

Figure 2. Experimental setup and location of wave gauges

The free surface elevation was measured at fifteen stations in the flume using eight wave gauges in two shifts, as sketched in Fig. 2. Stations 1 to 8 correspond to the first shift (8 gauges) while stations 9 to 15 to the second shift. The first measuring point (station 1) was common to the two shifts, enabling the repeatability of the two experiments to be checked. This gauge was also used as the reference gauge for the incident waves. Wave gauges 2 to 5 were set up in the horizontal part of the flume to allow measurements of wave reflection from the bar by applying the non-linear method proposed by Lin and Huang (2004). The other gauges were arranged symmetrically with respect to the bar at about 1m intervals to evaluate the transformations of waves passing the obstacle. The exact spacing between the wave gauges is reported in Fig. 2. The wave signal at station 15 was used as reference for the transmitted waves. In each run, data were recorded at a sampling frequency of 35Hz.

Experimental tests

Different wave conditions were tested. The transformation of waves passing over the submerged structure was evaluated for both regular and random waves. In order to evaluate also the effect of the sea level rise, two offshore water depths, h=0.51m and h=0.56m, were considered and the submergence of the bar was equal to 0.10cm and 0.15cm, respectively. For each water depth, 36 monochromatic waves were generated by combining 4 different wave periods (T=1.0s, T=1.5s, T=2.0s and T=2.5s) and 9 different wave heights from a minimum value of 2cm to a maximum value of 15cm. For regular waves, tests lasted 300s.

Irregular waves were generated as JONSWAP spectra with a peak enhancement factor equal to 3.3. For each peak period (T_p=1.0s, T_p=1.5s, T_p=2.0s and T_p=2.5s) 3 significant wave heights were selected in order to have different wave conditions: non-breaking waves, spilling breakers and plunging breakers. A total of 12 random waves was studied, for each water depth, h=0.51m and h=0.56m. For larger periods, the selected wave heights were smaller because it was attempted to keep the non-linearity parameter, ϵ=a/h (where a is the wave amplitude), nearly the same in the shallowest part of the flume for the four wave periods. For random waves, the duration of the experimental tests depends on the peak period to ensure the generation of at least 500 waves during each test. Therefore, the runs lasted 600s for T_p=1.0s, 900s for T_p=1.5s, 1200s for T_p=2.0s and 1500s for T_p=2.5s.

DATA ANALYSIS

The free surface elevation can be represented as a superposition of statistically independent waves in which the phases are random assuming that the sea state follows a Gaussian distribution. Consequently, the sea surface can be fully described by the continuous density energy spectrum. The spectral analysis of the collected data was carried out with the use of a standard Fast Fourier Transform (FFT) package. The time series of the wave gauges have been processed by removing the first part of each signal, having a duration equal to the sum of the time needed by the first wave to reach the probe and of the ramping-up time (equal to 10s for all the tests). The average signal over the entire recording period was removed from each time series. The power spectra of the signals were computed by applying the Welch's method. For monochromatic waves, each record was divided into 8 parts of 2048 data points with 50% overlapping. For random waves, data were recorded for different durations depending on the peak period. The number of segments into which the original signal is divided changed depending on the test duration with a minimum of 24 segments. The percentage of window overlapping was set equal to 50%. The window size depended on the entire length of the signal, hence the spectral estimates are obtained with different frequency resolutions.

The presence of the structure induces non-linear interactions between waves and their phases are not statistically independent. The deviation from a Gaussian distribution requires the application of higher order spectra, such as the bispectrum, in order to study the generation of harmonic components due to the non-linear interactions. The bispectrum was introduced by Hasselmann et al. (1963) to examine wave non-linearity in intermediate water depths. Bispectral analysis is shown to be a very useful diagnostic tool in experimental studies of non-linear wave interactions and can be used to investigate non-linearity even in shallow water, where wave's non-linearities can become very strong (Elgar and Guza, 1985; Eldeberky, 1996).

Bispectral analysis

The bispectrum $B(f_l, f_m)$ is a spectral representation of third-order statistics that can be used to analyse the non-linear interactions between a triad of frequencies f_l, f_m and fn that satisfies $f_l + f_m = f_n$. The digital (discrete) bispectrum, appropriate for discretely sampled data, is (Kim and Powers, 1979):

$$B(f_l, f_m) = < C(f_l)C(f_m)C^*(f_n) > \quad (1)$$

where < . > denotes the ensemble average; $C(f_l)$ and $C(f_m)$ are the complex Fourier coefficients of two frequencies f_l and f_m and $C^*(f_n)$ the complex conjugate of their sum.

If the three components are statistically independent, there is no phase correlation and $B(f_l, f_m) = 0$. In that case, the third component f_n is not bound to f_l and f_m but freely propagating. On the other hand, a non-zero bispectrum $B(f_l, f_m)$ indicates that (part of) the variance at f_n is bound to the energies at f_l and f_m.

The bispectrum can be efficiently computed using symmetry properties, in which it can be uniquely described by its values in a bi-frequency octant. For a digital time series with Nyquist frequency f_N, the bispectrum is uniquely defined within a triangle in (f_l, f_m)-space (bi-frequency plan) with vertices at ($f_l = 0, f_m = 0$), ($f_l = f_N, f_m = 0$), and ($f_l = f_{N/2}, f_m = f_{N/2}$).

The bispectrum can be used to identify coupled modes; however, it does not give a measure of the intensity of non-linear interactions because its value depends on the amplitudes of the three wave components involved in the interaction. Therefore, it is convenient to cast the bispectrum into its normalized magnitude and phase, the so-called bicoherence and biphase, given respectively by (Kim and Powers, 1979):

$$b^2(f_l, f_m) = \frac{|B(f_l,f_m)|^2}{\langle |C(f_l)C(f_m)|^2\rangle \langle |C(f_n)|^2\rangle} \quad (2)$$

$$\beta(f_l, f_m) = arctan\left(\frac{Im\{B(f_l,f_m)\}}{Re\{B(f_l,f_m)\}}\right) \quad (3)$$

In a random wave field with statistically independent components, the phases are randomly distributed between $-\pi$ and π, and thus the biphase-values tend to be scattered between $-\pi$ and π. The bicoherence is independent of the wave amplitude, unlike the bispectrum. For the bicoherence normalization given in (2), the bicoherence value is bounded by zero and unity ($0 \leq b^2 \leq 1$) with $b^2 = 0$ for statistically uncorrelated waves and $b^2 = 1$ for fully coupled waves. For a three-wave system, Kim and Powers (1979) showed that $b^2(f_l, f_m)$ represents the fraction of the total energy at the sum-frequency (f_n) due to the non-linear interaction.

For a finite-length time series even a truly Gaussian process will have a non-zero bispectrum. A 95% significance level on zero bicoherence is given by Haubrich (1965) as:

$$b^2_{95\%} \geq 6/d.o.f. \quad (4)$$

where d.o.f. is the number of degrees of freedom in the bispectral estimates.

Skewness and asymmetry

The non-linear shape of a wave can be described by its skewness Sk (asymmetry with respect to the horizontal axis) and asymmetry As (asymmetry with respect to the vertical axis), as shown in Fig. 3.

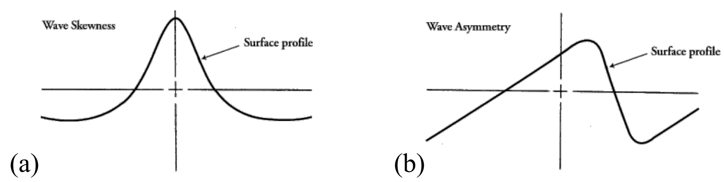

Figure 3. Sketch of the non-linear shape parameters skewness (a) and asymmetry (b) for a wave.

Skewness and asymmetry are third-order statistics (Elgar and Guza, 1985), which are proportional to the real and imaginary parts of the bispectrum, respectively, normalized by the variance $E(f_l)$. Sk and As are respectively computed as (de Wit et al., 2020):

$$Sk = \frac{6\sum_{l=l_{min}}^{N/2} Re\{B(f_l,f_l)\} + 12\sum_{m=l_{min}}^{N/2}\sum_{l=m+1}^{N-m} Re\{B(f_l,f_m)\}}{\left(\sum_{l=l_{min}}^{N} 2E(f_l)\right)^{3/2}} \tag{5}$$

$$As = \frac{6\sum_{l=l_{min}}^{N/2} Im\{B(f_l,f_l)\} + 12\sum_{m=l_{min}}^{N/2}\sum_{l=m+1}^{N-m} Im\{B(f_l,f_m)\}}{\left(\sum_{l=l_{min}}^{N} 2E(f_l)\right)^{3/2}} \tag{6}$$

The skewness and asymmetry represent overall measures of non-linearity and indicate the deviation of the wave profile statistics from the Gaussian distribution. Positive values of skewness and negligible asymmetry correspond to Stokes type wave forms. Negative values of asymmetry indicate waves which are pitched forward, while positive values correspond to pitched backward waves.

These parameters are used in the analysis of the experimental data presented in the following sections.

EXPERIMENTAL RESULTS AND CONCLUSIONS

Some of the main results about the transformation of waves passing over the submerged bar are presented in the present work. The measurements recorded by the wave gauges have been processed and analysed both in the time and frequency domains.

At first, the monochromatic waves are studied because they are easier to analyse and they provide quite valuable information about the generation of super-harmonics. Monochromatic waves are useful to simulate narrow banded spectra, which are typical of sea swell conditions. Their study is also the starting point to better understand the more complex mechanisms related to the propagation of random waves. Random waves are considered, where the interactions between frequency components within the spectra cause also the generation of sub-harmonics. Bispectral analyses have been also applied to the free surface records to better understand the exchange of energy between frequency components.

The spatial evolution of regular and random wave data has been analysed both in the time and in the frequency domain with spectral and bispectral analyses. The effect of different parameters on the wave propagation has been evaluated, such as the bar submergence and the wave characteristics. The non-linear interaction process over the bar is mostly influenced by the relative water depth over the bar crest (h/L) because, in shallow water conditions (h/L<0.05), waves are non-dispersive and triad interactions are nearly resonant.

Some collected and elaborated data are showed. In particular, for regular waves, the spatial evolution of the amplitudes of harmonic components of multiple frequencies is showed; for random waves, the resulted normalized spectra and bispectra and the effect of some non-linear parameters on the wave propagation are showed.

Spatial evolution for regular waves

In Fig. 4 the spatial evolution of the amplitudes of harmonic components of multiple frequencies is showed, for different periods and breaking and non-breaking conditions.

The regular waves in non-breaking conditions with a lower period, T=1s, show few changes in the wave shape and in the spatial evolution of the amplitudes of harmonic components of multiple frequencies.

As the wave period increases, an increasing amount of energy is transferred to higher frequencies. For T=2.5s the energy carried by the first harmonic decreases on the horizontal crest and the third harmonic becomes prevalent behind the bar.

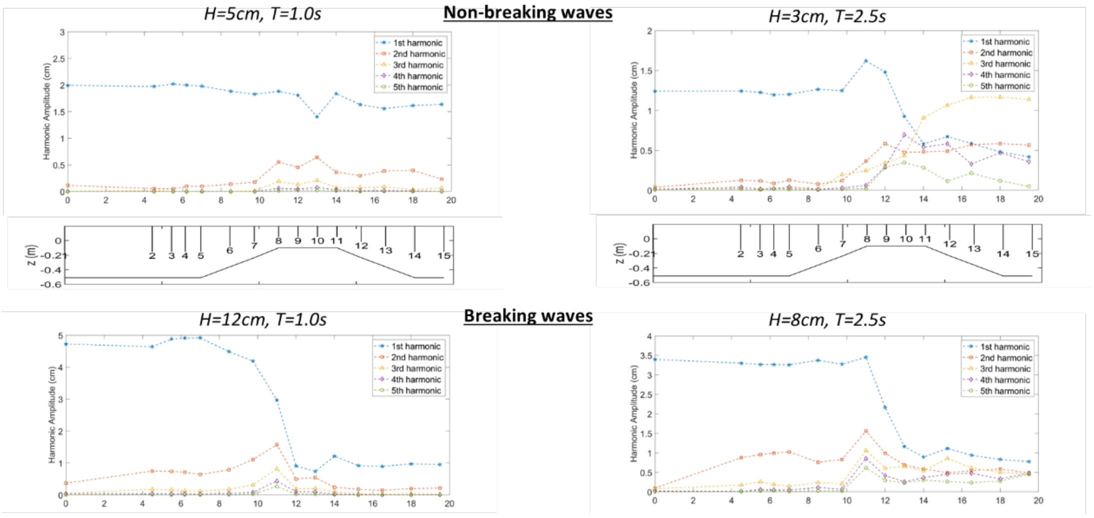

Figure 4. Spatial evolution of the amplitude of the first five harmonic components of multiple frequencies for different periods and breaking conditions.

For breaking waves, the energy dissipation reduces proportionally the energy carried by each harmonic. This implies that the amplitude reduction is stronger for the first and the second harmonics and less energy is transferred to higher harmonics as waves propagate over the bar.

The increase in water depth has a different effect on the harmonic generation of non-breaking and breaking waves. For non-breaking waves, higher submergence reduces the number of harmonics involved in the interactions and less energy is shifted toward higher frequencies. For breaking waves, as the submergence of the bar increases, the intensity of wave breaking decreases and more energy is available for non-linear couplings between frequencies.

Spatial evolution of spectra and bispectra for random waves

In order to evaluate the effect of wave breaking, spectral and bispectral estimates obtained for two random waves with the same peak period ($T_p=2.5s$) but different significant wave height H_s are normalized and plotted together and showed in Fig. 5.

The shape of the normalized spectra is very similar for the non-breaking and the plunging waves during propagation over the upslope and at the beginning of the horizontal crest (stations 7 and 9), even if the wave breaking reduces the spectral peak of the primary component (station 9) and it involves a greater amount of energy transferred to frequencies lower than $0.5f_p$. At stations 11 and 14 the spectrum of the plunging wave has lower peaks at higher frequencies and the energy is distributed over a wider range of frequencies.

The comparison between the corresponding values of the bicoherence b^2, i.e. the normalized bispectrum, shows that for the breaking wave triad interactions at stations 11 and 14 are less concentrated around the frequencies multiple of the primary component (which is more affected by the energy dissipation due to wave breaking) and they are distributed over wider frequency ranges. Therefore, wave breaking reduces the intensity of the non-linear interactions and it involves a greater number of frequency components in the interactions.

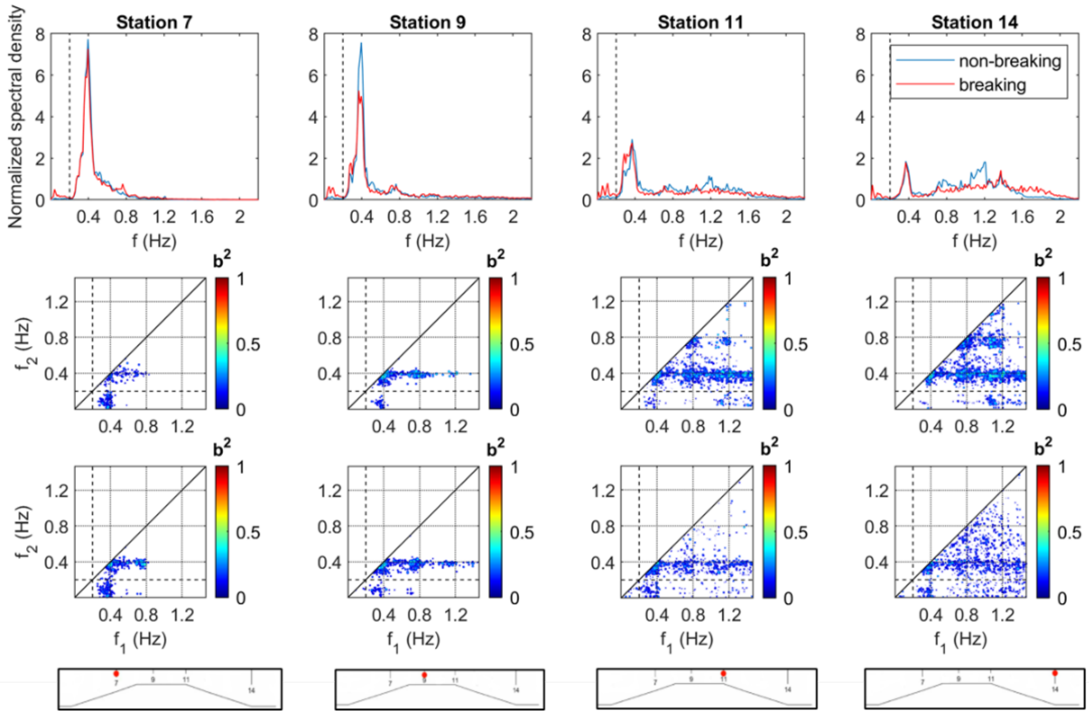

Figure 5. Comparison of normalized energy spectra (upper line) for the non-breaking wave with Hs=3cm (green line) and the plunging wave with Hs=8cm (red line) at stations 7, 9, 11 and 14; h_o=0.51m, T_p=2.5s. Absolute bispectrum (cm^3/Hz^2) at stations 7, 9, 11 and 14 for the non-breaking wave (center line) and for the plunging wave (lower line)

Non-linearity parameters

The values of the bicoherence are analysed as a function of the relative depth over the bar crest $k'_p h'$. As shown in Fig. 6a, the bicoherence decreases as the influence of finite depth decreases.

The bicoherence computed for all the frequency pairs of the bispectrum $\bar{b}^2_{95}$ is less dependent on the wave height as $k'_p h'$ increases, as shown in Fig. 6b.

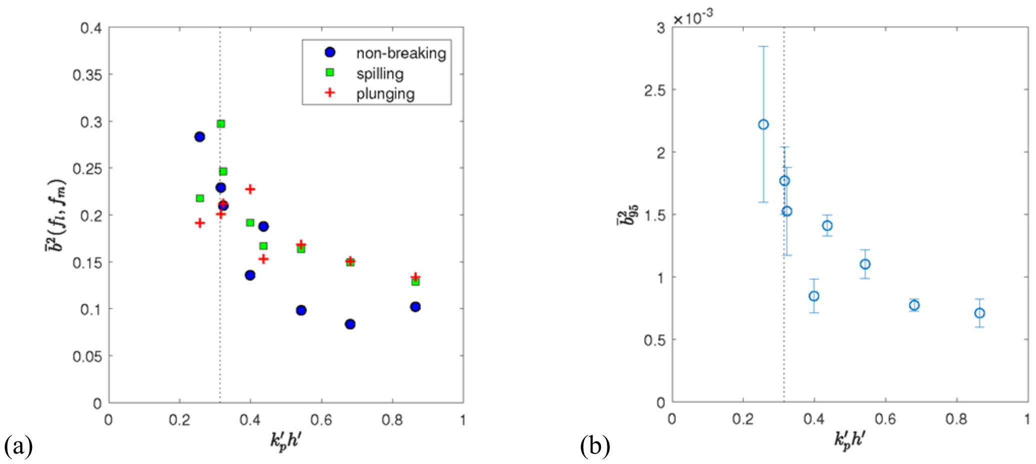

Figure 6. Normalized bicoherence (a) and bicoherence computed for all the frequency pairs of the bispectrum (b) plotted in dependence of the relative depth over the bar crest.

The skewness and the asymmetry over the bar crest also show a dependence on $k'_p h'$, as shown in Fig. 7. Larger absolute values are associated with higher waves.

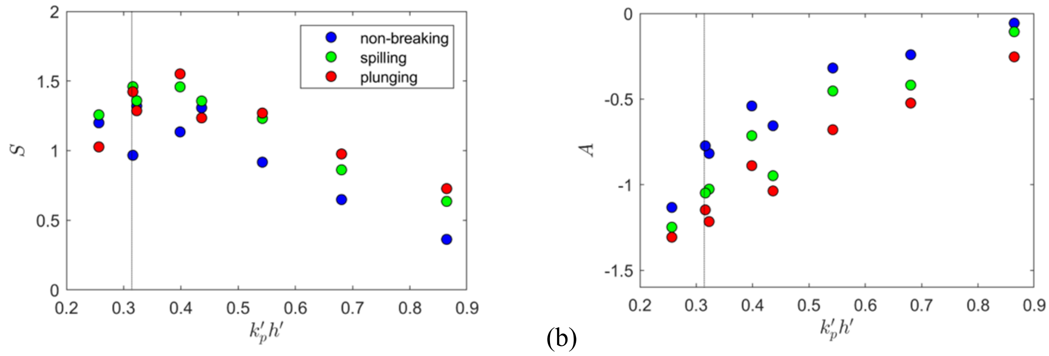

Figure 7. Skewness (a) and asymmetry (b) plotted in dependence of the relative depth over the bar crest.

REFERENCES

Beji, S., Battjes, J.A., 1993. Experimental investigation of wave propagation over a bar. *Coastal Engineering*, 19, 151–162.

Beji, S., Battjes, J.A., 1994. Numerical simulation of nonlinear wave propagation over a bar. *Coastal Engineering*, 23, 1–16.

Calabrese, M., Buccino, M., 2009. Estimating power spectral density behind low crested breakwaters, in: *Coastal Structures 2007*,World Scientific Publishing Company. doi:10.1142/9789814282024-0071.

Cappietti, L., Clementi, E., Aminti, P., Lamberti, A., 2007. Piling-up and filtration at low crested breakwaters of different permeability, in: *Coastal Engineering 2006*, World Scientific Publishing Company. doi:10.1142/9789812709554_0415.

Carevic, D., Loncar, G., Prsic, M., 2013. Wave parameters after smooth submerged breakwater. *Coastal Engineering*, 79, 32–41. doi:10.1016/j.coastaleng.2013.04.004.

Corvaro, S., Seta, E., Mancinelli, A., Brocchini, M., 2014. Flow dynamics on a porous medium. *Coastal Engineering*, 91, 280–298. doi:10.1016/j.coastaleng.2014.06.001.

Eldeberky, Y., 1996. *Nonlinear Transformation of Wave Spectra in the Nearshore Zone*. Ph.D. thesis. Delft University of Technology.

Elgar, S., Guza, R.T., 1985. Observations of bispectra of shoaling surface gravity waves. *J. Fluid Mech.*, 161, 425–448.

Elgar, S., Guza, R.T., 1986. Nonlinear model predictions of bispectra of shoaling surface gravity waves. *Journal of Fluid Mechanics*, 167, 1. doi:10.1017/s0022112086002690.

Elgar, S., Guza, R.T., Raubenheimer, B., Herbers, T.H.C., Gallagher, E.L., 1997. Spectral evolution of shoaling and breaking waves on a barred beach. *Journal of Geophysical Research: Oceans*, 102, 15797–15805. doi:10.1029/97jc01010.

Freilich, M.H., Guza, R.T., 1984. Nonlinear effects on shoaling surface gravity waves. *Philos. Trans. R. Soc. London*, Ser. A 311, 1–41.

Garcia, N., Lara, J., Losada, I., 2004. 2-d numerical analysis of near-field flow at low-crested permeable breakwaters. *Coastal Engineering*, 51, 991–1020. doi:10.1016/j.coastaleng.2004.07.017.

Goda, Y., Okazaki, K., Kagawa, M., 1999. Generation and evolution of harmonic wave components by abrupt depth changes, in: *Proceedings of Coastal Structures '99*, Balkema. pp. 649–658.

Hasselmann, K., 1962. On the non-linear energy transfer in a gravity wave spectrum, part i. general theory. *J. Fluid Mech.*, 12, 481–500.

Hasselmann, K., Munk, W., McDonald, G., 1963. Bispectra of ocean waves, in: *Proceedings of the Symposium on Time Series Analysis*, Wiley, New York. pp. 125–139.

Haubrich, R.A., 1965. Earth noise, 5 to 500 millicycles per second: 1. spectral stationarity, normality, and nonlinearity. *Journal of Geophysical Research*, 70, 1415–1427. doi:10.1029/jz070i006p01415.

IPCC, 2023. *Climate Change 2023: Synthesis Report. Contribution of Working Groups I, II and III to the Sixth Assessment Reportof the Intergovernmental Panel on Climate Change*. [Core Writing Team, H. Lee and J. Romero (eds.)], Geneva, Switzerland. pp. 35–115. doi:10.59327/IPCC/AR6-9789291691647.

Kim, Y.C., Powers, E.J., 1979. Digital bispectral analysis and its application to the non-linear wave interactions. *IEEE Trans. on Plasma Sc.*, 1, 120–131.

Lamberti, A., Zanuttigh, B., Martinelli, L., 2007. Wave overtopping and transmission: An interpretation of spectral change at low crested rubble mound structures, in: *Coastal Engineering 2006*, World Scientific Publishing Company. doi:10.1142/9789812709554_0388.

Lara, J., Garcia, N., Losada, I., 2006. Rans modelling applied to random wave interaction with submerged permeable structures. *Coastal Engineering*, 53, 395–417. doi:10.1016/j.coastaleng.2005.11.003.

Liberatore, G., Petti, M., 1992. Wave transformations over a submerged bar: Experiments and theoretical interpretations. *Coastal Engineering*, 18, 447–459.

Lin, C.Y., Huang, C.J., 2004. Decomposition of incident and reflected higher harmonic waves using four wave gauges. *Coastal Engineering*, 51, 395–406. doi:10.1016/j.coastaleng.2004.04.004.

Losada, I.J., Patterson, M.D., Losada, M.A., 1997. Harmonic generation past a submerged porous step. *Coastal Engineering*, 31, 281–304.

Marini, F., Corvaro, S., Rocchi, S., Lorenzoni, C., Mancinelli, A., 2022. Semi-analytical model for the evaluation of shoreline recession due to waves and sea level rise. *Water*, 14, 1305. doi:10.3390/w14081305.

Marini, F., Mancinelli, A., Corvaro, S., Rocchi, S., Lorenzoni, C., 2020. Coastal submerged structures adaptation to sea level rise over different beach profiles. *Italian journal of engineering geology and environment* , 87–98doi:10.4408/IJEGE.2020-01.S-10.

Massel, S., 1983. Harmonic generation by waves propagating over a submerged step. *Coastal Engineering*, 7, 357–380.

Masselink, G., 1998. Field investigation of wave propagation over a bar and the consequent generation of secondary waves. *Coastal Engineering*, 33, 1–9. doi:10.1016/s0378-3839(97)00032-x.

van derMeer, J.W., Briganti, R., Zanuttigh, B.,Wang, B., 2005. Wave transmission and reflection at low-crested structures: Design formulae, oblique wave attack and spectral change. *Coastal Engineering*, 52, 915–929. doi:10.1016/j.coastaleng.2005.09.005.

van der Meer, J.W., Regeling, H.J., de Waal, J.P., 2000. Wave transmission: spectral changes and its effects on run-up and overtopping, in: ASCE, *Proc. 27th ICCE*, Sydney, Australia.

Phillips, O.M., 1960. On the dynamics of unsteady gravity waves of finite amplitude. part 1. *J. Fluid Mech*,. 9, 193–217.

Sénéchal, N., Bonneton, P., Dupuis, H., 2002. Field experiment on secondary wave generation on a barred beach and the consequent evolution of energy dissipation on the beach face. *Coastal Engineering*, 46, 233–247. doi:10.1016/s0378-3839(02)00095-9.

de Wit, F., Tissier, M., Reniers, A., 2020. The relationship between sea-swell bound wave height and wave shape. *Journal of Marine Science and Engineering*, 8, 643. doi:10.3390/jmse8090643.

Zanuttigh, B., Martinelli, L., Lamberti, A., 2008. Wave overtopping and piling-up at permeable low crested structures. *Coastal Engineering*, 55, 484–498. doi:10.1016/j.coastaleng.2008.01.004.

Zou, Q., Peng, Z., 2011. Evolution of wave shape over a low-crested structure. *Coastal Engineering*, 58, 478–488. doi:10.1016/j.coastaleng.2011.01.001.